ANSWER KEY

Algebra II
Course Workbook
with Regents Questions

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Regents Exam Notation

A code next to each Regents Question answer number indicates from which Common Core (CC) or Next Generation (NG) Regents exam or sampler the question came. For example, CC AUG '18 [25] means the question appeared on the August 2018 exam as question 25.

Practice Problems

Chapter 1. Linear Functions

1.1 Linear Systems in Three Variables [CC]

1. The first variable, x, is already isolated, so find the others by substitution: $5(3) + 4y = -9$ Eq. 2 $15 + 4y = -9$ $4y = -24$ $y = -6$ $-(3) + 4(-6) - 2z = -25$ Eq. 3 $-27 - 2z = -25$ $-2z = 2$ $z = -1$ $x = 3, y = -6, z = -1$	2. $x + 3y + z = 10$ Eq. 1 $\underline{-x - y - z = -2}$ Eq. 2 × −1 $2y = 8$ $y = 4$ We have already isolated y, so substitute: $(4) - 2z = 2$ Eq. 3 $-2z = -2$ $z = 1$ $x + (4) + (1) = 2$ Eq. 2 $x = -3$ $x = -3, y = 4, z = 1$
3. $2x - 4y + 5z = -33$ Eq. 1 $\underline{-2x + 2y - 3z = 19}$ Eq. 3 $-2y + 2z = -14$ $-y + z = -7$ Result A $4x - y = -5$ Eq. 2 $\underline{-4x + 4y - 6z = 38}$ Eq. 3 × 2 $3y - 6z = 33$ Result B $-3y + 3z = -21$ Result A × 3 $\underline{3y - 6z = 33}$ Result B $-3z = 12$ $z = -4$ $-y + (-4) = -7$ Result A $y = 3$ $4x - (3) = -5$ Eq. 2 $x = -\frac{1}{2}$ $x = -\frac{1}{2}, y = 3, z = -4$	4. $x - 2y + 3z = 7$ Eq. 1 $\underline{4x + 2y + 2z = 8}$ Eq. 2 × 2 $5x \quad + 5z = 15$ $x + z = 3$ Result A $x - 2y + 3z = 7$ Eq. 1 $\underline{-3x + 2y - 2z = -10}$ Eq. 3 $-2x \quad + z = -3$ Result B $2x + 2z = 6$ Result A × 2 $\underline{-2x + z = -3}$ Result B $3z = 3$ $z = 1$ $x + (1) = 3$ Result A $x = 2$ $2(2) + y + (1) = 4$ Eq. 2 $y = -1$ $x = 2, y = -1, z = 1$

5. $ax^2 + bx + c = y$

$a(-1)^2 + b(-1) + c = 9$
$a(2)^2 + b(2) + c = 3$
$a(5)^2 + b(5) + c = 15$

$a - b + c = 9$	Eq. 1
$4a + 2b + c = 3$	Eq. 2
$25a + 5b + c = 15$	Eq. 3

$a - b + c = 9$	Eq. 1
$\underline{-4a - 2b - c = -3}$	Eq. 2 × –1
$-3a - 3b = 6$	
$a + b = -2$	Result A

$a - b + c = 9$	Eq. 1
$\underline{-25a - 5b - c = -15}$	Eq. 3 × –1
$-24a - 6b = -6$	
$4a + b = 1$	Result B

$a + b = -2$	Result A
$\underline{-4a - b = -1}$	Result B × –1
$-3a = -3$	
$a = 1$	

$(1) + b = -2$	Result A
$b = -3$	

$(1) - (-3) + c = 9$	Eq. 1
$c = 5$	

$y = x^2 - 3x + 5$

6. $ax^2 + bx + c = y$

$a(-1)^2 + b(-1) + c = -2$
$a(1)^2 + b(1) + c = 0$
$a(2)^2 + b(2) + c = 7$

$a - b + c = -2$	Eq. 1
$a + b + c = 0$	Eq. 2
$4a + 2b + c = 7$	Eq. 3

$a - b + c = -2$	Eq. 1
$\underline{-a - b - c = 0}$	Eq. 2 × –1
$-2b = -2$	
$b = 1$	

$a - (1) + c = -2$	Eq. 1
$a + c = -1$	Result A

$4a + 2(1) + c = 7$	Eq. 3
$4a + c = 5$	Result B

$a + c = -1$	Result A
$\underline{-4a - c = -5}$	Result B × –1
$-3a = -6$	
$a = 2$	

$(2) - (1) + c = -2$	Eq. 1
$c = -3$	

$y = 2x^2 + x - 3$

Chapter 2. Irrational Expressions

2.1 Operations with Square Roots [CC]

1. $5\sqrt{3}+\sqrt{3}=6\sqrt{3}$	2. $3\sqrt{3}+2\sqrt{3}=5\sqrt{3}$
3. $4\sqrt{3}-2\sqrt{3}=2\sqrt{3}$	4. $5\sqrt{6}+2\sqrt{6}=7\sqrt{6}$
5. $3\sqrt{2}+2\sqrt{2}=5\sqrt{2}$	6. $5\sqrt{2}-4\sqrt{2}=\sqrt{2}$
7. $6\sqrt{2}-3\sqrt{2}=3\sqrt{2}$	8. $10\sqrt{2}-\sqrt{2}=9\sqrt{2}$
9. $5\sqrt{7}+6\sqrt{7}=11\sqrt{7}$	10. $30\sqrt{2}+6\sqrt{2}=36\sqrt{2}$
11. $5-2\sqrt{3}+3\sqrt{3}+6=11+\sqrt{3}$	12. $y\sqrt{3}-4\sqrt{2}-3y\sqrt{3}=-2y\sqrt{3}-4\sqrt{2}$
13. $\sqrt{90}=3\sqrt{10}$	14. $\sqrt{3600}-\sqrt{144}=60-12=48$
15. $6\sqrt{100}-21\sqrt{20}=60-42\sqrt{5}$	16. $3\sqrt{98}+12\sqrt{392}=21\sqrt{2}+168\sqrt{2}=189\sqrt{2}$
17. $9-5=4$	18. $5\sqrt{2}-20+2-4\sqrt{2}=-18+\sqrt{2}$
19. $10\sqrt{18x^7}=10\sqrt{9x^6}\sqrt{2x}=30x^3\sqrt{2x}$	20. $15\sqrt{2x^6y^2}=15x^3y\sqrt{2}$
21. $9x-18\sqrt{x}+9$	22. $c^2=\left(x+\sqrt{2}\right)^2+\left(x-\sqrt{2}\right)^2$ $c^2=x^2+2x\sqrt{2}+2+x^2-2x\sqrt{2}+2$ $c^2=2x^2+4$ $c=\sqrt{2x^2+4}$
23. $\sqrt{13}$	24. $\frac{\sqrt{28}}{2}=\frac{2\sqrt{7}}{2}=\sqrt{7}$
25. $5\sqrt{50}=25\sqrt{2}$	26. $\frac{15\sqrt{3}+3\sqrt{3}}{3}=\frac{18\sqrt{3}}{3}=6\sqrt{3}$
27. $\frac{4\sqrt{3}-15\sqrt{3}+10\sqrt{3}}{\sqrt{3}}=\frac{-\sqrt{3}}{\sqrt{3}}=-1$	28. $\frac{3\sqrt{3}+5\sqrt{3}}{2\sqrt{3}}=\frac{8\sqrt{3}}{2\sqrt{3}}=4$
29. $\frac{16\sqrt{21}}{2\sqrt{7}}-5\sqrt{12}=8\sqrt{3}-10\sqrt{3}=-2\sqrt{3}$	30. $\sqrt{18x^4y^3}=3x^2y\sqrt{2y}$

2.2 Rationalize Monomial Denominators [CC]

1. $\frac{3}{\sqrt{7}}\left(\frac{\sqrt{7}}{\sqrt{7}}\right)=\frac{3\sqrt{7}}{7}$	2. $\frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{2\sqrt{2}}{2}=\sqrt{2}$
3. $\frac{3\sqrt{5}}{2\sqrt{10}}\left(\frac{\sqrt{10}}{\sqrt{10}}\right)=\frac{3\sqrt{50}}{2\cdot 10}=\frac{15\sqrt{2}}{20}=\frac{3\sqrt{2}}{4}$	4. $\frac{3-\sqrt{8}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{3\sqrt{3}-\sqrt{24}}{3}=\frac{3\sqrt{3}-2\sqrt{6}}{3}$

5. $\frac{2}{\sqrt{3}} \times \frac{\sqrt{2}}{5} = \frac{2\sqrt{2}}{5\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2\sqrt{6}}{15}$	6. $\frac{\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5}}}{\sqrt{5}} = \frac{\frac{2}{\sqrt{5}}}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{2}{5}$

2.3 Rationalize Binomial Denominators

1. $\frac{\sqrt{5}}{7-\sqrt{5}}\left(\frac{7+\sqrt{5}}{7+\sqrt{5}}\right) = \frac{7\sqrt{5}+5}{49-5} = \frac{7\sqrt{5}+5}{44}$	2. $\frac{1}{3-\sqrt{7}}\left(\frac{3+\sqrt{7}}{3+\sqrt{7}}\right) = \frac{3+\sqrt{7}}{9-7} = \frac{3+\sqrt{7}}{2}$
3. $\frac{5}{4-\sqrt{11}}\left(\frac{4+\sqrt{11}}{4+\sqrt{11}}\right) = \frac{20+5\sqrt{11}}{16-11} =$ $\frac{20+5\sqrt{11}}{5} = 4+\sqrt{11}$	4. $\frac{4}{5-\sqrt{13}}\left(\frac{5+\sqrt{13}}{5+\sqrt{13}}\right) = \frac{20+4\sqrt{13}}{25-13} =$ $\frac{20+4\sqrt{13}}{12} = \frac{5+\sqrt{13}}{3}$
5. $\frac{\sqrt{2}}{\sqrt{14}+4}\left(\frac{\sqrt{14}-4}{\sqrt{14}-4}\right) = \frac{\sqrt{28}-4\sqrt{2}}{14-16} =$ $\frac{2\sqrt{7}-4\sqrt{2}}{-2} = -\sqrt{7}+2\sqrt{2}$	6. $\frac{\sqrt{3}}{\sqrt{3}+5}\left(\frac{\sqrt{3}-5}{\sqrt{3}-5}\right) = \frac{3-5\sqrt{3}}{3-25} = \frac{3-5\sqrt{3}}{-22} =$ $-\frac{3-5\sqrt{3}}{22}$
7. $\frac{2-\sqrt{2}}{2+\sqrt{2}}\left(\frac{2-\sqrt{2}}{2-\sqrt{2}}\right) = \frac{4-4\sqrt{2}+2}{4-2} = \frac{6-4\sqrt{2}}{2}$ $= 3-2\sqrt{2}$	8. $\frac{\sqrt{3}+5}{\sqrt{3}-5}\left(\frac{\sqrt{3}+5}{\sqrt{3}+5}\right) = \frac{3+10\sqrt{3}+25}{3-25} =$ $\frac{28+10\sqrt{3}}{-22} = -\frac{14+5\sqrt{3}}{11}$
9. $\frac{\sqrt{6}+8}{\sqrt{2}+\sqrt{3}}\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}\right) =$ $\frac{\sqrt{12}-\sqrt{18}+8\sqrt{2}-8\sqrt{3}}{2-3} =$ $\frac{2\sqrt{3}-3\sqrt{2}+8\sqrt{2}-8\sqrt{3}}{-1} =$ $-(5\sqrt{2}-6\sqrt{3}) = 6\sqrt{3}-5\sqrt{2}$	10. $\frac{\sqrt{xy}}{\sqrt{x}-\sqrt{y}}\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right) = \frac{x\sqrt{y}+y\sqrt{x}}{x-y}$
11. $A = lw$ $2 = (\sqrt{5}-1)w$ $w = \frac{2}{\sqrt{5}-1}$ $w = \frac{2}{\sqrt{5}-1}\left(\frac{\sqrt{5}+1}{\sqrt{5}+1}\right) = \frac{2\sqrt{5}+2}{4} = \frac{\sqrt{5}+1}{2}$	12. $A = \frac{1}{2}bh$ $8+12\sqrt{2} = \frac{1}{2}(6+2\sqrt{2})h$ $8+12\sqrt{2} = (3+\sqrt{2})h$ $h = \frac{8+12\sqrt{2}}{3+\sqrt{2}}$ $h = \frac{8+12\sqrt{2}}{3+\sqrt{2}}\left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right) = \frac{24+28\sqrt{2}-24}{9-2} =$ $\frac{28\sqrt{2}}{7} = 4\sqrt{2}$

CHAPTER 3. QUADRATIC FUNCTIONS

3.1 Factor a Trinomial by Grouping

1. $6x^2 + x - 2 =$ $6x^2 - 3x + 4x - 2 =$ $3x(2x - 1) + 2(2x - 1) =$ $(3x + 2)(2x - 1)$	2. $12x^2 + 5x - 2 =$ $12x^2 - 3x + 8x - 2 =$ $3x(4x - 1) + 2(4x - 1) =$ $(3x + 2)(4x - 1)$
3. $12x^2 - 29x + 15 =$ $12x^2 - 9x - 20x + 15 =$ $3x(4x - 3) - 5(4x - 3) =$ $(3x - 5)(4x - 3)$	4. $6x^2 - 11x + 4 =$ $6x^2 - 3x - 8x + 4 =$ $3x(2x - 1) - 4(2x - 1) =$ $(3x - 4)(2x - 1)$
5. $15x^2 + 14x - 8 =$ $15x^2 + 20x - 6x - 8 =$ $5x(3x + 4) - 2(3x + 4) =$ $(5x - 2)(3x + 4)$	6. $-10x^2 - 29x - 10 =$ $-10x^2 - 25x - 4x - 10 =$ $-5x(2x + 5) - 2(2x + 5) =$ $(-5x - 2)(2x + 5)$
7. $4x^2 + 12x + 9 =$ $4x^2 + 6x + 6x + 9 =$ $2x(2x + 3) + 3(2x + 3) =$ $(2x + 3)(2x + 3)$ The square root is $2x + 3$.	8. First, write in standard form: $6x^2 - 57x - 30 =$ $6x^2 - 60x + 3x - 30 =$ $6x(x - 10) + 3(x - 10) =$ $(6x + 3)(x - 10)$
9. $2x^2 + 4x - 3x - 6 =$ $2x(x + 2) - 3(x + 2) =$ $(2x - 3)(x + 2)$ So, $(2x - 3)$	

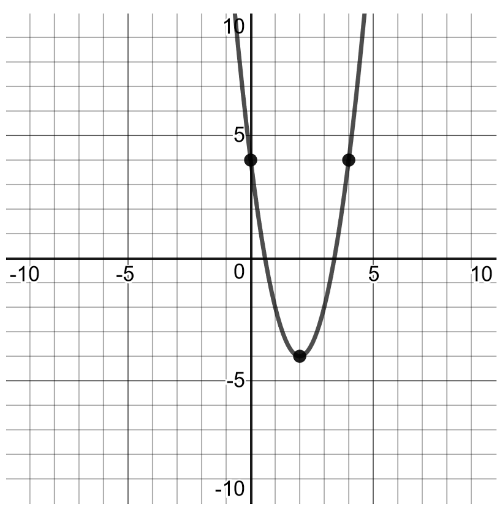
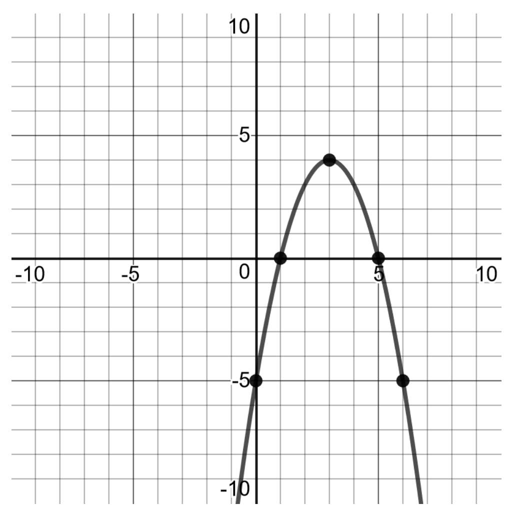
3.2 Solve Quadratics with $a \neq 1$

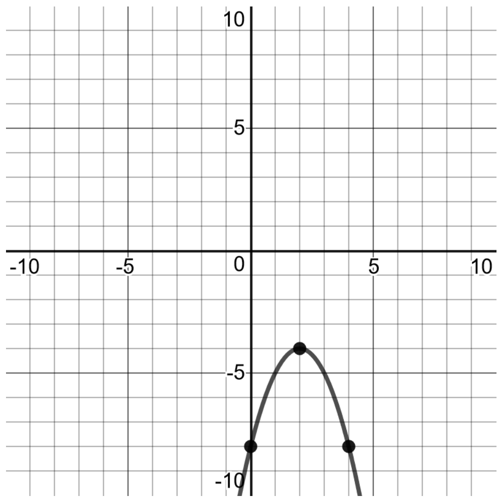
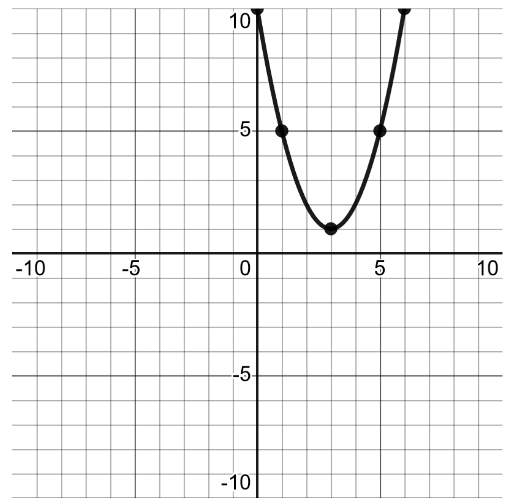
1. $10x^2 + 9x + 2 = 0$ $n^2 + 9n + 20 = 0$ $(n + 5)(n + 4) = 0$ $n = -5$ or $n = -4$ $x = -\frac{5}{10}$ or $x = -\frac{4}{10}$ $\left\{-\frac{1}{2}, -\frac{2}{5}\right\}$	2. $2x^2 - 3x - 2 = 0$ $n^2 - 3n - 4 = 0$ $(n + 1)(n - 4) = 0$ $n = -1$ or $n = 4$ $x = -\frac{1}{2}$ or $x = \frac{4}{2}$ $\left\{-\frac{1}{2}, 2\right\}$
3. $12x^2 + 29x + 15 = 0$ $n^2 + 29n + 180 = 0$ $(n + 20)(n + 9) = 0$ $n = -20$ or $n = -9$ $x = -\frac{20}{12}$ or $x = -\frac{9}{12}$ $\left\{-\frac{5}{3}, -\frac{3}{4}\right\}$	4. $4x^2 + 109x + 225 = 0$ $n^2 + 109n + 900 = 0$ $(n + 100)(n + 9) = 0$ $n = -100$ or $n = -9$ $x = -\frac{100}{4}$ or $x = -\frac{9}{4}$ $\left\{-25, -\frac{9}{4}\right\}$

5. $ac = 20$ and $b = 9$, so use 5 and 4 $10x^2 + 5x + 4x + 2 = 0$ $5x(2x+1) + 2(2x+1) = 0$ $(5x+2)(2x+1) = 0$ $5x + 2 = 0$ $\quad$ $2x + 1 = 0$ $x = -\frac{2}{5}$ $\quad$ $x = -\frac{1}{2}$ $\left\{-\frac{1}{2}, -\frac{2}{5}\right\}$	6. $ac = 24$ and $b = -14$; use -12 and -2 $3x^2 - 12x - 2x + 8 = 0$ $3x(x-4) - 2(x-4) = 0$ $(3x-2)(x-4) = 0$ $3x - 2 = 0$ $\quad$ $x - 4 = 0$ $x = \frac{2}{3}$ $\quad$ $x = 4$ $\left\{\frac{2}{3}, 4\right\}$
7. $ac = -4$ and $b = -3$, so use -4 and 1 $2x^2 - 4x + x - 2 = 0$ $2x(x-2) + 1(x-2) = 0$ $(2x+1)(x-2) = 0$ $2x + 1 = 0$ $\quad$ $x - 2 = 0$ $x = -\frac{1}{2}$ $\quad$ $x = 2$ $\left\{-\frac{1}{2}, 2\right\}$	8. $4x(x-1) = 15$ $4x^2 - 4x = 15$ $4x^2 - 4x - 15 = 0$ $ac = -60$ and $b = -4$; use 6 and -10 $4x^2 + 6x - 10x - 15 = 0$ $2x(2x+3) - 5(2x+3) = 0$ $(2x-5)(2x+3) = 0$ $2x - 5 = 0$ $\quad$ $2x + 3 = 0$ $x = \frac{5}{2}$ $\quad$ $x = -\frac{3}{2}$ $\left\{-\frac{3}{2}, \frac{5}{2}\right\}$
9. $4x^2 + 8x - 12 = 0$ $x^2 + 2x - 3 = 0$ $x^2 + 2x = 3$ $\quad$ $\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$ $x^2 + 2x + 1 = 3 + 1$ $(x+1)^2 = 4$ $x + 1 = \pm\sqrt{4} = \pm 2$ $x = -1 \pm 2$ $\{-3, 1\}$	10. $3x^2 - 18x - 21 = 0$ $x^2 - 6x - 7 = 0$ $x^2 - 6x = 7$ $\quad$ $\left(\frac{b}{2}\right)^2 = \left(-\frac{6}{2}\right)^2 = 9$ $x^2 - 6x + 9 = 7 + 9$ $(x-3)^2 = 16$ $x - 3 = \pm\sqrt{16} = \pm 4$ $x = 3 \pm 4$ $\{-1, 7\}$
11. $4x^2 + 8x = 45$ $x^2 + 2x = \frac{45}{4}$ $\quad$ $\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$ $x^2 + 2x + 1 = \frac{45}{4} + \frac{4}{4}$ $(x+1)^2 = \frac{49}{4}$ $x + 1 = \pm\sqrt{\frac{49}{4}} = \pm\frac{7}{2}$ $x = -1 \pm \frac{7}{2} = -\frac{2}{2} \pm \frac{7}{2}$ $\left\{-\frac{9}{2}, \frac{5}{2}\right\}$	12. $3x^2 - 12x + 2 = 0$ $x^2 - 4x + \frac{2}{3} = 0$ $x^2 - 4x = -\frac{2}{3}$ $\quad$ $\left(\frac{b}{2}\right)^2 = \left(-\frac{4}{2}\right)^2 = 4$ $x^2 - 4x + 4 = -\frac{2}{3} + \frac{12}{3}$ $(x-2)^2 = \frac{10}{3}$ $x - 2 = \pm\sqrt{\frac{10}{3}} = \pm\frac{\sqrt{30}}{3}$ $x = 2 \pm \frac{\sqrt{30}}{3}$ $\left\{2 - \frac{\sqrt{30}}{3}, 2 + \frac{\sqrt{30}}{3}\right\}$

13. $x = \dfrac{-9 \pm \sqrt{9^2 - 4(10)(2)}}{2(10)}$ $x = \dfrac{-9 \pm \sqrt{1}}{20} = \dfrac{-9 \pm 1}{20}$ $\left\{-\frac{1}{2}, -\frac{2}{5}\right\}$	14. $x = \dfrac{-4 \pm \sqrt{4^2 - 4(-3)(-1)}}{2(-3)}$ $x = \dfrac{-4 \pm \sqrt{4}}{-6} = \dfrac{-4 \pm 2}{-6}$ $\left\{\frac{1}{3}, 1\right\}$
15. $x = \dfrac{-8 \pm \sqrt{8^2 - 4(4)(-9)}}{2(4)}$ $x = \dfrac{-8 \pm \sqrt{208}}{8} = \dfrac{-8 \pm 4\sqrt{13}}{8} = -1 \pm \dfrac{\sqrt{13}}{2}$ $\left\{-1 - \frac{\sqrt{13}}{2}, -1 + \frac{\sqrt{13}}{2}\right\}$	16. $x = \dfrac{12 \pm \sqrt{(-12)^2 - 4(3)(2)}}{2(3)}$ $x = \dfrac{12 \pm \sqrt{120}}{6} = \dfrac{12 \pm 2\sqrt{30}}{6} = 2 \pm \dfrac{\sqrt{30}}{3}$ $\left\{2 - \frac{\sqrt{30}}{3}, 2 + \frac{\sqrt{30}}{3}\right\}$

3.3 Graphs of Quadratic Functions

1. $x = \dfrac{-(-8)}{2(-2)} = -2$ $y = -2(-2)^2 - 8(-2) + 3 = 11$ $x = -2$ and $(-2,11)$	2. $x = \dfrac{-(-2)}{2(-1)} = -1$ $y = -(-1)^2 - 2(-1) + 1 = 2$ $x = -1$ and $(-1,2)$
3. For a parabola opening down, the maximum value is at the vertex. $x = \dfrac{-6}{2(-3)} = 1$ $y = -3(1)^2 + 6(1) - 2 = 1$ Maximum is 1	4. For a parabola opening up, the minimum value is at the vertex. $x = \dfrac{-(-20)}{2(5)} = 2$ $y = 5(2)^2 - 20(2) + 14 = -6$ Minimum is −6
5. 	6. 

7. 	8. 

3.4 Vertex Form and Transformations

1. (3)	
2. $a = 1$ $h = -\frac{b}{2a} = -3$ $k = (-3)^2 + 6(-3) + 10 = 1$ $y = (x+3)^2 + 1$ vertex: (–3,1)	3. $y = x^2 + 10x + 21$ $y - 21 = x^2 + 10x$ $\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = 25$ $y + 4 = x^2 + 10x + 25$ $y + 4 = (x+5)^2$ $y = (x+5)^2 - 4$ vertex: (–5,–4)
4. $y = (x+5)^2 + 3$ $y = (x+5)(x+5) + 3$ $y = x^2 + 10x + 25 + 3$ $y = x^2 + 10x + 28$	5. $y = -2(x-4)^2 - 5$ $y = -2(x-4)(x-4) - 5$ $y = -2(x^2 - 8x + 16) - 5$ $y = -2x^2 + 16x - 32 - 5$ $y = -2x^2 + 16x - 37$

3.5 Focus and Directrix (CC)

1. Vertex is $(2, -4)$ $p = \frac{1}{4a} = \frac{1}{4} \div \frac{1}{16} = 4$ Since $p > 0$, the directrix is below the vertex. Directrix is $y = -4 - 4 = -8$	2. $h = 0$ and $k = \frac{3+5}{2} = 4$ $p = 3 - 4 = -1$ $a = \frac{1}{4p} = -\frac{1}{4}$ $y = -\frac{1}{4}x^2 + 4$

3. $h = 4$ and $k = \frac{2-4}{2} = -1$ $p = 2 - (-1) = 3$ $a = \frac{1}{4p} = \frac{1}{12}$ $y = \frac{1}{12}(x-4)^2 - 1$	4. $x^2 + 6x = -4y - 5$ $x^2 + 6x + 9 = -4y - 5 + 9$ $(x+3)^2 = -4y + 4$ $(x+3)^2 - 4 = -4y$ $y = -\frac{1}{4}(x+3)^2 + 1$ Vertex is $(-3,1)$ $p = \frac{1}{4a} = \frac{1}{4} \div \left(-\frac{1}{4}\right) = -1$ Since $p < 0$, the focus is 1 unit below the vertex, at $(-3,0)$, and the directrix is 1 unit above the vertex, at $y = 2$.
5. Vertex is $(-1,2)$. $p = 8 \div 4 = 2$ The equation of the directrix is $y = k - p$, so $y = 2 - 2$, or $y = 0$	6. Vertex is $(-3,1)$ $p = -20 \div 4 = -5$ The equation of the directrix is $y = k - p$, or $y = 6$.
7. $p = 1 - (-3) = 4$ $(x-2)^2 = 16(y-1)$	8. Vertex is $(2,1)$. $4p = 2$, so $p = \frac{1}{2}$. Focus is $\frac{1}{2}$ unit above the vertex, at $\left(2, \frac{3}{2}\right)$. Directrix is $y = k - p$, or $y = \frac{1}{2}$.

Chapter 4. Imaginary Numbers

4.1 Set of Complex Numbers

1. $\sqrt{-25} = \sqrt{25}\sqrt{-1} = 5i$	2. $\sqrt{100}\sqrt{-1}\sqrt{3} = 10i\sqrt{3}$
3. $2 + \sqrt{-12} = 2 + \sqrt{4}\sqrt{-1}\sqrt{3} =$ $2 + 2i\sqrt{3}$	4. $-8 + \frac{3}{4}\sqrt{16}\sqrt{-1}\sqrt{5} = -8 + 3i\sqrt{5}$
5. $\sqrt{36}\sqrt{x^{16}}\sqrt{-1}\sqrt{5} = 6x^8 i\sqrt{5}$	6. $(3i)^3 = (3^3)(i^3) = 27(-i) = -27i$
7. $(2i)^4 = (2^4)(i^4) = 16(1) = 16$	8. $(-3)^3 i^{10} = (-27)(-1) = 27$

4.2 Operations with Complex Numbers

1. $8 - 2i$	2. $(3 + 2i)(2 - i) =$ $6 - 3i + 4i - 2(-1) =$ $8 + i$
3. $(3 - 7i)(3 - 7i) =$ $9 - 21i - 21i + 49(-1) =$ $-40 - 42i$	4. $(2\sqrt{2} + 5i)(5\sqrt{2} - 2i) =$ $20 - 4i\sqrt{2} + 25i\sqrt{2} - 10(-1) =$ $30 + 21i\sqrt{2}$
5. $(-1 + i)(-1 + i)(-1 + i) =$ $(1 - i - i - 1)(-1 + i) =$ $(-2i)(-1 + i) =$ $2i - 2(-1) =$ $2 + 2i$	6. $(3i)(2i)^2(2 + i) =$ $(3i)(-4)(2 + i) =$ $(-12i)(2 + i) =$ $-24i - 12(-1) =$ $12 - 24i$
7. $(x + i)(x + i) - (x - i)(x - i) =$ $(x^2 + 2xi - 1) - (x^2 - 2xi - 1) =$ $4xi$	8. $2xi(i - 4i^2) =$ $2xi(i + 4) =$ $-2x + 8xi$
9. $3x^2 + 48 = 3(x^2 + 16) =$ $3(x + 4i)(x - 4i)$	10. $-9x^2 - 81 = -9(x^2 + 9) =$ $-9(x + 3i)(x - 3i)$

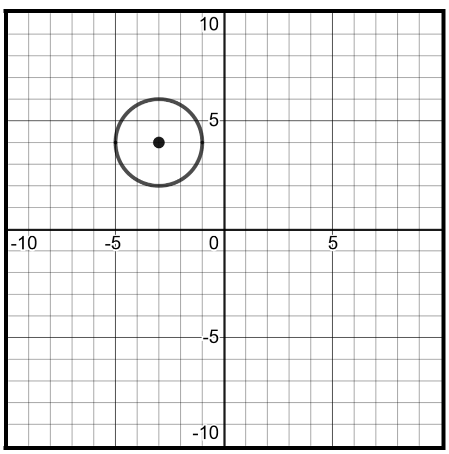
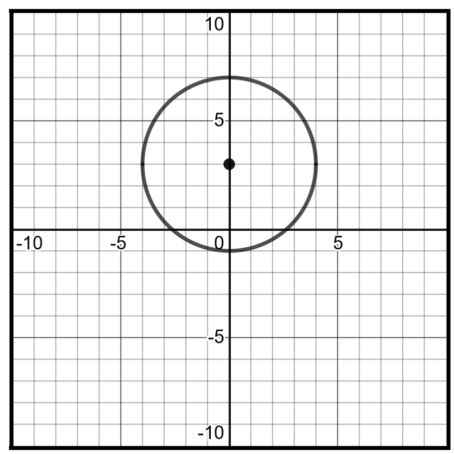
4.3 Imaginary Roots

1. $x = \pm\sqrt{-25} = \pm\sqrt{25}\sqrt{-1} = \pm 5i$ $\{-5i, 5i\}$	2. $x = \pm\sqrt{-16} = \pm\sqrt{16}\sqrt{-1} = \pm 4i$ $\{-4i, 4i\}$
3. $x^2 = \frac{3}{2}(-18) = -27$ $x = \pm\sqrt{-27} = \pm 3i\sqrt{3}$ $\{-3i\sqrt{3}, 3i\sqrt{3}\}$	4. $x + 2 = \pm\sqrt{-9}$ $x + 2 = \pm 3i$ $x = -2 \pm 3i$

5. $x^2 - 12x = -40$ $x^2 - 12x + 36 = -40 + 36$ $(x-6)^2 = -4$ $x - 6 = \pm\sqrt{-4}$ $x = 6 \pm 2i$	6. $x^2 + 8x = -25$ $x^2 + 8x + 16 = -25 + 16$ $(x+4)^2 = -9$ $x + 4 = \pm\sqrt{-9}$ $x = -4 \pm 3i$
7. $x^2 - 4x = -9$ $x^2 - 4x + 4 = -9 + 4$ $(x-2)^2 = -5$ $x - 2 = \pm\sqrt{-5}$ $x = 2 \pm i\sqrt{5}$	8. $n^2 + 6n + 12 = 0$ $n^2 + 6n = -12$ $n^2 + 6n + 9 = -12 + 9$ $(n+3)^2 = -3$ $n + 3 = \pm\sqrt{-3} = \pm i\sqrt{3}$ $n = -3 \pm i\sqrt{3}$ $x = -1 \pm \frac{i\sqrt{3}}{3}$
9. $\frac{-(-3) \pm\sqrt{(-3)^2 - 4(1)(7)}}{2(1)} = \frac{3 \pm\sqrt{-19}}{2} = \frac{3}{2}$ $\pm \frac{i\sqrt{19}}{2}$	10. $\frac{-(-12) \pm\sqrt{(-12)^2 - 4(4)(25)}}{2(4)} =$ $\frac{12 \pm\sqrt{-256}}{8} = \frac{12 \pm 16i}{8} = \frac{3}{2} \pm 2i$
11. $3x^2 - 4x + 5 = 0$ $\frac{-(-4) \pm\sqrt{(-4)^2 - 4(3)(5)}}{2(3)} = \frac{4 \pm\sqrt{-44}}{6} =$ $\frac{4 \pm 2i\sqrt{11}}{6} = \frac{2}{3} \pm \frac{i\sqrt{11}}{3}$	12. $-3x^2 + 2x - 2 = 0$ $\frac{-(2) \pm\sqrt{(2)^2 - 4(-3)(-2)}}{2(-3)} = \frac{-2 \pm\sqrt{-20}}{-6} =$ $\frac{-2 \pm 2i\sqrt{5}}{-6} = \frac{1}{3} \pm \frac{i\sqrt{5}}{3}$
13. $-2 - 3i$ (its conjugate)	14. $(a + bi)(a - bi)$ $= a^2 - abi + abi - (bi)^2$ $= a^2 + b^2$
15. The roots are $1 - 2i\sqrt{2}$ and $1 + 2i\sqrt{2}$. $\left(x - \left(1 - 2i\sqrt{2}\right)\right)\left(x - \left(1 + 2i\sqrt{2}\right)\right) = 0$ $\left((x-1) + 2i\sqrt{2}\right)\left((x-1) - 2i\sqrt{2}\right) = 0$ $(x-1)^2 - \left(2i\sqrt{2}\right)^2 = 0$ $(x^2 - 2x + 1) - (-8) = 0$ $x^2 - 2x + 9 = 0$ $f(x) = x^2 - 2x + 9$	

Chapter 5. Circles

5.1 Equations of Circles

1. Center is $(5, -2)$; radius is $\sqrt{36} = 6$.	2. Divide the equation by 3: $(x + 1)^2 + y^2 = 9$ Center is $(-1, 0)$ and radius is $\sqrt{9} = 3$.
3. $(x + 8)^2 + (y - 6)^2 = 25$	4. Circle B has its center at $(0, -5)$, so the center of circle A is $(2, -2)$.
5. $x^2 - 6x + y^2 + 14y + 42 = 0$ $x^2 - 6x + y^2 + 14y = -42$ $(x^2 - 6x + 9) + (y^2 + 14y + 49) =$ $-42 + 9 + 49$ $(x - 3)^2 + (y + 7)^2 = 16$ Center is $(3, -7)$, radius is $\sqrt{16} = 4$.	6. Divide by 2: $x^2 + 2x + y^2 - 10y - 23 = 0$ $x^2 + 2x + y^2 - 10y = 23$ $(x^2 + 2x + 1) + (y^2 - 10y + 25) =$ $23 + 1 + 25$ $(x + 1)^2 + (y - 5)^2 = 49$ Center is $(-1,5)$, radius is $\sqrt{49} = 7$.
7. 	8. 

5.2 Circle-Linear Systems

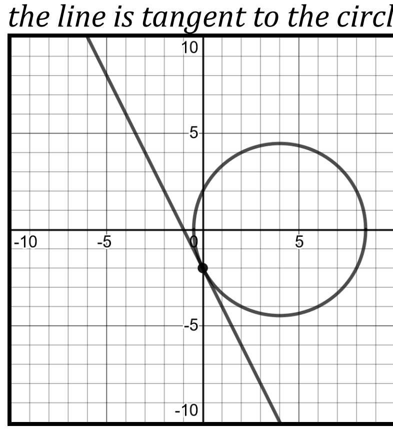
1. $x^2 + (-x)^2 = 36$ $x^2 + x^2 = 36$ $2x^2 = 36$ $x^2 = 18$ $x = \pm\sqrt{18} = \pm 3\sqrt{2}$ For $x = 3\sqrt{2}, y = -3\sqrt{2}$ For $x = -3\sqrt{2}, y = 3\sqrt{2}$ $(3\sqrt{2}, -3\sqrt{2})$ and $(-3\sqrt{2}, 3\sqrt{2})$	2. $(x - 1)^2 + (3x)^2 = 9$ $x^2 - 2x + 1 + 9x^2 = 9$ $10x^2 - 2x - 8 = 0$ $5x^2 - x - 4 = 0$ $x = \frac{1 \pm \sqrt{(-1)^2 - 4(5)(-4)}}{2(5)} = \frac{1 \pm 9}{10}$ $x = \{1, -0.8\}$ $y = 3(1) = 3$ $y = 3(-0.8) = -2.4$ $(1,3)$ and $(-0.8, -2.4)$

3. $x^2 + (-x + 2)^2 = 16$
$x^2 + x^2 - 4x + 4 = 16$
$2x^2 - 4x = 12$
$x^2 - 2x = 6$
$x^2 - 2x + 1 = 6 + 1$
$(x - 1)^2 = 7$
$x - 1 = \pm\sqrt{7}$
$x = 1 \pm \sqrt{7}$
$y = -\left(1 + \sqrt{7}\right) + 2 = 1 - \sqrt{7}$
$y = -\left(1 - \sqrt{7}\right) + 2 = 1 + \sqrt{7}$
$\left(1 + \sqrt{7},\ 1 - \sqrt{7}\right)$ and
$\left(1 - \sqrt{7},\ 1 + \sqrt{7}\right)$

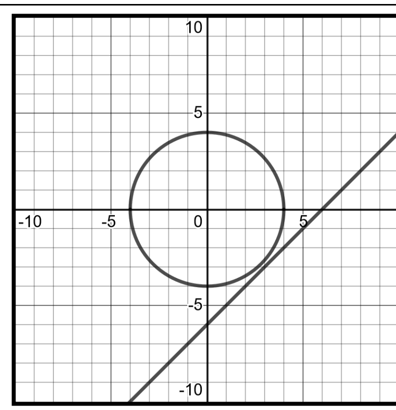
4. $(x + 2)^2 + (2x - 5 - 1)^2 = 25$
$(x + 2)^2 + (2x - 6)^2 = 25$
$x^2 + 4x + 4 + 4x^2 - 24x + 36 = 25$
$5x^2 - 20x + 15 = 0$
$x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$
$x = \{3,1\}$
$y = 2(3) - 5 = 1$
$y = 2(1) - 5 = -3$
$(3, 1)$ and $(1, -3)$

5. $(x - 2)^2 + \left(\frac{1}{2}x - 4\right)^2 = 9$
$x^2 - 4x + 4 + \frac{1}{4}x^2 - 4x + 16 = 9$
$\frac{5}{4}x^2 - 8x + 11 = 0$
$5x^2 - 32x + 44 = 0$ *multiply by 4*
$n^2 - 32n + 220 = 0$
$(n - 22)(n - 10) = 0$
$n = 22$ or $n = 10$
$x = \frac{22}{5} = 4.4$ or $x = \frac{10}{5} = 2$
$y = \frac{1}{2}(4.4) = 2.2$
$y = \frac{1}{2}(2) = 1$
$(4.4, 2.2)$ and $(2, 1)$

6. $-3y - 6 = 6x$
$y + 2 = -2x$ *divide by –3*
$y = -2x - 2$
$(x - 4)^2 + (-2x - 2)^2 = 20$
$x^2 - 8x + 16 + 4x^2 + 8x + 4 = 20$
$5x^2 = 0$
$x = 0$
$y = -2(0) - 2 = -2$
$(0, -2)$ *one point of intersection:*
the line is tangent to the circle



7. $x^2 + (x - 6)^2 = 16$
$x^2 + x^2 - 12x + 36 = 16$
$2x^2 - 12x + 20 = 0$
$x^2 - 6x + 10 = 0$
$x^2 - 6x = -10$
$x^2 - 6x + 9 = -10 + 9$
$(x - 3)^2 = -1$
$x - 3 = \pm\sqrt{-1}$
$x = 3 \pm i$
No real solutions, so the line and circle do not intersect.



6.1 Operations with Functions

1. a) $s(x) = 13x - 4$ b) $p(x) = 36x^2 - 11x - 5$	2. a) $d(x) = 3x^2 + 9x$ b) $q(x) = x + 4$
3. a) $2x + 6\sqrt{3}$ b) $10x\sqrt{3} + 15$ c) $\frac{2x+\sqrt{3}}{5\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2x\sqrt{3}+3}{15}$	4. a) $h(x) = 2(x^2 - 1) - 4(2x + 4) + 8$ $h(x) = 2x^2 - 2 - 8x - 16 + 8$ $h(x) = 2x^2 - 8x - 10$ b) $2x^2 - 8x - 10 = 0$ $x^2 - 4x - 5 = 0$ $(x + 1)(x - 5) = 0$ $\{-1, 5\}$
5. a) $P(x) = 6x - 170$ b) $6x - 170 > 0$ $6x > 170$ $x > 28.33$ Need to sell 29 headphones.	

6.2 Long Division

1. $5x-2$ $x+1\,\overline{)\,5x^2+3x-2}$ $-(5x^2+5x)$ $-2x-2$ $-(-2x-2)$ 0	2. $x-2$ $3x+1\,\overline{)\,3x^2-5x+1}$ $-(3x^2+x)$ $-6x+1$ $-(-6x-2)$ 3 Answer: $x - 2 + \frac{3}{3x+1}$
3. $x+2$ $x^2+x-6\,\overline{)\,x^3+3x^2-4x-12}$ $-(x^3+x^2-6x)$ $2x^2+2x-12$ $-(2x^2+2x-12)$ 0	4. $2x+3$ $x^2-4\,\overline{)\,2x^3+3x^2-8x-12}$ $-(2x^3 \quad -8x)$ $3x^2 \quad -12$ $-(3x^2 \quad -12)$ 0

5. $2x+3\,\overline{)\,2x^3+7x^2+2x+9}$ quotient x^2+2x-2 $-\left(2x^3+3x^2\right)$ $4x^2+2x$ $-\left(4x^2+6x\right)$ $-4x+9$ $-(-4x-6)$ 15 $x^2+2x-2+\dfrac{15}{2x+3}$	6. $x-3\,\overline{)\,x^3-2x^2+0x-4}$ quotient x^2+x+3 $-\left(x^3-3x^2\right)$ x^2+0x $-\left(x^2-3x\right)$ $3x-4$ $-(3x-9)$ 5 $x^2+x+3+\dfrac{5}{x-3}$
7. $x^3+2\,\overline{)\,x^6+2x^5+5x^3+4x^2+6}$ quotient x^3+2x^2+3 $-\left(x^6+2x^3\right)$ $2x^5+3x^3+4x^2$ $-\left(2x^5+4x^2\right)$ $3x^3+6$ $-\left(3x^3+6\right)$ 0	8. $x^2-4x+1\,\overline{)\,2x^3-5x^2+x-10}$ quotient $2x+3$ $-\left(2x^3-8x^2+2x\right)$ $3x^2-x-10$ $-\left(3x^2-12x+3\right)$ $11x-13$ $2x+3+\dfrac{11x-13}{x^2-4x+1}$

6.3 Synthetic Division

1. $\begin{array}{c\|rrrr} 2 & 3 & -4 & -7 & 6 \\ & & 6 & 4 & -6 \\ \hline & 3 & 2 & -3 & \| \; 0 \end{array}$ Ans: $3x^2+2x-3$	2. $\begin{array}{c\|rrrr} 4 & 2 & -5 & -11 & -4 \\ & & 8 & 12 & 4 \\ \hline & 2 & 3 & 1 & \| \; 0 \end{array}$ Ans: $2x^2+3x+1$
3. $\begin{array}{c\|rrrr} 1 & 3 & 1 & -6 & 2 \\ & & 3 & 4 & -2 \\ \hline & 3 & 4 & -2 & \| \; 0 \end{array}$ Ans: $3x^2+4x-2$	4. $\begin{array}{c\|rrrrr} -2 & 3 & 7 & -1 & -5 & 5 \\ & & -6 & -2 & 6 & -2 \\ \hline & 3 & 1 & -3 & 1 & \| \; 3 \end{array}$ Ans: $3x^3+x^2-3x+1+\dfrac{3}{x+2}$

<table>
<tr>
<td>5.
$\begin{array}{c|ccccc} 2 & 1 & 0 & -4 & -4 & 8 \\ & & 2 & 4 & 0 & -8 \\ \hline & 1 & 2 & 0 & -4 & \mid 0 \end{array}$
Ans: $x^3 + 2x^2 - 4$</td>
<td>6.
$\begin{array}{c|ccccc} -3 & 1 & 0 & -11 & -1 & 7 \\ & & -3 & 9 & 6 & -15 \\ \hline & 1 & -3 & -2 & 5 & \mid -8 \end{array}$
Ans: $x^3 - 3x^2 - 2x + 5 - \dfrac{8}{x+3}$</td>
</tr>
</table>

6.4 Remainder Theorem

<table>
<tr>
<td>1. $f(-2) = (-2)^3 + 2(-2)^2 + (-2) + 6$
$f(-2) = 4$, so the remainder is 4.
This can be checked by synthetic division:
$\begin{array}{c|cccc} -2 & 1 & 2 & 1 & 6 \\ & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & \mid 4 \end{array}$</td>
<td>2. Substitute each a to see if $P(a) = 0$.
$P(1) = 1^4 - 2(1^3) - 7(1^2) + 8(1) + 12$
$P(1) = 12$, so the correct answer is (3)</td>
</tr>
<tr>
<td>3. Enter the equation into the calculator as Y_1 and evaluate $Y_1(x)$ for each integer x between −3 and 3.
$P(-3) = 0, P(-2) = 0, P(-1) = -16,$
$P(0) = 0, P(1) = 0, P(2) = -40$, and
$P(3) = 0$
Therefore, the roots are $\{-3, -2, 0, 1, 3\}$</td>
<td>4. $\dfrac{a(x)}{x-2} = 3x + 13 + \dfrac{6}{x-2}$
Multiply both sides by $x - 2$.
$a(x) = 3x(x-2) + 13(x-2) + 6$
$a(x) = 3x^2 - 6x + 13x - 26 + 6$
$a(x) = 3x^2 + 7x - 20$</td>
</tr>
<tr>
<td>5.
a) $(-4)^3 + 3(-4)^2 + (-4)k - 24 = 0$
$-64 + 48 - 4k - 24 = 0$
$-40 - 4k = 0$
$k = -10$</td>
<td>b) Find the other factor by division:
$\begin{array}{c|cccc} -4 & 1 & 3 & -10 & -24 \\ & & -4 & 4 & 24 \\ \hline & 1 & -1 & -6 & \mid 0 \end{array}$
$(x^2 - x - 6)(x + 4) = 0$
$(x - 3)(x + 2)(x + 4) = 0$
$\{-4, -2, 3\}$</td>
</tr>
</table>

6.5 Factor Polynomials

<table>
<tr>
<td>1. $x^2(x+3) + 2(x+3)$
$(x^2 + 2)(x + 3)$</td>
<td>2. $2x^2(2x+5) - 5(2x+5)$
$(2x^2 - 5)(2x + 5)$</td>
</tr>
<tr>
<td>3. $x^2(x+3) - 4(x+3)$
$(x^2 - 4)(x + 3)$
$(x+2)(x-2)(x+3)$</td>
<td>4. $x^2(x-2) - 9(x-2)$
$(x^2 - 9)(x - 2)$
$(x+3)(x-3)(x-2)$</td>
</tr>
<tr>
<td>5. $x^2(x+2) - (x+2)$
$(x^2 - 1)(x + 2)$
$(x+1)(x-1)(x+2)$</td>
<td>6. $x^2(3x-5) - 16(3x-5)$
$(x^2 - 16)(3x - 5)$
$(x+4)(x-4)(3x-5)$</td>
</tr>
</table>

<table>
<tr><td>7. $a(a+b)+c(a+b)$
$(a+c)(a+b)$</td><td>8. a^3+a^2-ab-b
$a^2(a+1)-b(a+1)$
$(a^2-b)(a+1)$</td></tr>
<tr><td>9. $x^4-x^2-9x^2+9$
$x^2(x^2-1)-9(x^2-1)$
$(x^2-9)(x^2-1)$
$(x+3)(x-3)(x+1)(x-1)$</td><td>10. $8y^4-10y^2-28y^2+35$
$2y^2(4y^2-5)-7(4y^2-5)$
$(2y^2-7)(4y^2-5)$</td></tr>
<tr><td>11. $a^4-a^2b^2-4a^2b^2+4b^4$
$a^2(a^2-b^2)-4b^2(a^2-b^2)$
$(a^2-4b^2)(a^2-b^2)$
$(a+2b)(a-2b)(a+b)(a-b)$</td><td>12. $2x^4-2x^2y^2+x^2y^2-y^4$
$2x^2(x^2-y^2)+y^2(x^2-y^2)$
$(2x^2+y^2)(x^2-y^2)$
$(2x^2+y^2)(x+y)(x-y)$</td></tr>
<tr><td colspan="2">13. $x^4(2x-1)+5x^2(2x-1)+4(2x-1)$
$(x^4+5x^2+4)(2x-1)$
$(x^4+x^2+4x^2+4)(2x-1)$
$[x^2(x^2+1)+4(x^2+1)](2x-1)$
$(x^2+4)(x^2+1)(2x-1)$</td></tr>
<tr><td>14. $(x+5)(x^2-5x+25)$</td><td>15. $(b+4)(b^2-4b+16)$</td></tr>
<tr><td>16. $(y-6)(y^2+6y+36)$</td><td>17. $(3x-2)(9x^2+6x+4)$</td></tr>
<tr><td>18. $2(8x^3+27)=$
$2(2x+3)(4x^2-6x+9)$</td><td>19. $(xy^2-4)(x^2y^4+4xy^2+16)$</td></tr>
<tr><td>20. $(8x-7y)(64x^2+56xy+49y^2)$</td><td>21. $2(x^3+64y^3)=$
$2(x+4y)(x^2-4xy+16y^2)$</td></tr>
<tr><td>22. Let $u=2x^2$
$2u^2-3u-2$
$(2u+1)(u-2)$
$(4x^2+1)(2x^2-2)$
$2(4x^2+1)(x^2-1)$
$2(4x^2+1)(x+1)(x-1)$</td><td>23. Let $u=x-6$
x^4-u^2
$(x^2+u)(x^2-u)$
$(x^2+x-6)(x^2-x+6)$
$(x+3)(x-2)(x^2-x+6)$</td></tr>
<tr><td colspan="2">24. We can factor out the GCF of 2 from the middle terms:
$(x^5+2x)^2+2(x^5+2x)+1$
Now let $u=x^5+2x$
u^2+2u+1
$(u+1)(u+1)$
$(x^5+2x+1)(x^5+2x+1)$</td></tr>
</table>

6.6 Find Roots by Factoring

<table>
<tr><td>1. $x(x^2+x-2)=0$
$x(x+2)(x-1)=0$
$\{-2,0,1\}$</td><td>2. $x^2(2x-1)-4(2x-1)=0$
$(x^2-4)(2x-1)=0$
$(x+2)(x-2)(2x-1)=0$
$\left\{\frac{1}{2},\pm 2\right\}$</td></tr>
</table>

3. $4x^2(2x+1)-9(2x+1)=0$ $(4x^2-9)(2x+1)=0$ $(2x+3)(2x-3)(2x+1)=0$ $\left\{\pm\frac{3}{2},-\frac{1}{2}\right\}$	4. $x^3+5x^2-4x-20=0$ $x^2(x+5)-4(x+5)=0$ $(x^2-4)(x+5)=0$ $\{\pm 2,-5\}$
5. $(x^2+1)(x^2-4)=0$ $x^2+1=0$ or $x^2-4=0$ $\{\pm 2,\pm i\}$	6. $3x(x^4-16)=0$ $3x(x^2+4)(x^2-4)=0$ $3x(x^2+4)(x+2)(x-2)=0$ $\{0,\pm 2,\pm 2i\}$
7. $x(x^3+4x^2+4x+16)=0$ $x\big(x^2(x+4)+4(x+4)\big)=0$ $x(x^2+4)(x+4)=0$ $\{0,-4,\pm 2i\}$	8. $9x^2(x-10)+64(x-10)=0$ $(9x^2+64)(x-10)=0$ For the first factor, $x^2=-\frac{64}{9}$, so $x=\pm\frac{8}{3}i$ $\left\{10,\pm\frac{8}{3}i\right\}$
9. $(x+1)(x-2)(3x+1)(3x-2)=$ $(x^2-x-2)\,(3x+1)(3x-2)=$ $(3x^3-2x^2-7x-2)(3x-2)=$ $9x^4-12x^3-17x^2+8x+4$ $f(x)=9x^4-12x^3-17x^2+8x+4$	10. $x=-1+2i \quad \rightarrow \quad x+1-2i=0$ $x=-1-2i \quad \rightarrow \quad x+1+2i=0$ $x=2 \quad \rightarrow \quad x-2=0$ $(x+1-2i)(x+1+2i)(x-2)=$ $[(x+1)^2-(2i)^2](x-2)=$ $(x^2+2x+5)(x-2)=$ x^3+x-10 To get a leading coefficient of 4, vertically dilate the function by multiplying by 4: $g(x)=4x^3+4x-40$

6.7 Root Theorems

1. Possible roots are $\pm 1,\pm 2,\pm 4,\pm 5,\pm 10,\pm 20$. $(-1)^3-8(-1)^2+11(-1)+20=0$ $f(-1)=0$, so -1 is a root and $(x+1)$ is a factor. By synthetic division, $\begin{array}{r\|rrrr} \boxed{-1} & 1 & -8 & 11 & 20 \\ & & -1 & 9 & -20 \\ \hline & 1 & -9 & 20 & \|\ 0 \end{array}$ $(x+1)(x^2-9x+20)$ $(x+1)(x-4)(x-5)$ Roots are -1, 4, and 5.	2. Possible roots are $\pm 1,\pm 2,\pm 3,\pm 6,\pm\frac{1}{3},\pm\frac{2}{3}$. $3(1)^3-10(1)^2+1+6=0$ $g(1)=0$, so 1 is a root and $(x-1)$ is a factor. By synthetic division, $\begin{array}{r\|rrrr} \boxed{1} & 3 & -10 & 1 & 6 \\ & & 3 & -7 & -6 \\ \hline & 3 & -7 & -6 & \|\ 0 \end{array}$ $(x-1)(3x^2-7x-6)$ $(x-1)(3x^2-9x+2x-6)$ $(x-1)(x-3)(3x+2)$ [by grouping] Roots are 1, 3, and $-\frac{2}{3}$.

3. Possible roots are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24, \pm\frac{1}{2}, \pm\frac{3}{2}$.

$2(-2)^3 - (-2)^2 - 22(-2) - 24 = 0$

$h(-2) = 0$, so –2 is a root and $(x + 2)$ is a factor. By synthetic division,

$$\begin{array}{r|rrrr} \boxed{-2} & 2 & -1 & -22 & -24 \\ & & -4 & 10 & 24 \\ \hline & 2 & -5 & -12 & | \ 0 \end{array}$$

$(x + 2)(2x^2 - 5x - 12)$

$(x + 2)(2x^2 - 8x + 3x - 12)$

$(x + 2)(2x + 3)(x - 4)$ [by grouping]

Roots are –2, $-\frac{3}{2}$, and 4.

4. Possible roots are $\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$.

$2(-1)^4 + (-1)^3 - 19(-1)^2 - 9(-1) + 9 = 0$

$f(-1) = 0$, so –1 is a root and $(x + 1)$ is a factor. By synthetic division,

$$\begin{array}{r|rrrrr} \boxed{-1} & 2 & 1 & -19 & -9 & 9 \\ & & -2 & 1 & 18 & -9 \\ \hline & 2 & -1 & -18 & 9 & | \ 0 \end{array}$$

$(x + 1)(2x^3 - x^2 - 18x + 9)$

$(x + 1)(x^2 - 9)(2x - 1)$ [by grouping]

$(x + 1)(x + 3)(x - 3)(2x - 1)$

Roots are $-1, -3, 3$, and $\frac{1}{2}$.

5. $P(x)$ is cubic, so there are 3 roots.
$P(x)$ has two sign changes, so there are at most 2 positive real roots.
$P(-x) = -2x^3 + 3x^2 + 10x + 1$ has one sign change, so there is 1 negative real root.

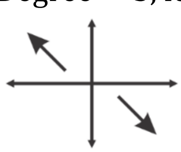
Positive Real Roots	Negative Real Roots	Imaginary Roots	Total Roots
2	1	0	3
0	1	2	3

6. $h(x)$ is a fifth-degree (quintic) polynomial, so there are 5 roots.
$h(x)$ has 3 sign changes, so there are at most 3 positive real roots.
$h(-x) = x^5 + x^4 + x^2 + x + 1$ has no sign changes, so 0 negative real roots.

Positive Real Roots	Negative Real Roots	Imaginary Roots	Total Roots
3	0	2	5
1	0	4	5

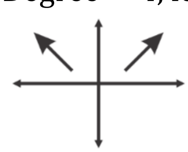
6.8 Properties of Polynomial Graphs

1. Degree = 5, leading coefficient = −3.

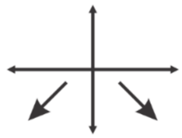
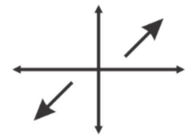


As $x \to -\infty, f(x) \to \infty$,
and as $x \to \infty, f(x) \to -\infty$

2. Degree = 4, leading coefficient = 5.

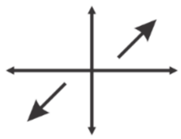


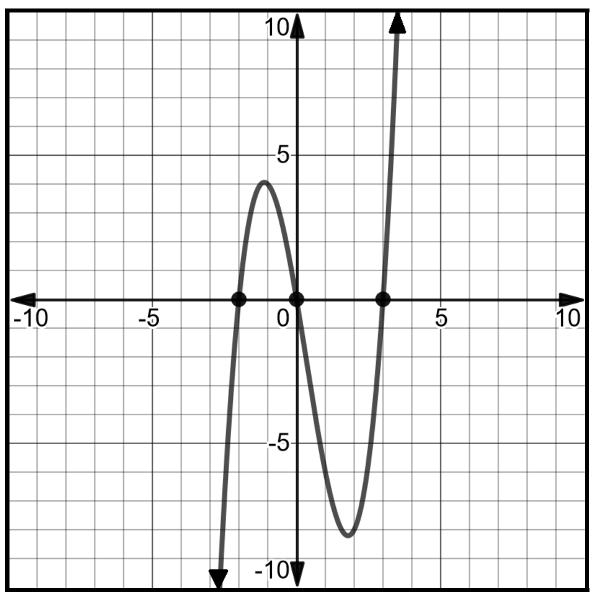
As $x \to -\infty, f(x) \to \infty$,
and as $x \to \infty, f(x) \to \infty$

3. Degree $= 4$, leading coefficient $= -16$.  As $x \to -\infty$, $f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to -\infty$	4. Degree $= 7$, leading coefficient $= 1$.  As $x \to -\infty$, $f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to \infty$
5. $h(0) = -4(-2)(-5)^2 = 200$, so the y-intercept is $(0, 200)$. $x^2 - 2 = 0 \quad \rightarrow \quad x = \pm\sqrt{2}$ $2x - 5 = 0 \quad \rightarrow \quad x = \frac{5}{2}$ *[double root]* so the x-intercepts are $(-\sqrt{2}, 0)$, $(\sqrt{2}, 0)$, and $\left(\frac{5}{2}, 0\right)$.	6. $k(0) = (0)(1)(4)^2 = 0$, so the y-intercept is $(0, 0)$. $5x = 0 \quad \rightarrow \quad x = 0$ $x^2 + 1 = 0 \rightarrow$ imaginary roots $\pm i$ $x + 4 = 0 \quad \rightarrow \quad x = -4$ *[double root]* so the x-intercepts are $(-4, 0)$ and $(0, 0)$.
7. $(-2.31, 0)$, $(-0.76, 0)$, and $(0.57, 0)$	
8. relative maximum at $(3, 5)$, relative minimum at $(5, 1)$. decreasing over $3 < x < 5$ increasing over $x < 3$ and $x > 5$	9. relative maximum at $(-2.5, 1.25)$, relative minima at $(-3, 0)$ and $(-2, 0)$. decreasing over $x < -3$ and $-2.5 < x < -2$ increasing over $3 < x < -2.5$ and $x > -2$

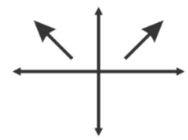
6.9 Graph Polynomial Functions

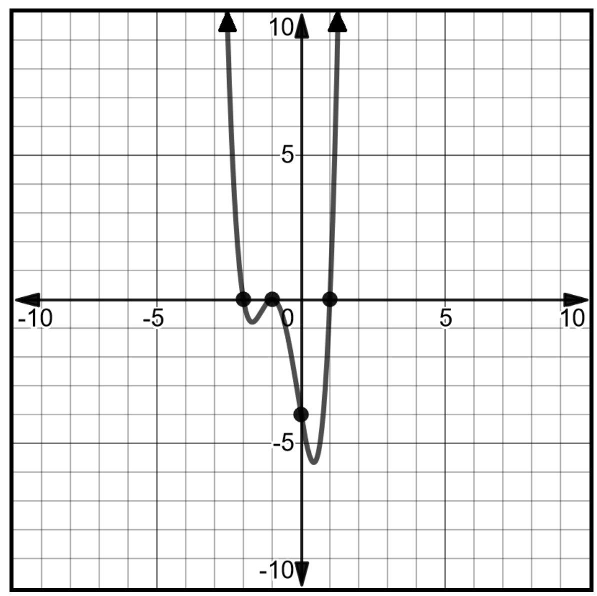
1. x-intercepts at -2, 0, and 3.
   y-intercept at $f(0) = 0$.
   Degree of 3 (odd), leading coefficient of 1 (positive), so end behavior of





2. x-intercepts at -2, -1, and 1, with -1 as a double root.
   y-intercept at $f(0) = -4$.
   Degree of 4 (even), leading coefficient of 2 (positive), so end behavior of





3. $c(x) = x^3 - 13x + 12$

4. $q(x) = 2x^4 + 3x^3 - 3x^2 - 2x$

5.

f(-2)	f(-1)	f(0)	f(1)	f(2)	
-24	-6	4	6	0	
18	10	2	-6		*first*
-8	-8	-8			*second*

Second degree (quadratic) function
$f(x) = -4x^2 + 6x + 4$

6.

	$f(-3)$	$f(-2)$	$f(-1)$	$f(0)$	$f(1)$	$f(2)$	$f(3)$
	65	0	-15	-16	-15	0	65
first	-65	-15	-1	1	15	65	
second	50	14	2	14	50		
third	-36	-12	12	36			
fourth	24	24	24				

Fourth degree (quartic) function
$g(x) = x^4 - 16$

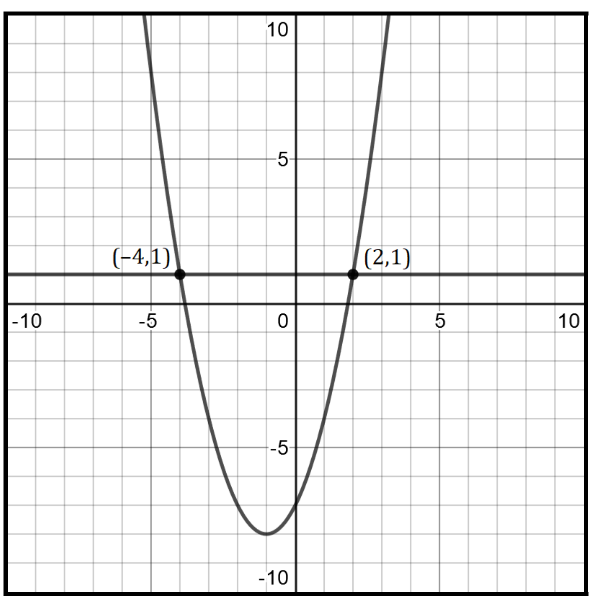
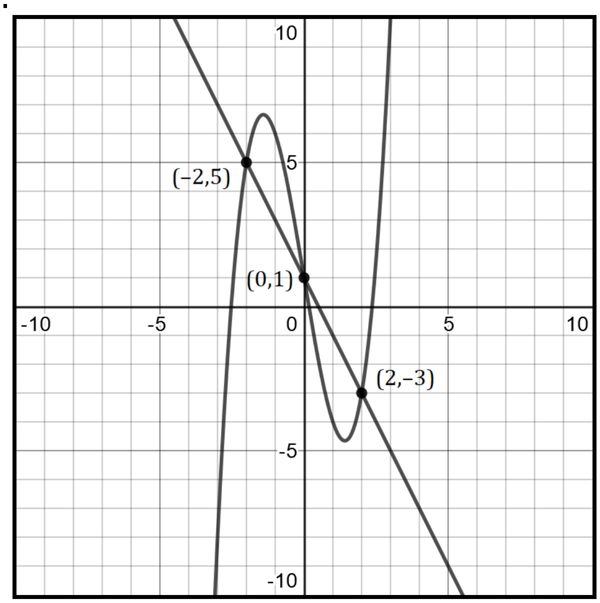
6.10 Polynomial Transformations

1. (3)
2. (2)
3. (1)
4. The graph shifts 2 units to the right and 3 units up.
5. $g(x) = f(x-2) =$
$(x-2)(x+2)(x-5)$
6. Start with $y = 2x^4 - 3x^3 + x - 5$.
For a vertical dilation by a factor of 3, multiply the equation by 3, giving us
$y = 6x^4 - 9x^3 + 3x - 15$.
To reflect over the y-axis, replace each x with $-x$, giving us
$y = 6(-x)^4 - 9(-x)^3 + 3(-x) - 15$, or $y = 6x^4 + 9x^3 - 3x - 15$.
Finally, translate 6 unit up by adding 6:
$n(x) = 6x^4 + 9x^3 - 3x - 9$

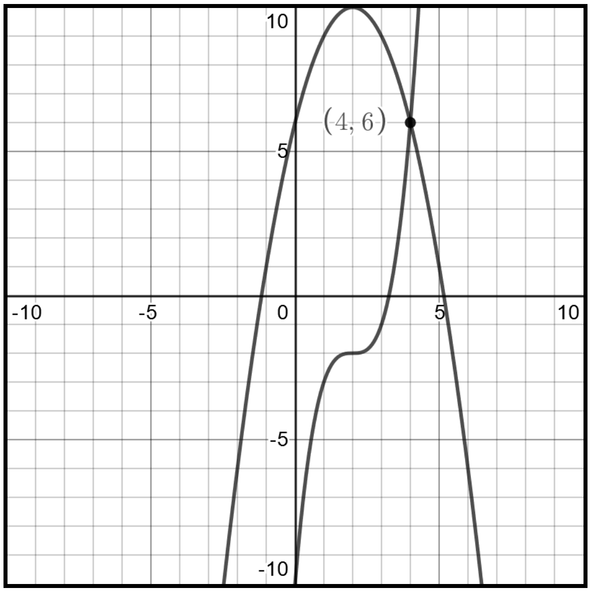
6.11 Systems of Polynomial Functions

1. $x^2 + 2x - 1 = 3x + 5$
$x^2 - x - 6 = 0$
$(x+2)(x-3) = 0$
$x = \{-2,3\}$
$y = 3(-2) + 5 = -1$
$y = 3(3) + 5 = 14$
$(-2,-1)$ and $(3,14)$

2. $y + 3x = 1 \quad \rightarrow \quad y = -3x + 1$
$x^2 + 7x + 22 = -3x + 1$
$x^2 + 10x + 21 = 0$
$(x+7)(x+3) = 0$
$x = \{-7,-3\}$
$y = -3(-7) + 1 = 22$
$y = -3(-3) + 1 = 10$
$(-7,22)$ and $(-3,10)$

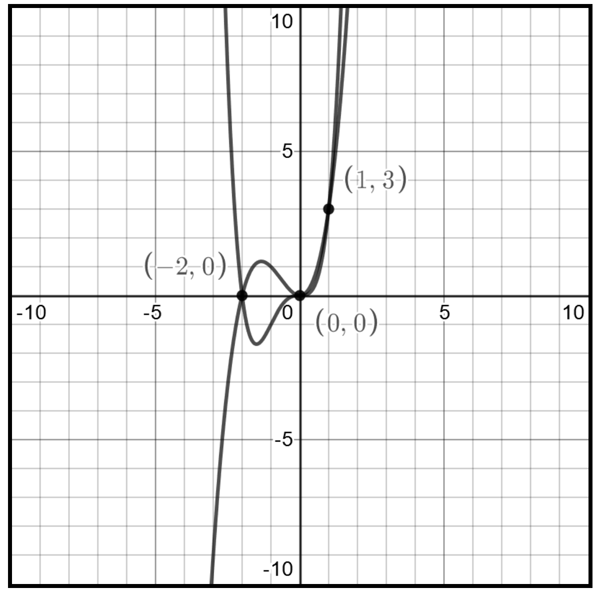
3. $x^2 + 2x = y + 7 \rightarrow y = x^2 + 2x - 7$ $x^2 + 2x - 7 = 2x + 1$ $x^2 = 8$ $x = \pm\sqrt{8} = \pm 2\sqrt{2}$ $y = 2(2\sqrt{2}) + 1 = 1 + 4\sqrt{2}$ $y = 2(-2\sqrt{2}) + 1 = 1 - 4\sqrt{2}$ $(2\sqrt{2}, 1 + 4\sqrt{2})$ and $(-2\sqrt{2}, 1 - 4\sqrt{2})$	4. $x^3 - 6x + 1 = -2x + 1$ $x^3 - 4x = 0$ $x(x^2 - 4) = 0$ $x(x + 2)(x - 2) = 0$ $x = \{0, -2, 2\}$ $y = -2(0) + 1 = 1$ $y = -2(-2) + 1 = 5$ $y = -2(2) + 1 = -3$ $(0,1), (-2,5), (2,-3)$
5. $y + 2 = 3x \rightarrow y = 3x - 2$ $x^2 - 3x + 9 = 3x - 2$ $x^2 - 6x + 11 = 0$ $x^2 - 6x = -11$ $x^2 - 6x + 9 = -11 + 9$ $(x - 3)^2 = -2$ $x - 3 = \pm\sqrt{-2}$ $x = 3 \pm i\sqrt{2}$ $y = 3(3 + i\sqrt{2}) - 2$ $y = 7 + 3i\sqrt{2}$ $y = 3(3 - i\sqrt{2}) - 2$ $y = 7 - 3i\sqrt{2}$ $(3 + i\sqrt{2}, 7 + 3i\sqrt{2})$ and $(3 - i\sqrt{2}, 7 - 3i\sqrt{2})$	6. $x^3 + 5x^2 + 2 = 7x^2 - 5x + 2$ $x^3 - 2x^2 + 5x = 0$ $x(x^2 - 2x + 5) = 0$ $x = 0 \qquad x^2 - 2x + 5 = 0$ $x^2 - 2x = -5$ $x^2 - 2x + 1 = -5 + 1$ $(x - 1)^2 = -4$ $x - 1 = \pm\sqrt{-4}$ $x = 1 \pm 2i$ $y = 7(0)^2 - 5(0) + 2 = 2$ $y = 7(1 + 2i)^2 - 5(1 + 2i) + 2 =$ $7(1 + 4i - 4) - 5 - 10i + 2 =$ $-21 + 28i - 3 - 10i = -24 + 18i$ $y = 7(1 - 2i)^2 - 5(1 - 2i) + 2 =$ $7(1 - 4i - 4) - 5 + 10i + 2 =$ $-21 - 28i - 3 + 10i = -24 - 18i$ $(0,2), (1 + 2i, -24 + 18i)$, and $(1 - 2i, -24 - 18i)$
7.  $(-4,1), (2,1)$	8.  $(-2,5), (0,1), (2,-3)$

9.



$(4, 6)$

10.



$(-2, 0), (0,0), (1,3)$

6.12 Polynomial Identities [CC]

1.	$(x+a)(x+b)$	
	$= x^2 + ax + bx + ab$	multiply the binomials
	$= x^2 + (a+b)x + ab$	distributive property
2.	$(x^2-1)^2 + (2x)^2$	
	$= x^4 - 2x^2 + 1 + (2x)^2$	expand the square of the binomial
	$= x^4 - 2x^2 + 1 + 4x^2$	Power Rule
	$= x^4 + 2x^2 + 1$	combine like terms
	$= (x^2+1)^2$	rewrite as the square of a binomial
3.	$(a+b)^3$	
	$= (a+b)(a+b)(a+b)$	rewrite the cube as a product
	$= (a^2 + 2ab + b^2)(a+b)$	multiply the first two binomials
	$= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3$	multiply the trinomial and binomial
	$= a^3 + 3a^2b + 3ab^2 + b^3$	combine like terms
4.	$(a+b)^2 + (a-b)^2$	
	$= (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2)$	expand both squares of binomials
	$= 2a^2 + 2b^2$	combine like terms
	$= 2(a^2 + b^2)$	distributive property
5.	$(a+b)(a-b)[(a+b)^2 - 2ab]$	
	$= (a+b)(a-b)(a^2 + 2ab + b^2 - 2ab)$	expand the square of the binomial
	$= (a+b)(a-b)(a^2+b^2)$	combine like terms
	$= (a^2-b^2)(a^2+b^2)$	express as a difference of two squares
	$= a^4 - b^4$	express as a difference of two squares

6. $\left(x^2 + y^2 + \sqrt{2}xy\right)\left(x^2 + y^2 - \sqrt{2}xy\right)$

$= x^4 + x^2y^2 - \sqrt{2}x^2y +$ multiply by distributing each term

$x^2y^2 + y^4 - \sqrt{2}xy^2 +$

$\sqrt{2}x^2y + \sqrt{2}xy^2 - 2x^2y^2$

$= x^4 + y^4$ combine like terms

Chapter 7. Radicals and Rational Exponents

7.1 nth Roots

1. 11	2. 6
3. 2.91	4. $\sqrt[4]{2\cdot\boxed{3\cdot3\cdot3\cdot3}} = 3\sqrt[4]{2}$
5. $x^2 = 25$ $\sqrt{x^2} = \pm\sqrt{25}$ $x = \pm 5$	6. $\sqrt[5]{x^5} = \sqrt[5]{243}$ $x = 3$
7. $\sqrt[6]{x^6} = \pm\sqrt[6]{46{,}656}$ $x = \pm 6$	8. $\sqrt[3]{x^3} = \sqrt[3]{515}$ $x \approx 8.02$
9. $4y^2$	10. $x^4y^5\sqrt[3]{y}$
11. $ab^2c\sqrt[5]{4a^3b^4}$	12. $\lvert x^3\rvert y^4$

7.2 Operations with Radicals

1. $9\sqrt[4]{2}$	2. $10\sqrt[3]{10}$
3. $\sqrt[3]{64} = 4$	4. $(\sqrt[4]{8})(3\sqrt[4]{6}) - (2\sqrt[4]{3}) = 3\sqrt[4]{48} - 2\sqrt[4]{3}$ $= 6\sqrt[4]{3} - 2\sqrt[4]{3} = 4\sqrt[4]{3}$
5. $\frac{3}{\sqrt[3]{9}}\cdot\frac{(\sqrt[3]{9})^2}{(\sqrt[3]{9})^2} = \frac{3\sqrt[3]{81}}{9} = \frac{\cancel{9}\sqrt[3]{3}}{\cancel{9}} = \sqrt[3]{3}$	6. $\frac{6\sqrt[4]{8}}{\sqrt[4]{16}} = \frac{6}{\sqrt[4]{2}}$ $\frac{6}{\sqrt[4]{2}}\cdot\frac{(\sqrt[4]{2})^3}{(\sqrt[4]{2})^3} = \frac{6(\sqrt[4]{2})^3}{2} = 3(\sqrt[4]{2})^3 = 3\sqrt[4]{8}$

7.3 Solve Equations with Radicals

1. $(\sqrt[3]{x})^3 = 7^3$ $x = 343$	2. $3\sqrt[4]{x} = 18$ $\sqrt[4]{x} = 6$ $(\sqrt[4]{x})^4 = 6^4$ $x = 1{,}296$
3. $x - 4 = 49$ $x = 53$	4. $\sqrt{2x-1} = 3$ $2x - 1 = 9$ $x = 5$

5. $\sqrt[3]{2x+3} = 3$ $2x + 3 = 27$ $2x = 24$ $x = 12$	6. $\left(\sqrt{x-a}\right)^2 = b^2$ $x - a = b^2$ $x = b^2 + a$
7. $x^2 - 3x + 3 = 1$ $x^2 - 3x + 2 = 0$ $(x-1)(x-2) = 0$ $\{1,2\}$	8. $x + 3 = (x+3)^2$ $x + 3 = x^2 + 6x + 9$ $x^2 + 5x + 6 = 0$ $(x+2)(x+3) = 0$ $\{-2,-3\}$
9. $2x - 4 = (x-2)^2$ $2x - 4 = x^2 - 4x + 4$ $x^2 - 6x + 8 = 0$ $(x-2)(x-4) = 0$ $\{2,4\}$	10. $x^2 = 4(2x-3)$ $x^2 - 8x + 12 = 0$ $(x-2)(x-6) = 0$ $\{2,6\}$
11. $9x + 10 = x^2$ $x^2 - 9x - 10 = 0$ $(x+1)(x-10) = 0$ $\{\cancel{-1}, 10\}$ –1 is extraneous	12. $5x + 29 = (x+3)^2$ $5x + 29 = x^2 + 6x + 9$ $x^2 + x - 20 = 0$ $(x+5)(x-4) = 0$ $\{\cancel{-5}, 4\}$ –5 is extraneous
13. $x^3 - 2x^2 - 5 = (x-1)^3$ $x^3 - 2x^2 - 5 = x^3 - 3x^2 + 3x - 1$ $x^2 - 3x - 4 = 0$ $(x+1)(x-4) = 0$ $\{-1, 4\}$	14. $15x^4 + 81 = (2x)^4$ $15x^4 + 81 = 16x^4$ $x^4 - 81 = 0$ $x^4 = 81$ $\{\cancel{-3}, 3\}$ –3 is extraneous
15. $\sqrt{x+4} = \sqrt{x-3} + 1$ $\left(\sqrt{x+4}\right)^2 = \left(\sqrt{x-3} + 1\right)^2$ $x + 4 = x - 3 + 2\sqrt{x-3} + 1$ $2\sqrt{x-3} = 6$ $\sqrt{x-3} = 3$ $\left(\sqrt{x-3}\right)^2 = 3^2$ $x - 3 = 9$ $x = 12$	16. $\left(\sqrt{x+4}\right)^2 = \left(\sqrt{5x} - 2\right)^2$ $x + 4 = 5x - 4\sqrt{5x} + 4$ $-4x = -4\sqrt{5x}$ $x = \sqrt{5x}$ $x^2 = 5x$ $x^2 - 5x = 0$ $x(x-5) - 0$ $\{\cancel{0}, 5\}$ 0 is extraneous
17. $\left(\sqrt{x+3} + 1\right)^2 = \left(\sqrt{-2x}\right)^2$ $x + 3 + 2\sqrt{x+3} + 1 = -2x$ $2\sqrt{x+3} = -3x - 4$ $\left(2\sqrt{x+3}\right)^2 = (-3x-4)^2$ $4(x+3) = 9x^2 + 24x + 16$ $4x + 12 = 9x^2 + 24x + 16$ $9x^2 + 20x + 4 = 0$ $n^2 + 20n + 36 = 0$ $(n+18)(n+2) = 0$ $x = \frac{n}{9} = \left\{-2, \cancel{-\frac{2}{9}}\right\}$ $-\frac{2}{9}$ is extraneous	18. $\left(\sqrt{x+1} - \sqrt{x-4}\right)^2 = x - 7$ $x + 1 - 2\sqrt{x+1}\sqrt{x-4} + x - 4 = x - 7$ $2\sqrt{x+1}\sqrt{x-4} = x + 4$ $2\sqrt{x^2 - 3x - 4} = x + 4$ $4(x^2 - 3x - 4) = (x+4)^2$ $4x^2 - 12x - 16 = x^2 + 8x + 16$ $3x^2 - 20x - 32 = 0$ $n^2 - 20n - 96 = 0$ $(n-24)(n+4) = 0$ $x = \frac{n}{3} = \left\{8, \cancel{-\frac{4}{3}}\right\}$ $-\frac{4}{3}$ is extraneous

7.4 Graphs of Radical Functions

1. (1)

2. (2)

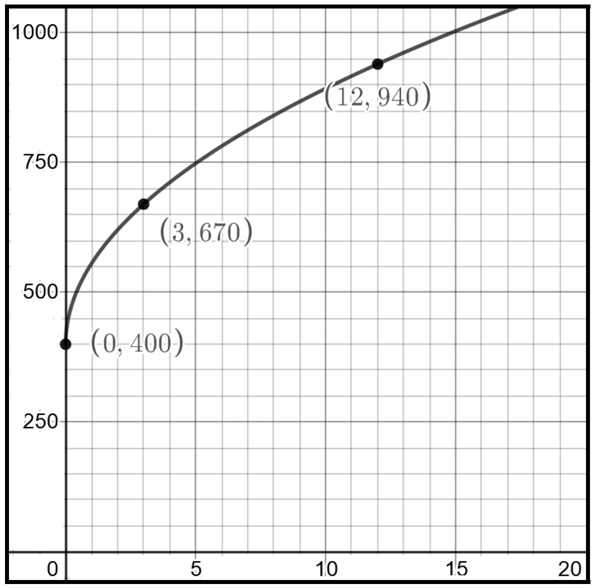
3. a) dilation by a factor of 2, translations 2 units left and 4 units down (in any order)
   b) $g(x) = 2\sqrt{x+2} - 4$

4.

(a)

x	y
0	400
3	670
6	781.8
9	867.7
12	940
15	1003.7

(b)



(c) 670
(d) 12

7.5 Negative Exponents

1. $\dfrac{1}{p^7}$

2. $\dfrac{1}{(5x)^2} = \dfrac{1}{25x^2}$

3. $1 + \dfrac{1}{3^2} = 1\frac{1}{9}$

4. $2^{-2} - 2^0 + 2^2 = \frac{1}{4} - 1 + 4 = 3\frac{1}{4}$

5. $-\dfrac{2m^3}{n^5}$

6. $5x^3$

7. $-\dfrac{3bc}{5}$

8. $\left(\dfrac{5y}{2x}\right)^3 = \dfrac{125y^3}{8x^3}$

9. $\left(\frac{3}{4}\right)^2 \cdot 4^2 = 9$

10. $\dfrac{a^6}{b^5}$

11. $\dfrac{3^{-2}}{(-2)^{-3}} = \dfrac{(-2)^3}{3^2} = -\dfrac{8}{9}$

12. $\dfrac{1}{5^{-2}a^3b^{-4}} = \dfrac{25b^4}{a^3}$

13. $\frac{y^6}{x^3}$	14. $\frac{x^4y^5}{3}$
15. $\frac{y^7}{2x^2}$	16. $\frac{3x^{-4}y^5}{(2x^3y^{-7})^{-2}} = \frac{3y^5(2x^3y^{-7})^2}{x^4} = \frac{3y^5(4x^6y^{-14})}{x^4} = \frac{12x^6y^{-9}}{x^4} = \frac{12x^2}{y^9}$

7.6 Rational Exponents

1. $x^{\frac{4}{5}}$	2. $x^{\frac{7}{4}}$
3. $3x^{-\frac{1}{5}}$	4. $\sqrt{x^3}$
5. $\frac{1}{\sqrt[3]{x^2}}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\right) = \frac{\sqrt[3]{x}}{x}$	6. $\left(\frac{27}{64}\right)^{-\frac{2}{3}} = \left(\frac{64}{27}\right)^{\frac{2}{3}} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$
7. $\left(\frac{x^2}{x^{\frac{1}{9}}}\right)^3 = \left(\frac{x^{\frac{18}{9}}}{x^{\frac{1}{9}}}\right)^3 = \left(x^{\frac{17}{9}}\right)^3 = x^{\frac{17}{3}} = \sqrt[3]{x^{17}}$	8. $\left(\frac{27x^4}{xy^{-\frac{2}{3}}}\right)^{\frac{1}{3}} = \left(27x^3y^{\frac{2}{3}}\right)^{\frac{1}{3}} = 3xy^{\frac{2}{9}} = 3x\sqrt[9]{y^2}$
9. $(9x^2y^6)^{-\frac{1}{2}} = \frac{1}{\sqrt{9x^2y^6}} = \frac{1}{3xy^3}$	10. $\left(x^{\frac{1}{2}}y^{-\frac{2}{3}}\right)^{-6} = x^{-3}y^4 = \frac{y^4}{x^3}$
11. $\left(\frac{x^{-5}}{x^{-9}}\right)^{\frac{1}{2}} = \left(\frac{x^9}{x^5}\right)^{\frac{1}{2}} = (x^4)^{\frac{1}{2}} = x^2$	12. $\frac{(m^6)^{-\frac{2}{3}}}{m^2} = \frac{1}{(m^6)^{\frac{2}{3}} \cdot m^2} = \frac{1}{m^4 \cdot m^2} = \frac{1}{m^6}$
13. $\frac{\sqrt[3]{x^2}}{\sqrt[6]{x}} = \frac{x^{\frac{2}{3}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{4}{6}}}{x^{\frac{1}{6}}} = x^{\frac{3}{6}} = x^{\frac{1}{2}} = \sqrt{x}$	14. $\frac{\sqrt{x^5}+x^3}{\sqrt[3]{x^7}} = \frac{x^{\frac{5}{2}}+x^3}{x^{\frac{7}{3}}} = \frac{x^{\frac{5}{2}}}{x^{\frac{7}{3}}} + \frac{x^3}{x^{\frac{7}{3}}} = \frac{x^{\frac{15}{6}}}{x^{\frac{14}{6}}} + \frac{x^{\frac{9}{3}}}{x^{\frac{7}{3}}} = x^{\frac{1}{6}} + x^{\frac{2}{3}} = \sqrt[6]{x} + \sqrt[3]{x^2}$
15. $2x^{\frac{3}{4}} + 5 = 133$ $2x^{\frac{3}{4}} = 128$ $x^{\frac{3}{4}} = 64$ $\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = 64^{\frac{4}{3}}$ $x = 256$	16. $x^{\frac{7}{10}} \cdot \sqrt{x} = 729$ $x^{\frac{7}{10}} \cdot x^{\frac{1}{2}} = 729$ $x^{\left(\frac{7}{10}+\frac{5}{10}\right)} = 729$ $x^{\frac{6}{5}} = 729$ $\left(x^{\frac{6}{5}}\right)^{\frac{5}{6}} = \pm\, 729^{\frac{5}{6}}$ $x = \pm 243$

17. $y^{\frac{2}{3}} = x^{\frac{5}{6}}$ $\left(y^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(x^{\frac{5}{6}}\right)^{\frac{3}{2}}$ $y = x^{\frac{5}{4}}$	18. $V = a^2\sqrt{b^{\frac{1}{3}}}$ $\frac{V}{a^2} = \sqrt{b^{\frac{1}{3}}}$ $\frac{V^2}{a^4} = b^{\frac{1}{3}}$ $\frac{V^6}{a^{12}} = b$

Chapter 8. Rational Functions

8.1 Undefined Expressions

1. $x + 2 = 0$, so $x = -2$	2. $3x + 1 = 0$, so $x = -\frac{1}{3}$
3. $x^2 - 4 = 0$ $x^2 = 4$ $x = \pm\sqrt{4} = \pm 2$	4. $x^2 - 4x - 12 = 0$ $(x+2)(x-6) = 0$ $x = \{-2,6\}$
5. $9 - x^2 = 0$ $-x^2 = -9$ $x^2 = 9$ $x = \pm 3$	6. $x^2 + 5x - 6 = 0$ $(x+6)(x-1) = 0$ $x = \{-6,1\}$
7. $x + 2 = 0$ or $x - 1 = 0$ $x = \{-2,1\}$	8. $2x^2 + 1 = 0$ $2x^2 = -1$ $x^2 = -\frac{1}{2}$ $x = \pm\sqrt{-\frac{1}{2}}$ (not real numbers) The expression is defined for all possible real values of *x*.

8.2 Simplify Rational Expressions

1. $x + 2$	2. $2x^2 + 3x + 1$
3. $3x - 9x^3$	4. $x^4 - 9x^2 + 1$
5. $3a^2b^2 - 6a$	6. $\frac{2x\cancel{(x-6)}}{\cancel{x-6}} = 2x$
7. $\frac{25\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} = \frac{25}{x+5}$	8. $\frac{\overset{2}{\cancel{6}}\cancel{(x+2)}}{\cancel{3}x\cancel{(x+2)}} = \frac{2}{x}$
9. $\frac{\cancel{(x+5)}(x+1)}{\cancel{(x+5)}(x-5)} = \frac{x+1}{x-5}$	10. $\frac{3x\cancel{(3x-5y)}}{(3x+5y)\cancel{(3x-5y)}} = \frac{3x}{3x+5y}$
11. $\frac{\cancel{(x+3)}(x-5)}{x\cancel{(x+3)}} = \frac{x-5}{x}$	12. $\frac{(x+2)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}} = \frac{x+2}{x-2}$
13. $\frac{2\left(x^2+5x-14\right)}{4(x+7)} = \frac{2\cancel{(x+7)}(x-2)}{4\cancel{(x+7)}} =$ $\frac{x-2}{2}$	14. $\frac{3-x}{2(x-3)} = \frac{-\cancel{(x-3)}}{2\cancel{(x-3)}} = -\frac{1}{2}$

15. $\frac{y-x}{(x+y)(x-y)} = \frac{-\cancel{(x-y)}}{(x+y)\cancel{(x-y)}} = -\frac{1}{x+y}$	16. $\frac{x(x-9)}{5x(9-x)} = \frac{x\cancel{(x-9)}}{-5x\cancel{(x-9)}} = -\frac{1}{5}$
17. $\frac{3y(y-4)}{y^2(4-y)} = \frac{3y\cancel{(y-4)}}{-y^2\cancel{(y-4)}} = -\frac{3}{y}$	18. $\frac{(xy+3)(xy-3)}{3-xy} = \frac{(xy+3)\cancel{(xy-3)}}{-\cancel{(xy-3)}} = -(xy+3) = -xy-3$
19. $\frac{x^2+8x+15}{x+5} = \frac{\cancel{(x+5)}(x+3)}{\cancel{x+5}} = x+3$	20. Base: $14x+21-2(4x+6)=6x+9$ $\frac{6x+9}{14x+21} = \frac{3\cancel{(2x+3)}}{7\cancel{(2x+3)}} = \frac{3}{7}$

8.3 Multiply and Divide Rational Expressions

1. $\frac{7x^2}{3}\cdot\frac{9}{14x} = \frac{x}{1}\cdot\frac{3}{2} = \frac{3x}{2}$	2. $\frac{4x^2}{7y^2}\cdot\frac{21y^3}{20x^4} = \frac{1}{1}\cdot\frac{3y}{5x^2} = \frac{3y}{5x^2}$
3. $\frac{x^2-1}{x}\cdot\frac{4x^2}{x+1} = \frac{\cancel{(x+1)}(x-1)}{x}\cdot\frac{4x^2}{\cancel{x+1}} = \frac{x-1}{1}\cdot\frac{4x}{1} = 4x(x-1) = 4x^2-4x$	4. $\frac{4x}{x-1}\cdot\frac{x^2-1}{3x+3} = \frac{4x}{\cancel{x-1}}\cdot\frac{\cancel{(x+1)}\cancel{(x-1)}}{3\cancel{(x+1)}} = \frac{4x}{3}$
5. $\frac{x^2-1}{x+1}\cdot\frac{x+3}{3x-3} = \frac{\cancel{(x+1)}\cancel{(x-1)}}{\cancel{x+1}}\cdot\frac{x+3}{3\cancel{(x-1)}} = \frac{x+3}{3}$	6. $\frac{x+2}{2}\cdot\frac{4x+20}{x^2+6x+8} = \frac{\cancel{x+2}}{2}\cdot\frac{4(x+5)}{(x+4)\cancel{(x+2)}} = \frac{2(x+5)}{x+4} = \frac{2x+10}{x+4}$
7. $\frac{x+2}{3x+3}\cdot\frac{x^2+5x+4}{2x+4} = \frac{\cancel{x+2}}{3\cancel{(x+1)}}\cdot\frac{(x+4)\cancel{(x+1)}}{2\cancel{(x+2)}} = \frac{x+4}{6}$	8. $\frac{x^2-9}{x^2+9x+18}\cdot\frac{x}{x^2-3x} = \frac{\cancel{(x+3)}\cancel{(x-3)}}{(x+6)\cancel{(x+3)}}\cdot\frac{x}{x\cancel{(x-3)}} = \frac{1}{x+6}$
9. $\frac{x}{x+3}\div\frac{3x}{x^2-9} = \frac{x}{x+3}\cdot\frac{x^2-9}{3x} = \frac{x}{\cancel{x+3}}\cdot\frac{\cancel{(x+3)}(x-3)}{3x} = \frac{x-3}{3}$	10. $\frac{x}{x+4}\div\frac{2x}{x^2-16} = \frac{x}{x+4}\cdot\frac{x^2-16}{2x} = \frac{x}{\cancel{x+4}}\cdot\frac{\cancel{(x+4)}(x-4)}{2x} = \frac{x-4}{2}$

11. $\frac{9x^2}{x^2+12x+36} \div \frac{12x}{x^2+6x} =$ $\frac{9x^2}{x^2+12x+36} \cdot \frac{x^2+6x}{12x} =$ $\frac{9x^2}{\cancel{(x+6)}(x+6)} \cdot \frac{x\cancel{(x+6)}}{12x} =$ $\frac{3x^2}{4(x+6)} = \frac{3x^2}{4x+24}$	12. $\frac{3x+6}{4x+12} \div \frac{x^2-4}{x+3} =$ $\frac{3x+6}{4x+12} \cdot \frac{x+3}{x^2-4} =$ $\frac{3\cancel{(x+2)}}{4\cancel{(x+3)}} \cdot \frac{\cancel{x+3}}{\cancel{(x+2)}(x-2)} =$ $\frac{3}{4(x-2)} = \frac{3}{4x-8}$
13. $\frac{2x^2-8x-42}{6x^2} \cdot \frac{x^2-3x}{x^2-9} =$ $\frac{2\cancel{(x+3)}(x-7)}{6x^2} \cdot \frac{x\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x-3)}} = \frac{x-7}{3x}$	14. $\frac{3x^2+9x}{x^2+5x+6} \cdot \frac{x^2-x-6}{x^2-9} =$ $\frac{3x\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x+2)}} \cdot \frac{\cancel{(x+2)}\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} = \frac{3x}{x+3}$
15. $\frac{x^2+9x+14}{x^2-49} \cdot \frac{x^2+x-56}{3x+6} =$ $\frac{\cancel{(x+7)}\cancel{(x+2)}}{\cancel{(x+7)}\cancel{(x-7)}} \cdot \frac{(x+8)\cancel{(x-7)}}{3\cancel{(x+2)}} = \frac{x+8}{3}$	16. $\frac{2x-6}{2x+4} \cdot \frac{x^2+2x}{x^2+2x-15} =$ $\frac{2\cancel{(x-3)}}{2\cancel{(x+2)}} \cdot \frac{x\cancel{(x+2)}}{(x+5)\cancel{(x-3)}} = \frac{x}{x+5}$
17. $\frac{x^2+2x-15}{x^2-4x-45} \cdot \frac{x^2-5x-36}{x^2+x-12} =$ $\frac{\cancel{(x+5)}\cancel{(x-3)}}{\cancel{(x+5)}\cancel{(x-9)}} \cdot \frac{\cancel{(x+4)}\cancel{(x-9)}}{\cancel{(x+4)}\cancel{(x-3)}} = 1$	18. $\frac{x^2+4x+3}{2x^2-x-10} \cdot \frac{2x^2+4x^3}{x^2+3x} \cdot \frac{x^2+4x+4}{x^2+3x+2} =$ $\frac{\cancel{(x+3)}\cancel{(x+1)}}{(2x-5)\cancel{(x+2)}} \cdot \frac{2x^2(1+2x)}{x\cancel{(x+3)}} \cdot \frac{\cancel{(x+2)}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x+1)}} =$ $\frac{2x(2x+1)}{2x-5} = \frac{4x^2+2x}{2x-5}$

8.4 Add and Subtract Rational Expressions

1. $\frac{8}{x^2+1}$	2. $\frac{4x^2}{x-2}$
3. $\frac{x+12}{2x+4}$	4. $\frac{4}{5x}$
5. $\frac{2y+10}{y+5} = \frac{2\cancel{(y+5)}}{\cancel{y+5}} = 2$	6. $\frac{x^2}{x+1} + \frac{6x+5}{x+1} = \frac{x^2+6x+5}{x+1} =$ $\frac{(x+5)\cancel{(x+1)}}{\cancel{x+1}} = x+5$

7. $\underline{2^2}\cdot 3\cdot \underline{7}\cdot \underline{a^3}\cdot b$ and $\underline{3^3}\cdot a^2\cdot \underline{b^2}$ gives us an LCM of $2^2\cdot 3^3\cdot 7\cdot a^3\cdot b^2$, or $756a^3b^2$	8. Factor as $(x+2)(x-2)$ and $(x+2)$. So, LCM is $(x+2)(x-2)$.
9. Factor as $2x^2$, $x(x+1)$, and $3x(x^2+1)$. So, LCM is $6x^2(x+1)(x^2+1)$.	10. Factor as $(x+4)(x-1)$, $(x-1)^2$, and $(x+4)(x+2)$. So, the LCM is $(x-1)^2(x+4)(x+2)$.
11. LCM is 12 $\frac{5x}{6}\left(\frac{2}{2}\right)+\frac{x}{4}\left(\frac{3}{3}\right)=\frac{10x}{12}+\frac{3x}{12}=\frac{13x}{12}$	12. LCM is $5x$ $\frac{3}{x}\left(\frac{5}{5}\right)-\frac{2}{5}\left(\frac{x}{x}\right)=\frac{15}{5x}-\frac{2x}{5x}=\frac{15-2x}{5x}$
13. LCM is $21n$ $\frac{3}{7n}\left(\frac{3}{3}\right)-\frac{7}{3n}\left(\frac{7}{7}\right)=\frac{9}{21n}-\frac{49}{21n}=-\frac{40}{21n}$	14. LCM is $2x$ $\frac{a}{x}\left(\frac{2}{2}\right)+\frac{b}{2x}=\frac{2a}{2x}+\frac{b}{2x}=\frac{2a+b}{2x}$
15. LCM is $3b$ $\frac{a}{b}\left(\frac{3}{3}\right)-\frac{1}{3}\left(\frac{b}{b}\right)=\frac{3a}{3b}-\frac{b}{3b}=\frac{3a-b}{3b}$	16. LCM is $12x^2$ $\frac{7}{12x}\left(\frac{x}{x}\right)-\frac{y}{6x^2}\left(\frac{2}{2}\right)=\frac{7x}{12x^2}-\frac{2y}{12x^2}=$ $\frac{7x-2y}{12x^2}$
17. $\frac{6}{y-5}-\frac{y+5}{y^2-25}=$ $\frac{6}{y-5}-\frac{y+5}{(y+5)(y-5)}=$ $\frac{6}{y-5}-\frac{1}{y-5}=\frac{5}{y-5}$	18. $\frac{5}{x(x+5)}+\frac{x}{x+5}$ LCM is $x(x+5)$ $\frac{5}{x(x+5)}+\frac{x}{x+5}\left(\frac{x}{x}\right)=$ $\frac{5}{x(x+5)}+\frac{x^2}{x(x+5)}=$ $\frac{x^2+5}{x(x+5)}=\frac{x^2+5}{x^2+5x}$
19. $\frac{4x}{(x+1)(x-1)}-\frac{3x}{2(x+1)}$ LCM is $2(x+1)(x-1)$ $\frac{4x}{(x+1)(x-1)}\left(\frac{2}{2}\right)-\frac{3x}{2(x+1)}\left(\frac{x-1}{x-1}\right)=$ $\frac{8x}{2(x+1)(x-1)}-\frac{3x^2-3x}{2(x+1)(x-1)}=$ $\frac{-3x^2+11x}{2(x+1)(x-1)}=\frac{-3x^2+11x}{2(x^2-1)}=$ $\frac{-3x^2+11x}{2x^2-2}$	20. $\frac{6x\cancel{(x+1)}}{\cancel{(x+1)}(x-2)}+\frac{(x+1)(x+1)}{3x(x-2)}=$ $\frac{6x}{(x-2)}+\frac{(x+1)(x+1)}{3x(x-2)}$ LCM is $3x(x-2)$ $\frac{6x}{(x-2)}\left(\frac{3x}{3x}\right)+\frac{(x+1)(x+1)}{3x(x-2)}=$ $\frac{18x^2}{3x(x-2)}+\frac{(x+1)(x+1)}{3x(x-2)}=$ $\frac{19x^2+2x+1}{3x(x-2)}=\frac{19x^2+2x+1}{3x^2-6x}$

21. $\frac{y-5}{1}+\frac{3}{y+2}=\frac{y-5}{1}\left(\frac{y+2}{y+2}\right)+\frac{3}{y+2}=$ $\frac{(y-5)(y+2)+3}{y+2}=\frac{y^2-3y-7}{y+2}$	22. $\frac{3}{a-1}+\frac{3}{1-a}=\frac{3}{a-1}-\frac{3}{a-1}=$ $\frac{0}{a-1}=0$
23. $\frac{3x}{2x-6}+\frac{9}{6-2x}=\frac{3x}{2x-6}-\frac{9}{2x-6}=$ $\frac{3x-9}{2x-6}=\frac{3\cancel{(x-3)}}{2\cancel{(x-3)}}=\frac{3}{2}$	24. $\frac{x}{x-1}-\frac{1}{2-2x}=\frac{x}{x-1}-\frac{1}{2(1-x)}=$ $\frac{x}{x-1}+\frac{1}{2(x-1)}=\frac{x}{x-1}\left(\frac{2}{2}\right)+\frac{1}{2(x-1)}=$ $\frac{2x}{2(x-1)}+\frac{1}{2(x-1)}=\frac{2x+1}{2(x-1)}=\frac{2x+1}{2x-2}$

8.5 Simplify Complex Fractions

1. $\frac{x^2}{\frac{1}{x}}=x^2\div\frac{1}{x}=x^2\cdot x=x^3$	2. $\frac{\frac{2x}{y}}{\frac{4x}{y^2}}=\frac{2x}{y}\div\frac{4x}{y^2}=\frac{2x}{y}\cdot\frac{y^2}{4x}=\frac{y}{2}$
3. LCM is xy $\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}\left(\frac{xy}{xy}\right)=\frac{y+x}{y-x}$	4. LCM is x^2 $\frac{1-\frac{1}{x^2}}{1+\frac{1}{x}}\left(\frac{x^2}{x^2}\right)=\frac{x^2-1}{x^2+x}=\frac{\cancel{(x+1)}(x-1)}{x\cancel{(x+1)}}=$ $\frac{x-1}{x}$
5. LCM is $(x+1)(x-1)$ $\frac{6-\frac{x}{x-1}}{4-\frac{x}{x+1}}\left(\frac{(x+1)(x-1)}{(x+1)(x-1)}\right)=$ $\frac{6(x+1)(x-1)-x(x+1)}{4(x+1)(x-1)-x(x-1)}=$ $\frac{6x^2-6-x^2-x}{4x^2-4-x^2+x}=\frac{5x^2-x-6}{3x^2+x-4}$ *[Note: both the numerator and denominator can be factored, but there are no common factors.]*	6. LCM is $x-5$ $1-\frac{1}{1-\frac{1}{x-5}}\left(\frac{x-5}{x-5}\right)=1-\frac{x-5}{x-5-1}$ $=1-\frac{x-5}{x-6}=\frac{x-6}{x-6}-\frac{x-5}{x-6}=\frac{-1}{x-6}$

8.6 Solve Rational Equations

1. $16\left(\frac{x}{16}\right)+16\left(\frac{1}{4}\right)=16\left(\frac{1}{2}\right)$ $x+4=8$ $x=4$	2. $6\left(\frac{x}{2}\right)+6\left(\frac{x}{6}\right)=6(2)$ $3x+x=12$ $4x=12$ $x=3$
3. $5\left(\frac{3}{5}x\right)+5\left(\frac{2}{5}\right)=5(4)$ $3x+2=20$ $3x=18$ $x=6$	4. $4\left(\frac{3}{4}x\right)+4(2)=4\left(\frac{5}{4}x\right)-4(6)$ $3x+8=5x-24$ $32=2x$ $16=x$
5. $12\left(\frac{3}{4}x\right)=12\left(\frac{1}{3}x\right)+12(5)$ $9x=4x+60$ $5x=60$ $x=12$	6. $6n\left(\frac{5}{n}\right)-6n\left(\frac{1}{2}\right)=6n\left(\frac{3}{6n}\right)$ $30-3n=3$ $-3n=-27$ $n=9$
7. $15\left(\frac{2x}{5}\right)+15\left(\frac{1}{3}\right)=15\left(\frac{7x-2}{15}\right)$ $6x+5=7x-2$ $7=x$	8. $x\left(\frac{2}{x}\right)-x(3)=x\left(\frac{26}{x}\right)$ $2-3x=26$ $-3x=24$ $x=-8$
9. $6\left(\frac{2x}{3}\right)+6\left(\frac{x}{6}\right)=6(5)$ $4x+x=30$ $5x=30$ $x=6$	10. $21\left(\frac{1}{7}\right)+21\left(\frac{2x}{3}\right)=21\left(\frac{15x-3}{21}\right)$ $3+14x=15x-3$ $6=x$
11. $6\left(\frac{x}{3}\right)+6\left(\frac{x+1}{2}\right)=6(x)$ $2x+3(x+1)=6x$ $2x+3x+3=6x$ $5x+3=6x$ $3=x$	12. $12x\left(\frac{8}{3x}\right)-12x\left(\frac{x-1}{12}\right)=12x\left(\frac{1}{6x}\right)$ $32-x^2+x=2$ $x^2-x-30=0$ $(x+5)(x-6)=0$ $x=\{-5,6\}$
13. $4\cdot\frac{3}{4}(x+3)=4(9)$ $3(x+3)=36$ $3x+9=36$ $3x=27$ $x=9$	14. $5\cdot\frac{3}{5}(x+2)=5(x-4)$ $3(x+2)=5(x-4)$ $3x+6=5x-20$ $26=2x$ $13=x$
15. $10\left(\frac{m}{5}\right)+10\left(\frac{3(m-1)}{2}\right)=10\cdot 2(m-3)$ $2m+15(m-1)=20(m-3)$ $2m+15m-15=20m-60$ $17m-15=20m-60$ $45=3m$ $15=m$	16. $x^2(1)-x^2\left(\frac{6}{x^2}\right)=x^2\left(\frac{1}{x}\right)$ $x^2-6=x$ $x^2-x-6=0$ $(x+2)(x-3)=0$ $x=\{-2,3\}$

17. $x(x+2)\left[\frac{4x}{x+2} - \frac{12}{x} = 1\right]$

$4x^2 - 12(x+2) = x(x+2)$

$4x^2 - 12x - 24 = x^2 + 2x$

$3x^2 - 14x - 24 = 0$

$(3x+4)(x-6) = 0$

$x = \left\{-\frac{4}{3}, 6\right\}$

18. $3x(x+1)\left[\frac{2}{3x} + \frac{4}{x} = \frac{7}{x+1}\right]$

$2(x+1) + 12(x+1) = 21x$

$2x + 2 + 12x + 12 = 21x$

$14x + 14 = 21x$

$14 = 7x$

$2 = x$

19. $3(x+3)(x-4)\left[\frac{3}{x+3} + \frac{2}{x-4} = \frac{4}{3}\right]$

$9(x-4) + 6(x+3) = 4(x+3)(x-4)$

$9x - 36 + 6x + 18 = 4(x^2 - x - 12)$

$15x - 18 = 4x^2 - 4x - 48$

$4x^2 - 19x - 30 = 0$

$(4x+5)(x-6) = 0$

$x = \left\{-\frac{5}{4}, 6\right\}$

20.

$(x+5)(x-5)\left[\frac{x}{x+5} + \frac{9}{x-5} = \frac{50}{(x+5)(x-5)}\right]$

$x(x-5) + 9(x+5) = 50$

$x^2 - 5x + 9x + 45 = 50$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = \{\cancel{-5}, 1\}$

-5 is an extraneous root, since

$x + 5 = (-5) + 5 = 0$

21.

$(x+3)(x-4)\left[\frac{x}{x-4} - \frac{1}{x+3} = \frac{28}{(x+3)(x-4)}\right]$

$x(x+3) - (x-4) = 28$

$x^2 + 3x - x + 4 = 28$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = \{-6, \cancel{4}\}$

4 is an extraneous root.

22. $x(x+1)\left[\frac{4}{x} - \frac{3}{x+1} = 7\right]$

$4(x+1) - 3x = 7x(x+1)$

$4x + 4 - 3x = 7x^2 + 7x$

$7x^2 + 6x - 4 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(7)(-4)}}{2(7)} = \frac{-6 \pm \sqrt{148}}{14} =$

$\frac{-6 \pm 2\sqrt{37}}{14} = \frac{-3 \pm \sqrt{37}}{7}$

23. $3x\left[\frac{x+3}{3} + \frac{x+3}{x} = 2\right]$

$x^2 + 3x + 3x + 9 = 6x$

$x^2 + 9 = 0$

$x^2 = -9$

$x = \pm 3i$

24. $x\left[x + \frac{5}{x} = 2\right]$

$x^2 + 5 = 2x$

$x^2 - 2x = -5$

$x^2 - 2x + 1 = -5 + 1$

$(x-1)^2 = -4$

$x - 1 = \pm\sqrt{-4}$

$x - 1 = \pm 2i$

$x = 1 \pm 2i$

25. $x\left[x = 2 - \frac{8}{x}\right]$ $x^2 = 2x - 8$ $x^2 - 2x = -8$ $x^2 - 2x + 1 = -8 + 1$ $(x-1)^2 = -7$ $x - 1 = \pm\sqrt{-7}$ $x - 1 = \pm i\sqrt{7}$ $x = 1 \pm i\sqrt{7}$	26. $x\left[2x + \frac{3}{x} = -2\right]$ $2x^2 + 3 = -2x$ $2x^2 + 2x + 3 = 0$ $x^2 + x + \frac{3}{2} = 0$ $x^2 + x = -\frac{3}{2}$ $x^2 + x + \frac{1}{4} = -\frac{3}{2} + \frac{1}{4}$ $\left(x + \frac{1}{2}\right)^2 = -\frac{5}{4}$ $x + \frac{1}{2} = \pm\sqrt{-\frac{5}{4}}$ $x = -\frac{1}{2} \pm \frac{i\sqrt{5}}{2}$
27. $72x\left[\frac{x}{8} + \frac{8}{9x} = 0\right]$ $9x^2 + 64 = 0$ $x^2 = -\frac{64}{9}$ $x = \pm\sqrt{-\frac{64}{9}}$ $x = \pm\frac{8i}{3}$	28. $x^2\left[2 + \frac{5}{x^2} = \frac{6}{x}\right]$ $2x^2 + 5 = 6x$ $2x^2 - 6x = -5$ $x^2 - 3x = -\frac{5}{2}$ $x^2 - 3x + \frac{9}{4} = -\frac{5}{2} + \frac{9}{4}$ $\left(x - \frac{3}{2}\right)^2 = -\frac{1}{4}$ $x - \frac{3}{2} = \pm\sqrt{-\frac{1}{4}}$ $x - \frac{3}{2} = \pm\frac{1}{2}i$ $x = \frac{3}{2} \pm \frac{1}{2}i$ or $x = \frac{3 \pm i}{2}$

8.7 Model Rational Expressions and Equations

1. $\frac{x - 3 + 7}{x + 7} = \frac{3}{4}$ $4(x+4) = 3(x+7)$ $4x + 16 = 3x + 21$ $x = 5$ Original fraction is $\frac{2}{5}$.	2. $x + \frac{64}{x} = 16$ $x\left[x + \frac{64}{x} = 16\right]$ $x^2 + 64 = 16x$ $x^2 - 16x + 64 = 0$ $(x-8)^2 = 0$ $x = 8$

3. $\frac{5}{x} = \frac{7}{2x} + 3$

$2x\left[\frac{5}{x} = \frac{7}{2x} + 3\right]$

$10 = 7 + 6x$

$3 = 6x$

$x = \frac{1}{2}$

4. $\frac{6}{x} + \frac{7}{x+2} = 1$

$x(x+2)\left[\frac{6}{x} + \frac{7}{x+2} = 1\right]$

$6(x+2) + 7x = x(x+2)$

$13x + 12 = x^2 + 2x$

$x^2 - 11x - 12 = 0$

$(x+1)(x-12) = 0$

$x = \{\cancel{-1}, 12\}$

Reject -1. Numbers are 12 and 14.

5. $\frac{number\ of\ pets}{number\ of\ students} = 2$

$\frac{1 \cdot 6 + 2 \cdot 10 + 4k + 5 \cdot 2}{22 + k} = 2$

$\frac{36 + 4k}{22 + k} = 2$

$36 + 4k = 2(22 + k)$

$36 + 4k = 44 + 2k$

$2k = 8$

$k = 4$

6. Written as a fraction, $2.25 = 2\frac{1}{4} = \frac{9}{4}$.

Therefore, $\frac{1}{R_T} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$.

$9x(x+3)\left[\frac{1}{x} + \frac{1}{x+3} = \frac{4}{9}\right]$

$9(x+3) + 9x = 4x(x+3)$

$9x + 27 + 9x = 4x^2 + 12x$

$4x^2 - 6x - 27 = 0$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(-27)}}{2(4)} = \frac{6 \pm \sqrt{468}}{8}$

Positive solution $x \approx 3.5$.

7. $\frac{1}{6} + \frac{1}{4} = \frac{1}{x}$

$12x\left[\frac{1}{6} + \frac{1}{4} = \frac{1}{x}\right]$

$2x + 3x = 12$

$5x = 12$

$x = \frac{12}{5} = 2.4$ hrs

8. $\frac{1}{20} + \frac{1}{30} = \frac{1}{x}$

$60x\left[\frac{1}{20} + \frac{1}{30} = \frac{1}{x}\right]$

$3x + 2x = 60$

$5x = 60$

$x = 12$ mins

$12 \times 50 = 600$ mins or 10 hrs

9. $\frac{1}{c} + \frac{1}{2c} = \frac{1}{5}$

$10c\left[\frac{1}{c} + \frac{1}{2c} = \frac{1}{5}\right]$

$10 + 5 = 2c$

$15 = 2c$

$c = \frac{15}{2} = 7.5$ hrs

10. $\frac{1}{x-5} + \frac{1}{x} = \frac{1}{6}$

$6x(x-5)\left[\frac{1}{x-5} + \frac{1}{x} = \frac{1}{6}\right]$

$6x + 6(x-5) = x(x-5)$

$6x + 6x - 30 = x^2 - 5x$

$x^2 - 17x + 30 = 0$

$x = \frac{17 \pm \sqrt{17^2 - 4(1)(30)}}{2(1)} = \frac{17 \pm \sqrt{169}}{2} = \frac{17 \pm 13}{2}$

$x = \{\cancel{2}, 15\}$

Reject $x = 2$ because $x - 5 > 0$.

Faster machine takes $15 - 5 = 10$ hrs.

11.

	D	R	T
moped	40	$x+20$	$\frac{40}{x+20}$
bicycle	15	x	$\frac{15}{x}$

$\frac{40}{x+20} = \frac{15}{x}$
$40x = 15(x+20)$
$40x = 15x + 300$
$25x = 300$
$x = 12$
Bicycle is 12 mph, moped is 32 mph

12.

	D	R	T
upstream	4	$5-c$	$\frac{4}{5-c}$
downstream	16	$5+c$	$\frac{16}{5+c}$

$\frac{4}{5-c} = \frac{16}{5+c}$
$4(5+c) = 16(5-c)$
$20 + 4c = 80 - 16c$
$20c = 60$
$c = 3$ mph

13.

	D	R	T
with wind	1656	$x+12$	$\frac{1656}{x+12}$
against wind	3168	$x-12$	$\frac{3168}{x-12}$

$\frac{1656}{x+12} = \left(\frac{1}{2}\right) \cdot \frac{3168}{x-12}$
$\frac{3312}{x+12} = \frac{3168}{x-12}$
$3312(x-12) = 3168(x+12)$
$3312x - 39744 = 3168x + 38016$
$144x = 77760$
$x = 540$ mph

14.

	D	R	T
car	120	$x-100$	$\frac{120}{x-100}$
train	120	x	$\frac{120}{x}$

75 mins = $\frac{5}{4}$ hours
$\frac{120}{x} = \frac{120}{x-100} - \frac{5}{4}$
$4x(x-100)\left[\frac{120}{x} = \frac{120}{x-100} - \frac{5}{4}\right]$
$480(x-100) = 480x - 5x(x-100)$
$480x - 48{,}000 = 480x - 5x^2 + 500x$
$5x^2 - 500x - 48{,}000 = 0$
$x^2 - 100x - 9{,}600 = 0$
$(x-160)(x+60) = 0$
$x = \{160, \cancel{-60}\}$ reject negative value
Train travels 160 mph.

8.8 Graphs of Rational Functions

1. $x = -2$ and $x = -9$

2. This is in $y = \frac{a}{x-h} + k$ form.
Vertical asymptote at $x = -1$.
Horizontal asymptote at $y = 3$.

3. Degrees of numerator and denominator are equal (2), and leading coefficients are both 1. Asymptote is at $y = \frac{1}{1} = 1$.	4. The degree of the numerator (1) is smaller than the degree of the denominator (2). So, there is an asymptote at $y = 0$.
5. There is no horizontal asymptote because the degree of the numerator (2) is greater than the degree of the denominator (1).	6. $2x + 1 = 0$ Vertical asymptote at $x = -\frac{1}{2}$. Horizontal asymptote at $y = \frac{4}{2} = 2$.
7. (2) Vertical asymptotes at –3 and 3, which are the roots of $x^2 - 9 = 0$, and a horizontal asymptote at 2.	
8. $4x = 0$ x-intercept is 0 $f(0) = \frac{4(0)}{2(0) + 1} = 0$ y-intercept is 0	9. $x - 1 = 0$ x-intercept is 1 $g(0) = \frac{0 - 1}{0^2 - 4} = \frac{1}{4}$ y-intercept is $\frac{1}{4}$
10. $x^2 + 11x + 18 = 0$ $(x + 2)(x + 9) = 0$ x-intercepts at -2 and -9 $h(0) = \frac{0^2 + 11(0) + 18}{0^2 + 3(0)} = \frac{18}{0}$ undefined There is no y-intercept	11. $\frac{x^2}{2x + 1} - 1 = 0$ $\frac{x^2}{2x + 1} = 1$ $x^2 = 2x + 1$ $x^2 - 2x = 1$ $x^2 - 2x + 1 = 2$ $(x - 1)^2 = 2$ $x - 1 = \pm\sqrt{2}$ $x = 1 \pm \sqrt{2}$ x-intercepts at $1 \pm \sqrt{2}$ $j(0) = \frac{0^2}{2(0) + 1} - 1 = -1$ y-intercept at –1

9.1 Solve Simple Exponential Equations

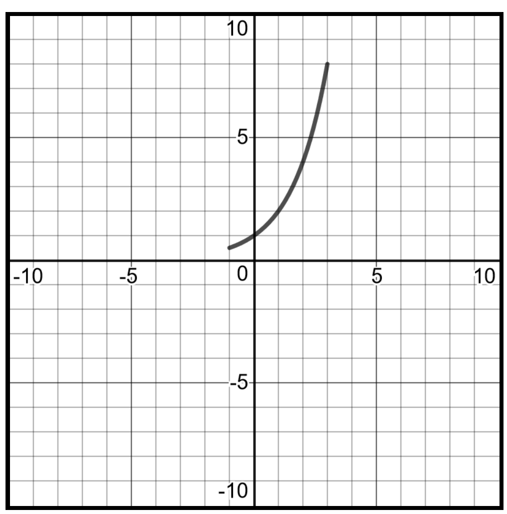
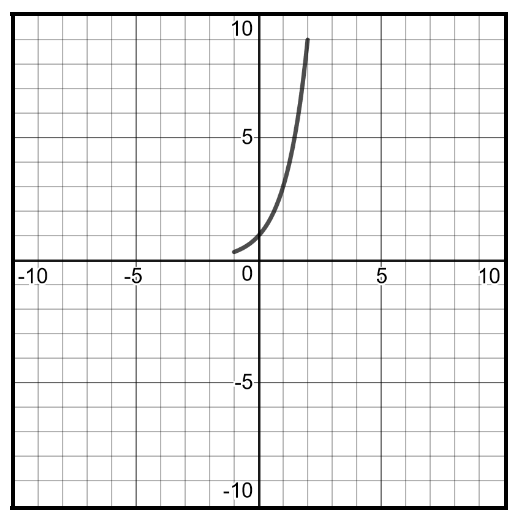
1. $4^x = 64$ $4^x = 4^3$ $x = 3$	2. $3^x = 81$ $3^x = 3^4$ $x = 4$
3. $5^x = 25$ $5^x = 5^2$ $x = 2$	4. $\left(\frac{1}{2}\right)^x = \frac{1}{8}$ $\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^3$ $x = 3$
5. $2^{x+1} = 2^3$ $x + 1 = 3$ $x = 2$	6. $3^{2x-2} = 3^4$ $2x - 2 = 4$ $x = 3$
7. $3^{x-3} = 3^0$ $x - 3 = 0$ $x = 3$	8. $4^{3x+5} = 4^2$ $3x + 5 = 2$ $x = -1$
9. $3^{x+1} = 27$ $3^{x+1} = 3^3$ $x + 1 = 3$ $x = 2$	10. $(2^2)^4 = 2^{3x-1}$ $2^8 = 2^{3x-1}$ $8 = 3x - 1$ $x = 3$
11. $2x = x + 4$ $x = 4$	12. $2^x = 2^{2(x+1)}$ $x = 2(x + 1)$ $x = 2x + 2$ $x = -2$
13. $2^{2x} = 2^{3x+1}$ $2x = 3x + 1$ $x = -1$	14. $2^{6x} = 2^{x+5}$ $6x = x + 5$ $x = 1$
15. $3^{x-5} = 3^{2(x-3)}$ $x - 5 = 2x - 6$ $x = 1$	16. $2^{3(x-2)} = 2^x$ $3x - 6 = x$ $x = 3$
17. $2^1 = 2^{2x+1}$ $1 = 2x + 1$ $x = 0$	18. $2^{3(x-2)} = 2^{2x}$ $3x - 6 = 2x$ $x = 6$

9.2 Rewrite Exponential Expressions

1. $5^{2x} = (5^2)^x = 25^x$	2. $10(1.1)^{5x} = 10(1.1^5)^x = 10(1.61051)^x$
3. $2^{3x+2} = (2^3)^x \cdot 2^2 = 4(8)^x$	4. $4(3)^{x+1} = 4(3)^x(3^1) = 12(3)^x$

5. $\frac{3^{5x+1}}{9^x} = \frac{3^{5x+1}}{3^{2x}} = 3^{5x+1-2x} = 3^{3x+1}$ $= 3^{3x} \cdot 3^1 = 3(3^{3x}) = 3(27)^x$	6. $3^{2x-3} = \frac{(3^2)^x}{3^3} = \frac{9^x}{27}$
7. $5\left(4^{\frac{x}{2}+2}\right) \cdot 3^{3x}$ $= 5\left(4^{\frac{1}{2}}\right)^x \cdot (4^2) \cdot (3^3)^x$ $= 5(\sqrt{4})^x \cdot (16) \cdot (27)^x$ $= (5 \cdot 16)(2^x)(27^x) = 80(54)^x$	8. $\frac{2^{4y+5}}{16^y} = \frac{2^{4y+5}}{2^{4y}} =$ $2^{4y+5-4y} = 2^5 = 32$
9. $4^x = \left(4^{\frac{1}{3}}\right)^{3x}$, so $k = 4^{\frac{1}{3}} = \sqrt[3]{4}$	10. $2^{x+3} - 2^x = 2^x(2^3) - 2^x = 2^x(2^3 - 1)$ $= 7 \cdot 2^x$, so $k = 7$
11. $\frac{1}{4}(2^x) = \frac{1}{4}\left(2^{4 \cdot \frac{x}{4}}\right) = \frac{1}{4}\left(16^{\frac{x}{4}}\right) = \frac{1}{4}\left(16^{\frac{x}{4}-2+2}\right) = \frac{1}{4}\left(16^{\frac{x}{4}-2}\right)(16^2) = \frac{1}{4}\left(16^{\frac{x}{4}-2}\right)(256)$ $= 64\left(16^{\frac{x}{4}-2}\right)$, so $k = 64$ and $b = 16$.	

9.3 Graphs of Exponential Functions

1. 	2. 

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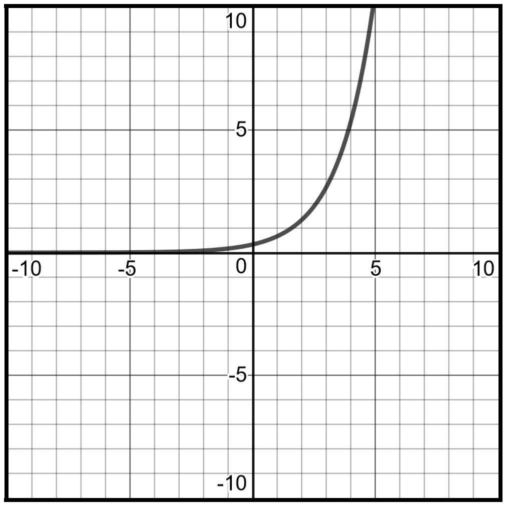
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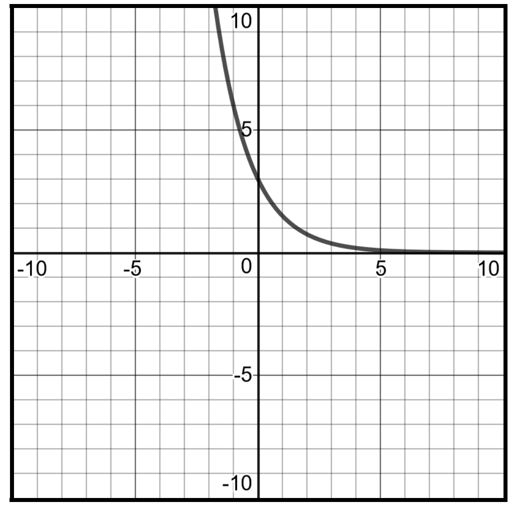
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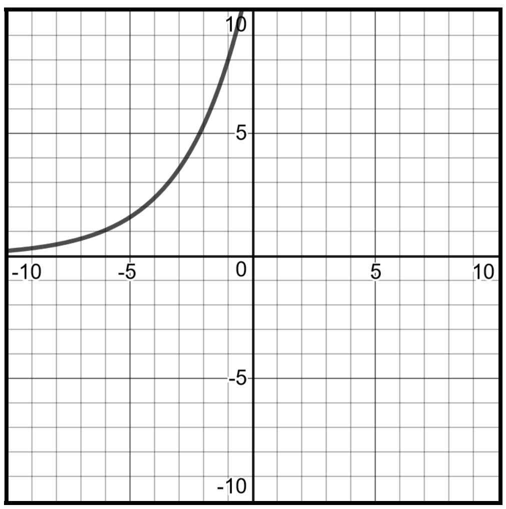
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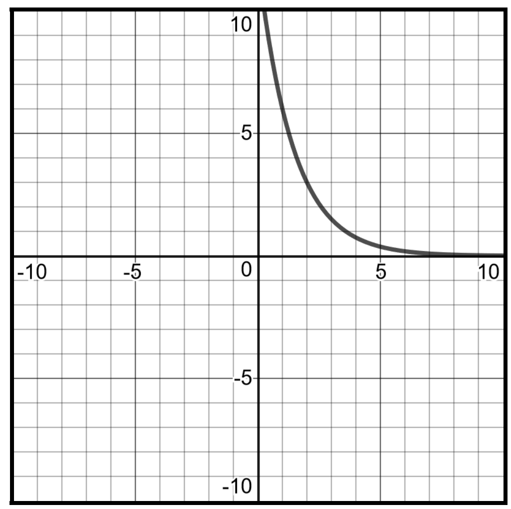
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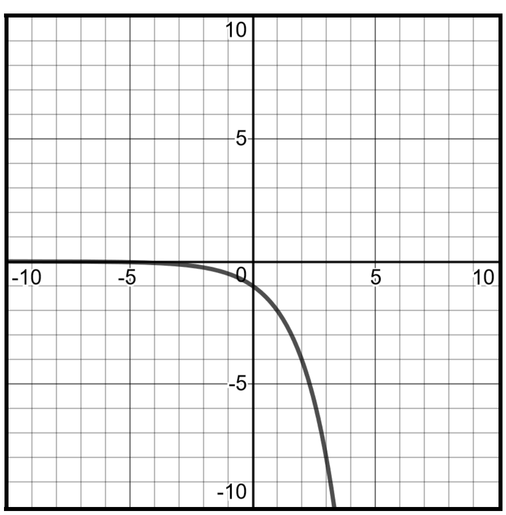
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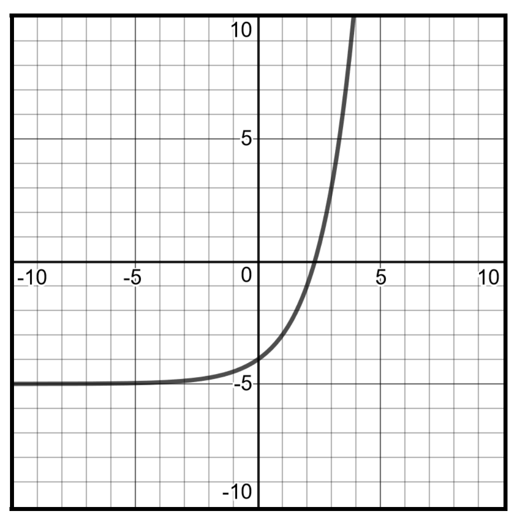
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9.4 Exponential Regression

1. $y = (0.25)^x$	
2. $y = 0.1(4)^x$	3. Enter $(10, 4.072)$ and $(15, 5.197)$. $y = 2.5(1.05)^x$
4. $y = 6.162(0.796)^x$	5. $f(x) = 1548.977(1.126)^x$ $f(10) \approx 5{,}079$

6. Shift the data down by 70°, so enter the data as

L1	0	5	10	15	20
L2	140	108	84	65	50

This gives us $y = 140(0.95)^x$. Shifting the function back up by adding 70° gives us $f(x) = 140(0.95)^x + 70$. After 30 minutes, the coffee is $f(30) \approx 100°$.

9.5 Exponential Growth or Decay

1. (3) because the base of the exponent t is less than 1.	2. $500(0.75)^5 \approx \$118.65$
3. $1(1.08)^{\frac{365}{10}} \approx 17$ feet	4. $250(0.85)^{\frac{24}{3}} \approx 68$ mg
5. "every 30 seconds" means twice a minute $5000(1.10)^{2 \cdot 60} \approx 463{,}545{,}344$ cells	6. $x(1.005)^{\frac{52}{2}} = 28{,}461.49$ $x = \dfrac{28{,}461.49}{1.005^{26}} \approx \$25{,}000$
7. $f(t) = 10(0.5)^{\frac{t}{29}}$	8. $278(0.5)^{\frac{18}{1.8}} \approx 0.27$ MBq

9.6 Rate Conversion

1. a) 2% b) $1.02^{\frac{1}{12}} = \sqrt[12]{1.02} \approx 1.0017$, or 0.17%	2. a) 15% b) $\sqrt[52]{1.15} \approx 1.00269$, so 0.27%
3. $P\left(1.033^{\frac{1}{12}}\right)^{12t} \approx P(1.00271)^m$ Note: $m = 12t$	4. $\sqrt[365]{0.75} \approx 0.99921$ $1 - 0.99921 = 0.00079$, so 0.08%

9.7 Continuous Growth or Decay

1. $f(t) = Pe^{rt}$ $f(t) = 25e^{0.1t}$ $f(5) = 25e^{(0.1)(5)} \approx 41$	2. $f(m) = Pe^{rm}$ $f(m) = 18e^{0.06m}$ $f(30) = 18e^{(0.06)(30)} = 109$
3. $A = Pe^{rt}$ $A = 800e^{(0.2)(10)} \approx 5911$	4. $A = Pe^{rt}$ $A = 1000e^{(1.16)(12)} = 1{,}110{,}143{,}673$

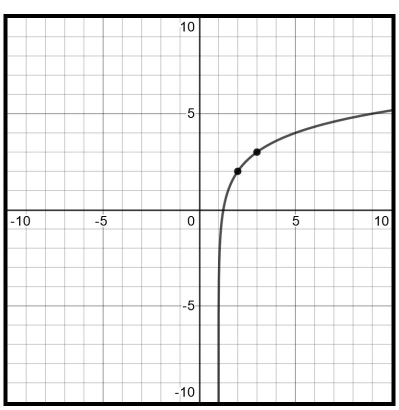
5. $A = Pe^{rt}$ $A = 8e^{(0.009)(20)} \approx 9.6$ billion	6. $A = Pe^{rt}$ a) $A = 170{,}000e^{(-0.004)(75)} \approx 125{,}939$ km³ b) $\frac{125{,}939}{170{,}000} \approx 0.74;\ 1 - 0.74 = 26\%$ loss
7. $A = Pe^{rt}$ $257 = Pe^{(0.05)(5)}$ $P = \frac{257}{e^{(0.05)(5)}} \approx 200$	8. $A = Pe^{rt}$ $50{,}000 = Pe^{(0.08)(10)}$ $P = \frac{50{,}000}{e^{(0.08)(10)}} \approx \$22{,}466.45$

Chapter 10. Logarithms

10.1 Introduction to Logarithms

<table>
<tr><td>1. $4^x = 64$, so $x = 3$</td><td>2. $5^x = \frac{1}{125}$, so $x = -3$</td></tr>
<tr><td>3. $6^x = 1$, so $x = 0$</td><td>4. $k = \log_2 1$ means $2^k = 1$, so $k = 0$</td></tr>
<tr><td>5. $x = 3^4 = 81$</td><td>6. $x = 5^2 = 25$ $\sqrt{x} = 5$</td></tr>
<tr><td>7. $x + 1 = 2^3$
$x + 1 = 8$
$x = 7$</td><td>8. $5x - 7 = 2^3$
$5x - 7 = 8$
$5x = 15$
$x = 3$</td></tr>
<tr><td>9. $\log_{10}(x + 2) = 4$
$x + 2 = 10^4$
$x + 2 = 10{,}000$
$x = 9{,}998$</td><td>10.
<table><tr><td>x</td><td>$\frac{1}{4}$</td><td>$\frac{1}{2}$</td><td>1</td><td>2</td><td>4</td></tr><tr><td>y</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr></table></td></tr>
<tr><td>11. $\log\frac{I_1}{I_2} = 8 - 5.5 = 2.5$
$\frac{I_1}{I_2} = 10^{2.5} \approx 316$ times as intense</td><td>12. $\log\frac{I_1}{I_2} = \frac{85-47}{10} = 3.8$
$\frac{I_1}{I_2} = 10^{3.8} \approx 6{,}310$ times as intense</td></tr>
</table>

10.2 Graphs of Log Functions

<table>
<tr><td colspan="2">1. a) (3) b) (2) c) (1) d) (4)</td></tr>
<tr><td>2. $g(x) = \log(x + 5) - 2$</td><td>3. $g(x) = \frac{1}{2}\log_2 x + 3$</td></tr>
<tr><td>4.

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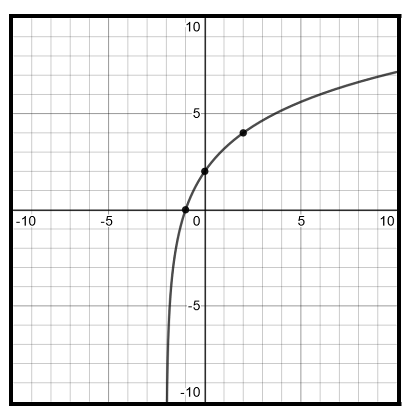
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</table>

6. y-intercept is $\log(0+1)+2=2$ To find the x-intercept: $\log(x+1)+2=0$ $\log(x+1)=-2$ $x+1=10^{-2}$ $x+1=0.01$ $x=-0.99$	7. y-intercept is $2\log_2(0+5)\approx 4.64$ To find the x-intercept: $2\log_2(x+5)=0$ $\log_2(x+5)=0$ $x+5=2^0=1$ $x=-4$
8. $\log 0$ is undefined, so there is no y-intercept. $\log x-2=0$ $\log x=2$ $x=10^2=100$	9. $\log(-5)$ is undefined, so there is no y-intercept. $\log_3(x-5)-3=0$ $\log_3(x-5)=3$ $x-5=3^3=27$ $x=32$

10.3 Properties of Logarithms

1. $2\log x+\log y$	2. $3\log a-\log b$
3. $\log 2+\log x+3\log y$	4. $\frac{1}{2}(\log x+\log y)$
5. $2x\log 4+\frac{1}{2}\log y$	6. $\log 5\,(2x)^2-\log(x+1)^3=$ $\log 5+2\log(2x)-3\log(x+1)=$ $\log 5+2\log 2+2\log x-3\log(x+1)$
7. $2\log_3 10-\log_3 20$ $=\log_3 10^2-\log_3 20$ $=\log_3\frac{10^2}{20}$ $=\log_3 5$	8. $3\log_b x+\log_b y-2\log_b z$ $=\log_b x^3+\log_b y-\log_b z^2$ $=\log_b\dfrac{x^3y}{z^2}$
9. $\frac{1}{2}\log x-2\log y$ $=\log\sqrt{x}-\log y^2$ $=\log\dfrac{\sqrt{x}}{y^2}$	10. $\dfrac{2\log x}{3}+\dfrac{3\log y}{4}$ $=\frac{2}{3}\log x+\frac{3}{4}\log y$ $=\log\sqrt[3]{x^2}+\log\sqrt[4]{y^3}$ $=\log\left(\sqrt[3]{x^2}\cdot\sqrt[4]{y^3}\right)$
11. $\log 3{,}000-\log 3=\log\frac{3{,}000}{3}=\log 1{,}000=3$	12. $\log_2 8+\log_2 2=\log_2(8\cdot 2)=\log_2 16=4$
13. (4) $\log\left(\frac{2I}{T}\right)=\log 2+\log I-\log T$	14. (2) $\log[P(1+r)^t]=\log P+t\log(1+r)$

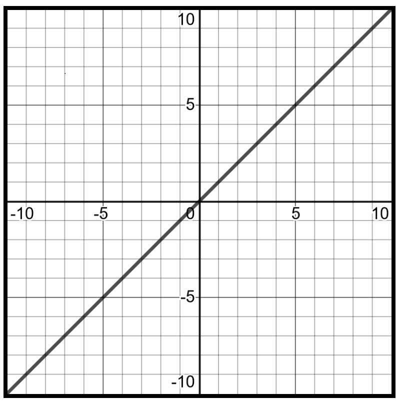
10.4 Use Logarithms to Solve Equations

1. $\log 2^x = \log 5$ $x \log 2 = \log 5$ $x = \frac{\log 5}{\log 2} \approx 2.32$	2. $\log 16^x = \log 88$ $x \log 16 = \log 88$ $x = \frac{\log 88}{\log 16} \approx 1.61$
3. $\log 13^x = \log 76$ $x \log 13 = \log 76$ $x = \frac{\log 76}{\log 13} \approx 1.69$	4. $\log 2^x = \log \frac{3}{16}$ $x \log 2 = \log \frac{3}{16}$ $x = \frac{\log \frac{3}{16}}{\log 2} \approx -2.42$
5. $20^x = 9$ $x \log 20 = \log 9$ $x = \frac{\log 9}{\log 20} \approx 0.73$	6. $3^x = 13$ $x \log 3 = \log 13$ $x = \frac{\log 13}{\log 3} \approx 2.33$
7. $8^x = 7$ $\log 8^x = \log 7$ $x \log 8 = \log 7$ $x = \frac{\log 7}{\log 8} \approx 0.94$	8. $\log 3^{2x-1} = \log 20$ $(2x-1) \log 3 = \log 20$ $2x - 1 = \frac{\log 20}{\log 3} \approx 2.7268$ $2x \approx 3.7268$ $x \approx 1.863$
9. $3(5^{x+1}) = 125$ $5^{x+1} = \frac{125}{3}$ $\log 5^{x+1} = \log \frac{125}{3}$ $(x+1) \log 5 = \log \frac{125}{3}$ $x + 1 = \frac{\log \frac{125}{3}}{\log 5} \approx 2.317$ $x \approx 1.317$	10. $1200(1.024)^t = 2400$ $(1.024)^t = 2$ $\log(1.024)^t = \log 2$ $t \log 1.024 = \log 2$ $t = \frac{\log 2}{\log 1.024} \approx 29.2$ weeks
11. $95(0.90)^t = 5$ $(0.90)^t = \frac{5}{95} = \frac{1}{19}$ $t \log 0.90 = \log \frac{1}{19}$ $t = \frac{\log \frac{1}{19}}{\log 0.90} \approx 27.9$ hours	
12. $97.656 = \frac{5^x}{2^x}$ $97.656 = \left(\frac{5}{2}\right)^x$ $\log 97.656 = \log 2.5^x$ $\log 97.656 = x \log 2.5$ $x = \frac{\log 97.656}{\log 2.5} \approx 5.0$	13. $\frac{551}{5} = \frac{9^x}{5^x}$ $110.2 = \left(\frac{9}{5}\right)^x$ $\log 110.2 = \log 1.8^x$ $\log 110.2 = x \log 1.8$ $x = \frac{\log 110.2}{\log 1.8} \approx 8.0$

14. $40{,}353{,}607 = (7^x)\left(7^{\frac{x}{2}}\right)$ $40{,}353{,}607 = 7^{\frac{3x}{2}}$ $\log 40{,}353{,}607 = \log 7^{\frac{3x}{2}}$ $\log 40{,}353{,}607 = \frac{3}{2} x \log 7$ $x = \frac{2 \log 40{,}353{,}607}{3 \log 7} = 6$	15. $3.72(18)^x = 25^x$ $\log [3.72(18)^x] = \log [25^x]$ $\log 3.72 + \log 18^x = \log 25^x$ $\log 3.72 + x \log 18 = x \log 25$ $\log 3.72 = x \log 25 - x \log 18$ $\log 3.72 = x(\log 25 - \log 18)$ $x = \frac{\log 3.72}{\log 25 - \log 18} \approx 4.0$

10.5 Natural Logarithms

1. Since $\ln e^x = x$, this is simply graphed as the line $y = x$.



2. $\ln e^x = \ln 15$ $x = \ln 15 \approx 2.71$	3. $\ln e^{2x} = \ln 13$ $2x = \ln 13$ $x = \frac{\ln 13}{2} \approx 1.28$
4. $4e^{2x} = 8$ $e^{2x} = 2$ $\ln e^{2x} = \ln 2$ $2x = \ln 2$ $x = \frac{\ln 2}{2} \approx 0.35$	5. $\frac{2050}{1500} = e^{4x}$ $\ln \frac{2050}{1500} = \ln e^{4x}$ $\ln \frac{2050}{1500} = 4x$ $x = \frac{\ln \frac{2050}{1500}}{4} \approx 0.078$
6. Use the power rule: $a \ln x = \ln x^a$. $\ln x^2 = \ln 16$ $x^2 = 16$ $x = [\cancel{-4}, 4\}$ $f(x) = \ln x$ is restricted to $x > 0$, so reject –4. $x = 4$	7. Use the properties of logarithms: $\ln(2x-3) + \ln(x+2) = 2 \ln x$ $\ln[(2x-3)(x+2)] = \ln x^2$ $(2x-3)(x+2) = x^2$ $2x^2 + x - 6 = x^2$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = \{\cancel{-3}, 2\}$ *[reject $x = -3$]* $x = 2$

8. $2\ln x = \ln 9 + \ln e$ *[ln e = 1]*
$\ln x^2 = \ln 9e$
$x^2 = 9e$
$x = 3\sqrt{e} \approx 4.95$ *[reject $-3\sqrt{e}$]*

9. $\ln e^{kt} = \ln 100^{2t}$
$kt = 2t \cdot \ln 100$
$k = 2\ln 100 \approx 9.21$

10. $900 = 800e^{4r}$
$1.125 = e^{4r}$
$\ln 1.125 = \ln e^{4r}$
$\ln 1.125 = 4r$
$r = \frac{\ln 1.125}{4} \approx 2.9\%$

11. $2a = ae^{0.025t}$
$2 = e^{0.025t}$
$\ln 2 = \ln e^{0.025t}$
$\ln 2 = 0.025t$
$t = \frac{\ln 2}{0.025} \approx 28$ hours

12. a) $f(10) = 40e^{-0.02877(10)} + 60 \approx 90°$
$f(20) = 40e^{-0.02877(20)} + 60 \approx 82.5°$
b) $70 = 40e^{-0.02877t} + 60$
$10 = 40e^{-0.02877t}$
$0.25 = e^{-0.02877t}$
$\ln 0.25 = \ln e^{-0.02877t}$
$\ln 0.25 = -0.02877t$
$-1.38629 \approx -0.02877t$
$t \approx 48$ minutes

13. $c(t) = ae^{rt}$
$c(1) = 1000 = ae^{r}$
$c(7) = 12{,}000 = ae^{7r}$
$a = \frac{1000}{e^r}$
$a = \frac{12{,}000}{e^{7r}}$
$\frac{1000}{e^r} = \frac{12{,}000}{e^{7r}}$
$\frac{e^{7r}}{e^r} = \frac{12{,}000}{1000}$
$e^{6r} = 12$

$\ln(e^{6r}) = \ln 12$
$6r = \ln 12$
$r = \frac{\ln 12}{6} \approx 0.4142$
$1000 = ae^{r}$
$1000 = ae^{0.4142}$
$a = \frac{1000}{e^{0.4142}} \approx 660.9$
$c(t) = 660.9e^{0.4142t}$

11.1 Periodic Compound Interest

1. $A = 500\left(1+\frac{0.04}{12}\right)^{(12)(3)} \approx \$563.64$	2. $A = 500\left(1+\frac{0.04}{365}\right)^{(365)(3)} \approx \$563.74$
3. a) $\frac{0.03}{4} = 0.0075 = 0.75\%$ b) $A(t) = 2000\left(1+\frac{0.03}{4}\right)^{4t}$ $A(5) \approx \$2322.37$	4. a) $\frac{0.05}{12} \approx 0.42\%$ b) $A(t) = 850\left(1+\frac{0.05}{12}\right)^{12t}$ 21 months = 1.75 years $A(1.75) = \$927.56$
5. a) $f(t) = 2000(1.02)^{4t}$ b) $2000(1.02)^{4(1.5)} \approx 2252$ c) $(1.02)^4 - 1 \approx 0.0824 = 8.24\%$	6. a) $0.001 \times 365 = 0.365 = 36.5\%$ b) $100(1.001)^{365} \approx \$144$ c) $(1.001)^{365} - 1 \approx 1.44 = 44\%$
7. $1403.60 = P\left(1+\frac{0.068}{12}\right)^{60}$ $P = \frac{1403.60}{\left(1+\frac{0.068}{12}\right)^{60}} \approx \$1{,}000.00$	8. $1{,}000{,}000 = P\left(1+\frac{0.0365}{365}\right)^{365\cdot 20}$ $1{,}000{,}000 = P(1.0001)^{7300}$ $P = \frac{1{,}000{,}000}{(1.0001)^{7300}} \approx \$481{,}926.58$
9. $\left(1+\frac{0.04}{4}\right)^{4t} = 2$ $(1.01)^{4t} = 2$ → $\log(1.01)^{4t} = \log 2$ $4t\log 1.01 = \log 2$ $t = \frac{\log 2}{4\log 1.01} \approx 17.4$ years	
10. $8{,}000\left(1+\frac{0.06}{12}\right)^{12t} = 10{,}000$ $\left(1+\frac{0.06}{12}\right)^{12t} = 1.25$ $(1.005)^{12t} = 1.25$ $\log(1.005)^{12t} = \log 1.25$ $12t\log 1.005 = \log 1.25$ $t = \frac{\log 1.25}{12\log 1.005} \approx 3.7$ years	11. $8{,}000\left(1+\frac{0.07}{4}\right)^{4t} = 10{,}000$ $\left(1+\frac{0.07}{4}\right)^{4t} = 1.25$ $(1.0175)^{4t} = 1.25$ $\log(1.0175)^{4t} = \log 1.25$ $4t\log 1.0175 = \log 1.25$ $t = \frac{\log 1.25}{4\log 1.0175} \approx 3.2$ years

11.2 Continuous Compound Interest

1. $A = 5000e^{(0.03)(4)} = \$5637.48$	2. $A = 550e^{(0.066)(10)} = \$1064.14$
3. a) $1000(1.025)^3 \approx \$1076.89$ b) $1000\left(1+\frac{0.025}{12}\right)^{(12)(3)} \approx \$1077.80$ c) $1000e^{(0.025)(3)} \approx \$1077.88$	4. $30{,}000 = Pe^{(0.05)(6)}$ $P = \frac{30{,}000}{e^{0.3}} \approx \$22{,}225$

5. $A = Pe^{rt}$ $\frac{A}{P} = e^{rt}$ $1.25 = e^{3r}$ $\ln 1.25 = \ln e^{3r}$ $\ln 1.25 = 3r$ $r = \frac{\ln 1.25}{3} \approx 7.4\%$	6. $A = Pe^{rt}$ $3000 = 2500e^{5r}$ $1.2 = e^{5r}$ $\ln 1.2 = \ln e^{5r}$ $\ln 1.2 = 5r$ $r = \frac{\ln 1.2}{5} \approx 3.6\%$
7. $90{,}000e^{0.10t} = 80{,}000e^{0.11t}$ $1.125e^{0.10t} = e^{0.11t}$ $1.125 = \frac{e^{0.11t}}{e^{0.10t}}$ $1.125 = e^{0.01t}$ $\ln 1.125 = 0.01t$ $t = \frac{\ln 1.125}{0.01} \approx 11.8$ years	

11.3 Regular Contributions

1. a) $A = P(1+r)^t = 1000(1.04)^5 = \$1{,}216.65$ b) $V = \frac{C - C(1+r)^t}{1-(1+r)} = \frac{200 - 200(1.04)^5}{1-(1.04)} = \$1{,}083.26$ c) $F = A + V = 1216.65 + 1083.26 = \$2{,}299.91$
2. For parts a) and b), $1 + \frac{r}{n} = 1 + \frac{0.03}{4} = 1.0075$ a) $A = P\left(1+\frac{r}{n}\right)^{nt} = 5000(1.0075)^{24} = \$5{,}982.07$ b) $V = \frac{C - C\left(1+\frac{r}{n}\right)^{nt}}{1-\left(1+\frac{r}{n}\right)} = \frac{500 - 500(1.0075)^{24}}{1-(1.0075)} = \$13{,}094.24$ c) $F = A + V = 5{,}982.07 + 13{,}094.24 = \$19{,}076.30$

11.4 Evaluate Loan Formulas

1. a) $M = 300{,}000 \cdot \frac{0.0025(1.0025)^{360}}{(1.0025)^{360} - 1}$ $M \approx \$1{,}265$ b) $M = 275{,}000 \cdot \frac{0.0025(1.0025)^{360}}{(1.0025)^{360} - 1}$ $M \approx \$1{,}159$ She can reduce her payments by \$106.
2. $M = 650{,}000 \cdot \frac{\left(\frac{0.025}{12}\right)\left(1+\frac{0.025}{12}\right)^{300}}{\left(1+\frac{0.025}{12}\right)^{300} - 1} = 650{,}000 \cdot \frac{(0.00208)(1.00208)^{300}}{(1.00208)^{300} - 1}$ $M \approx \$2{,}915$

3.	$1500 = (500{,}000 - x) \cdot \dfrac{0.002(1.002)^{360}}{(1.002)^{360} - 1}$ $1500 \cdot \dfrac{(1.002)^{360} - 1}{0.002(1.002)^{360}} = 500{,}000 - x$ $x = -1500 \cdot \dfrac{(1.002)^{360} - 1}{0.002(1.002)^{360}} + 500{,}000$ $x \approx 115{,}326.78$, so he should make a down payment of \$115,327.
4.	$1386.50 = 250{,}000 \cdot \dfrac{\left(\frac{0.03}{12}\right)\left(1 + \frac{0.03}{12}\right)^n}{\left(1 + \frac{0.03}{12}\right)^n - 1}$ $1386.50 = 250{,}000 \cdot \dfrac{(0.0025)(1.0025)^n}{(1.0025)^n - 1}$ $1386.50 = 625 \cdot \dfrac{(1.0025)^n}{(1.0025)^n - 1}$ $2.2184 = \dfrac{(1.0025)^n}{(1.0025)^n - 1}$ $2.2184(1.0025)^n - 2.2184 = (1.0025)^n$ $1.2184(1.0025)^n = 2.2184$ $(1.0025)^n = \dfrac{2.2184}{1.2184}$ $n \log 1.0025 = \log \dfrac{2.2184}{1.2184}$ $n = \dfrac{\log \frac{2.2184}{1.2184}}{\log 1.0025} \approx 240$ mos, or 20 yrs

Chapter 12. Trigonometric Functions

12.1 Trigonometric Ratios

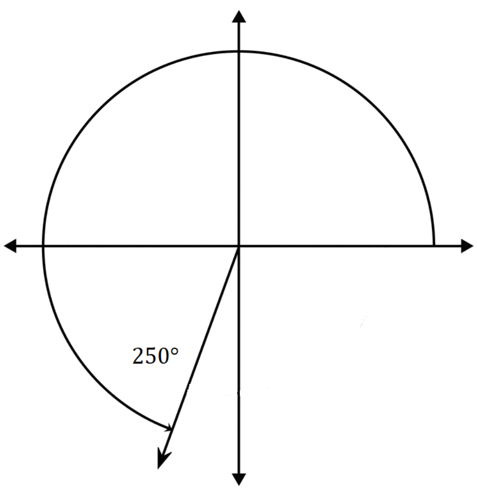
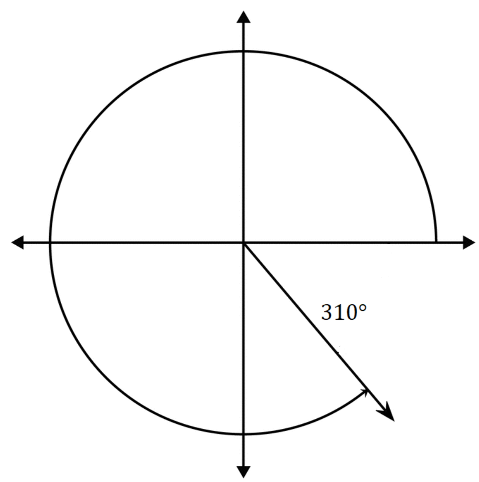
1. $\cot A = \frac{adj}{opp} = \frac{2}{4} = \frac{1}{2}$	2. $\csc A = \frac{hyp}{opp} = \frac{25}{7}$
3. $\csc 45° = \frac{1}{\sin 45°} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$	4. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{4}}{\frac{\sqrt{15}}{4}} = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$
5. $\cot 60° = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	6. $\csc 60° = \frac{1}{\sin 60°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
7. $\sec 35° = \frac{1}{\cos 35°} \approx 1.221$	8. $\csc 35° = \frac{1}{\sin 35°} \approx 1.743$
9. sin and cos are cofunctions, so $\sin x = \cos(90° - x)$. Therefore, $\cos 20° \approx 0.9397$.	10. $\sec x = \csc(90° - x)$, so $a = 90 - 28 = 62°$
11. a) $\theta = \frac{360}{12} = 30°$ b) $\cot\theta = \frac{h}{\frac{x}{2}} = \frac{2h}{x}$, so $h = \frac{1}{2}x\cot\theta$ c) $\cot 30° = \sqrt{3}$, so $h = \frac{\sqrt{3}}{2}x$ d) $A_{\triangle} = \frac{1}{2}bh = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$ e) $A_{⬡} = 6\left(\frac{\sqrt{3}}{4}x^2\right) = \frac{3\sqrt{3}}{2}x^2$	

12.2 Radians

1. a) $45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$ rad b) $270 \cdot \frac{\pi}{180} = \frac{3\pi}{2}$ rad c) $150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$ rad d) $-210 \cdot \frac{\pi}{180} = -\frac{7\pi}{6}$ rad	2. a) $\frac{\pi}{6} \cdot \frac{180}{\pi} = 30°$ b) $\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225°$ c) $-\frac{3\pi}{5} \cdot \frac{180}{\pi} = -108°$ d) $\frac{5\pi}{9} \cdot \frac{180}{\pi} = 100°$ e) $\frac{8\pi}{5} \cdot \frac{180}{\pi} = 288°$
3. $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \approx 0.866$	4. $\csc\left(-\frac{5\pi}{6}\right) = \frac{1}{\sin\left(-\frac{5\pi}{6}\right)} = -2$

5. cofunctions of complementary angles are equal, so $\frac{\pi}{6} + x = \frac{\pi}{2}$ $\left[\frac{\pi}{2} \text{ rad } = 90°\right]$ $x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$ rad	6. $\theta + \left(\theta + \frac{\pi}{3}\right) = \frac{\pi}{2}$ *[cofunctions]* $2\theta + \frac{\pi}{3} = \frac{\pi}{2}$ $2\theta = \frac{\pi}{6}$ $\theta = \frac{\pi}{12}$
7. $L = \frac{\pi}{4} \cdot 12 = 3\pi \approx 9.4$ inches	8. $L = 2 \cdot 4 = 8$ inches
9. $8\pi = \theta \cdot 10$ $\theta = \frac{4\pi}{5}$ rad	10. $65 = 5r$ $r = 13$ feet

12.3 Unit Circle

1. (4)	2. (2)
3. (4)	4. (2) $(\cos 30°, \sin 30°) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
5.  $-\cos 70°$	6.  $-\sin 50°$
7. $\tan 50°$ *[230° − 180° = 50°]*	8. $\sin 135° = \sin 45° = \frac{\sqrt{2}}{2}$
9. $\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$	10. $\sin\frac{3\pi}{2} + \cos\frac{2\pi}{3} = -1 + \left(-\frac{1}{2}\right) = -\frac{3}{2}$
11. $\cos 120° = -\cos 60° = -\frac{1}{2}$	12. $\sin 145° = \sin 35° \approx 0.574$
13. $(-\cos 30°, -\sin 30°) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	14. $400° - 360° = 40°$ $(\cos 40°, \sin 40°) \approx (0.766, 0.643)$

12.4 Solve Simple Trigonometric Equations

1. $\cos\theta = \frac{1}{2}$ $\cos^{-1}\left(\frac{1}{2}\right) = 60°$	2. $\cos\theta = \frac{\sqrt{2}}{2}$ $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45°$
3. $\cos\theta = \frac{\sqrt{3}}{2}$ $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$ $m\angle\theta = 180° + 30° = 210°$	4. $\cos\theta = 0.6$ $\cos^{-1}(0.6) \approx 53°$ $m\angle\theta \approx 360° - 53° \approx 307°$
5. Points are in Quadrants I and IV $R = \cos^{-1}(0.25) \approx 75.5°$ $360° - 75.5° = 284.5°$ Solutions: 75.5° and 284.5°	6. Points are in Quadrants II and III $R = \cos^{-1}(0.75) \approx 41.4°$ $180° - 41.4° = 138.6°$ and $180° + 41.4° = 221.4°$ Solutions: 138.6° and 221.4°
7. Points are in Quadrants I and II $R = \sin^{-1}(0.99) \approx 81.9°$ $180° - 81.9° = 98.1°$ Solutions: 81.9° and 98.1°	8. Points are in Quadrants III and IV $R = \sin^{-1}\left(\frac{1}{3}\right) \approx 19.5°$ $180° + 19.5° = 199.5°$ and $360° - 19.5° = 340.5°$ Solutions: 199.5° and 340.5°

12.5 Circles of Any Radius

1. $\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$	2. $\tan\theta = \frac{y}{x} = \frac{-\sqrt{2}}{-\sqrt{2}} = 1$
3. $r = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5$ $\sin\theta = \frac{y}{r} = \frac{4}{5}$	4. $r = \sqrt{(-4)^2 + 0^2} = \sqrt{16} = 4$ $\sec\theta = \frac{r}{x} = \frac{4}{-4} = -1$
5. $r = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$ $\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$	6. $r = \sqrt{(-8)^2 + 5^2} = \sqrt{89}$ $\sec\theta = \frac{r}{x} = -\frac{\sqrt{89}}{8}$
7. $r = \sqrt{(-7)^2 + 5^2} = \sqrt{74}$ $\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{74}} = \frac{5\sqrt{74}}{74}$ $\cos\theta = \frac{x}{r} = -\frac{7}{\sqrt{74}} = -\frac{7\sqrt{74}}{74}$ $\tan\theta = \frac{y}{x} = -\frac{5}{7}$	8. $r = \sqrt{3^2 + (-7)^2} = \sqrt{58}$ $\csc\theta = \frac{r}{y} = -\frac{\sqrt{58}}{7}$ $\sec\theta = \frac{r}{x} = \frac{\sqrt{58}}{3}$ $\cot\theta = \frac{x}{y} = -\frac{3}{7}$

9. In Quadrant I, $x > 0$ and $y > 0$. $\sin\theta = \frac{y}{r}$, so $y = 4$ and $r = 5$ $x^2 + 4^2 = 5^2$, so $x = 3$ $\cot\theta = \frac{x}{y} = \frac{3}{4}$	10. In Quadrant IV, $x > 0$ and $y < 0$. $\cos\theta = \frac{x}{r}$, so $x = \sqrt{3}$ and $r = 2$ $\left(\sqrt{3}\right)^2 + y^2 = 2^2$, so $y = -1$ $\sin\theta = \frac{y}{r} = -\frac{1}{2}$
11. In Quadrant III, $x < 0$ and $y < 0$. $\sec\theta = \frac{r}{x}$, so $x = -2$ and $r = 5$ $(-2)^2 + y^2 = 5^2$, so $y = -\sqrt{21}$ $\sin\theta = \frac{y}{r} = -\frac{\sqrt{21}}{5}$	12. $\cos\theta = \frac{x}{r}$, so $x = \sqrt{2}$ and $r = 3$ $\left(\sqrt{2}\right)^2 + y^2 = 3^2$, so $y = -\sqrt{7}$ *[reject $y = \sqrt{7}$ because $y = \sin\theta < 0$]* $\tan\theta = \frac{y}{x} = -\frac{\sqrt{7}}{\sqrt{2}} = -\frac{\sqrt{14}}{2}$
13. In Quadrant III, $x < 0$ and $y < 0$. $\tan\theta = \frac{y}{x}$, so $x = -5$ and $y = -3$ $5^2 + 3^2 = r^2$, so $r = \sqrt{34}$ *[reject $r = -\sqrt{34}$ because $r > 0$]* $\sec\theta = \frac{r}{x} = -\frac{\sqrt{34}}{5}$	14. In Quadrant II, $x < 0$ and $y > 0$. $\cot\theta = \frac{x}{y}$, so $x = -3\sqrt{2}$ and $y = 2$ $\left(-3\sqrt{2}\right)^2 + 2^2 = r^2$, so $r = \sqrt{22}$. [reject $r = -\sqrt{22}$ because $r > 0$] $\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{22}} = \frac{2\sqrt{22}}{22} = \frac{\sqrt{22}}{11}$
15. $(x, y) = (r\cos\theta, r\sin\theta)$ $2\cos\frac{\pi}{6} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$ and $2\sin\frac{\pi}{6} = 2 \cdot \frac{1}{2} = 1$ $P\left(\sqrt{3}, 1\right)$	

12.6 Pythagorean Identity

1. $\sin^2\theta = 1 - \cos^2\theta$ $\sin^2\theta = 1 - \left(\frac{1}{2}\right)^2$ $\sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$ $\sin\theta = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$ (sin is negative in Quadrant IV)	2. $\cos^2\theta = 1 - \sin^2\theta$ $\cos^2\theta = 1 - \left(-\frac{\sqrt{2}}{2}\right)^2$ $\cos^2\theta = 1 - \frac{1}{2} = \frac{1}{2}$ $\cos\theta = -\sqrt{\frac{1}{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ (cos is negative in Quadrant III)
3. $\sin^2\theta = 1 - \cos^2\theta$ $\sin^2\theta = 1 - \left(\frac{1}{3}\right)^2$ $\sin^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$ $\sin\theta = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{3}$ (sin is positive in Quadrant I)	4. $\cos^2\theta = 1 - \sin^2\theta$ $\cos^2\theta = 1 - \left(\frac{\sqrt{3}}{4}\right)^2$ $\cos^2\theta = 1 - \frac{3}{16} = \frac{13}{16}$ $\cos\theta = -\sqrt{\frac{13}{16}} = -\frac{\sqrt{13}}{4}$ (cos is negative in Quadrant II)

5. If $\sec\theta = 2$, then $\cos\theta = \frac{1}{2}$ $\sin^2\theta = 1 - \left(\frac{1}{2}\right)^2$ $\sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$ $\sin\theta = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$ (sin is negative in Quadrant IV) $\csc\theta = \frac{1}{\sin\theta} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$	6. If $\csc\theta = -\sqrt{5}$, then $\sin\theta = -\frac{1}{\sqrt{5}}$ $\cos^2\theta = 1 - \left(-\frac{1}{\sqrt{5}}\right)^2$ $\cos^2\theta = 1 - \frac{1}{5} = \frac{4}{5}$ $\cos\theta = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}}$ (cos is negative in Quadrant III) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{1}{\sqrt{5}}}{-\frac{2}{\sqrt{5}}} = \frac{1}{2}$
7. $\cos^2\theta = 1 - \left(\frac{9}{10}\right)^2$ $\cos^2\theta = 1 - \frac{81}{100} = \frac{19}{100}$ $\cos\theta = \frac{\sqrt{19}}{10}$ (cos is positive)	8. $\sin^2\theta = 1 - \left(\frac{4}{9}\right)^2$ $\sin^2\theta = 1 - \frac{16}{81} = \frac{65}{81}$ $\sin\theta = -\frac{\sqrt{65}}{9}$ (sin is negative)

12.7 Simplify Trigonometric Expressions

1. $\sec x \cot x = \frac{1}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x} = \frac{1}{\sin x} = \csc x$	2. $\tan x \csc x = \frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cancel{\sin x}} = \frac{1}{\cos x} = \sec x$
3. $\sin x \cos x \tan x =$ $\sin x \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} = \sin^2 x$	4. $\frac{\sin\theta}{\csc\theta} = \frac{\sin\theta}{\frac{1}{\sin\theta}} = \sin^2\theta$
5. $\sin^2\theta \csc\theta = \sin^2\theta \cdot \frac{1}{\sin\theta} = \sin\theta$	6. $\frac{\tan\theta}{\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\sin\theta} = \frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}} = \frac{1}{\cos\theta} = \sec\theta$
7. $\frac{\cot x}{\csc x} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x} = \cos x$	8. $\frac{\cot x \sin x}{\sec x} = \frac{\frac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x}}{\frac{1}{\cos x}} = \cos^2 x$
9. $\frac{1 - \sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$	10. $\frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 1$
11. $\frac{\sin^2 x + \cos^2 x}{1 - \sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$	12. $\cos x\,(\cos x + 1) + \sin^2 x =$ $\cos^2 x + \cos x + \sin^2 x =$ $\cos x + 1$
13. $\cos x\,(\sec x - \cos x) =$ $\cos x\left(\frac{1}{\cos x} - \cos x\right) =$ $1 - \cos^2 x = \sin^2 x$	14. $\sin^2 x\,(1 + \cot^2 x) =$ $\sin^2 x\,(1 + \frac{\cos^2 x}{\sin^2 x}) =$ $\sin^2 x + \cos^2 x = 1$

15. $\frac{2-2\sin^2 x}{\cos x}=\frac{2(1-\sin^2 x)}{\cos x}=\frac{2\cos^2 x}{\cos x}=$ $2\cos x$	16. $\sec x-\tan x\sin x=$ $\frac{1}{\cos x}-\frac{\sin x}{\cos x}\cdot\sin x=\frac{1}{\cos x}-\frac{\sin^2 x}{\cos x}=$ $\frac{1-\sin^2 x}{\cos x}=\frac{\cos^2 x}{\cos x}=\cos x$
17. $\csc^2 x\,(1+\sin x)(1-\sin x)=$ $\frac{1}{\sin^2 x}(1-\sin^2 x)=\frac{\cos^2 x}{\sin^2 x}=\cot^2 x$	18. $\frac{\tan^2 x-\sec^2 x}{\cot^2 x-\csc^2 x}=\frac{\frac{\sin^2 x}{\cos^2 x}-\frac{1}{\cos^2 x}}{\frac{\cos^2 x}{\sin^2 x}-\frac{1}{\sin^2 x}}=\frac{\frac{\sin^2 x-1}{\cos^2 x}}{\frac{\cos^2 x-1}{\sin^2 x}}$ $=\frac{\frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}}}{\frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}}}=\frac{1}{1}=1$

12.8 Graphs of Parent Trig Functions

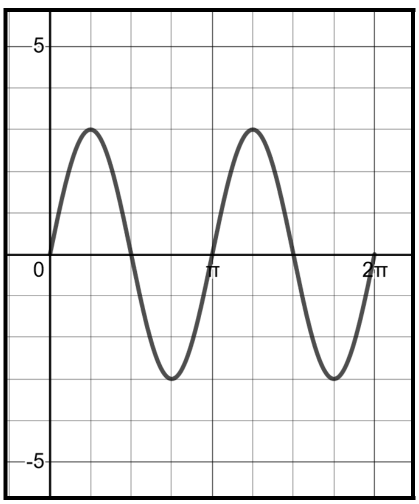
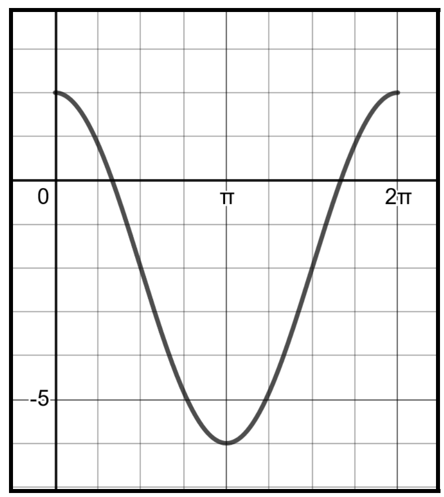
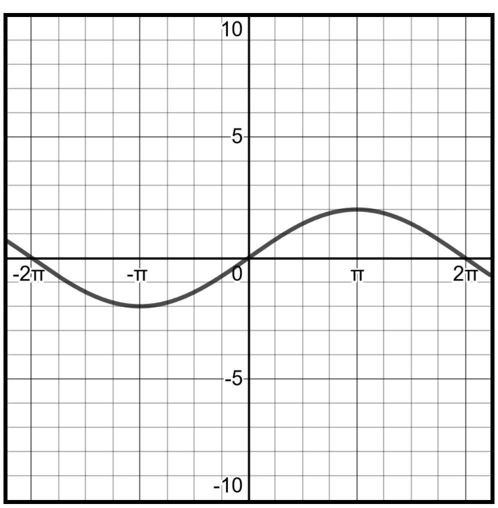
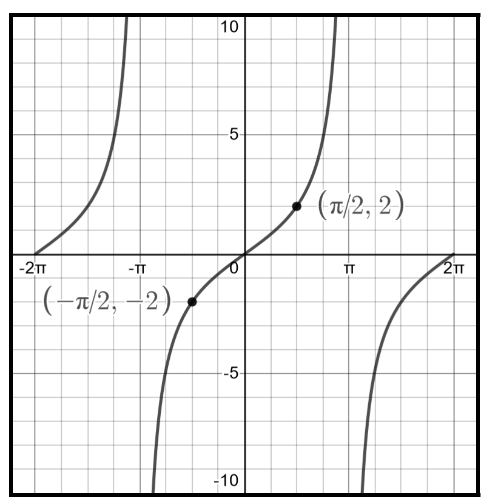
1. (4)	2. (3)
3. (3)	4. (4)
5. (2)	6. (4)

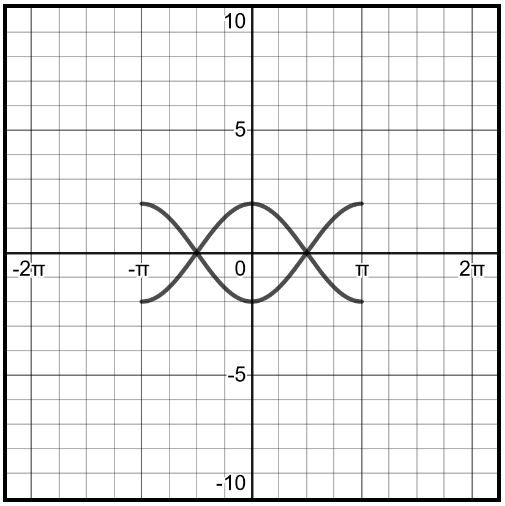
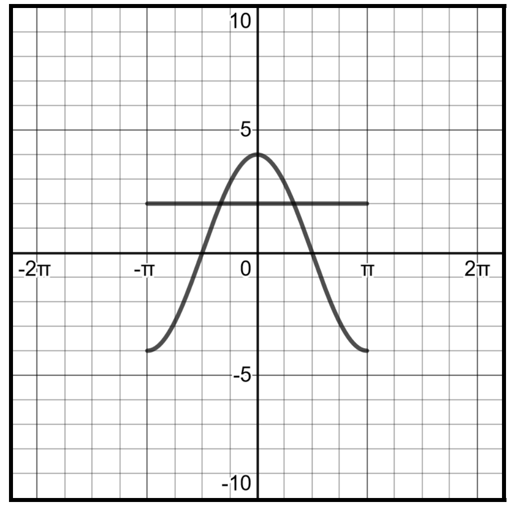
12.9 Trigonometric Transformations

1. amplitude is 2; period is 2π	
2. amplitude is 3; frequency is 2; each period has a length of π	
3. (3)	4. (2)
5. $p=\frac{2\pi}{\lvert 4\rvert}=\frac{\pi}{2}$	6. $f=\frac{2\pi}{\pi}=2$
7. $p=\frac{2\pi}{\lvert 2(-4)\rvert}=\frac{\pi}{4}$	8. $f=\frac{2\pi}{\frac{\pi}{6}}=2\pi\cdot\frac{6}{\pi}=12$
9. amplitude is $\frac{2}{3}$ frequency is 4 each period is $\frac{2\pi}{4}=\frac{\pi}{2}$ (or 90°) long	10. midline is $y=4$ frequency is $\lvert 2\cdot 3\rvert=6$ each period is $\frac{2\pi}{6}=\frac{\pi}{3}$ (or 60°) long
11. 3; There is no vertical shift and the amplitude is 3.	12. $-\frac{1}{3}$; There is no vertical shift and the amplitude is $\frac{1}{3}$.
13. 6; The amplitude is 2 plus there is a vertical shift of 4.	14. -5; The amplitude is 2 and there is a vertical shift of 3 units down. *[The negation of a does not affect the amplitude; it merely reflects the graph over the midline.]*

15. midline is $y = 0$ amplitude is 4 period is π phase shift is $\frac{\pi}{2}$ to the right	16. midline is $y = -5$ amplitude is $\frac{1}{2}$ period is $\frac{2\pi}{3}$ phase shift is $\frac{\pi}{6}$ to the left
17. (2)	18. If the sine graph is shifted left by $\frac{\pi}{2}$, then it will coincide with the cosine graph, so $h = \frac{\pi}{2}$.

12.10 Graph Trigonometric Functions

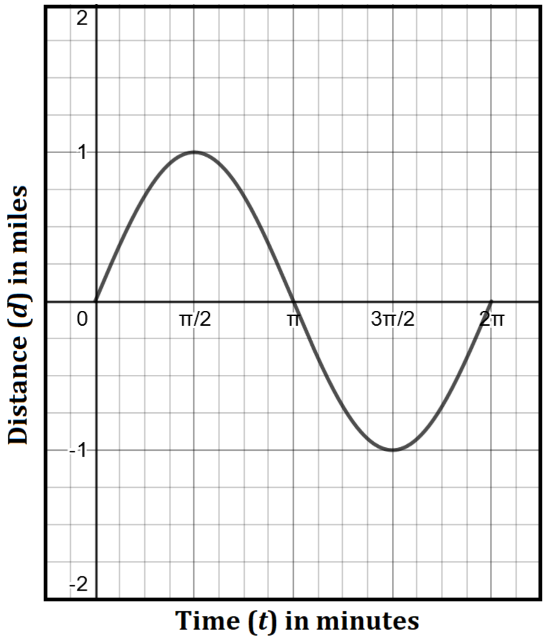
1.  $y = 3\sin 2x$	2.  $y = 4\cos x - 2$
3. 	4. 

5.  2 points of intersection. $g\left(\frac{\pi}{6}\right) = -2\cos\frac{\pi}{6} = -2 \cdot \frac{\sqrt{3}}{2} = -\sqrt{3}$	6.  Solve for *x*: $4\cos x \geq 2$ $\cos x \geq \frac{1}{2}$ $x \geq \cos^{-1}\frac{1}{2}$ $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

12.11 Model Trigonometric Functions

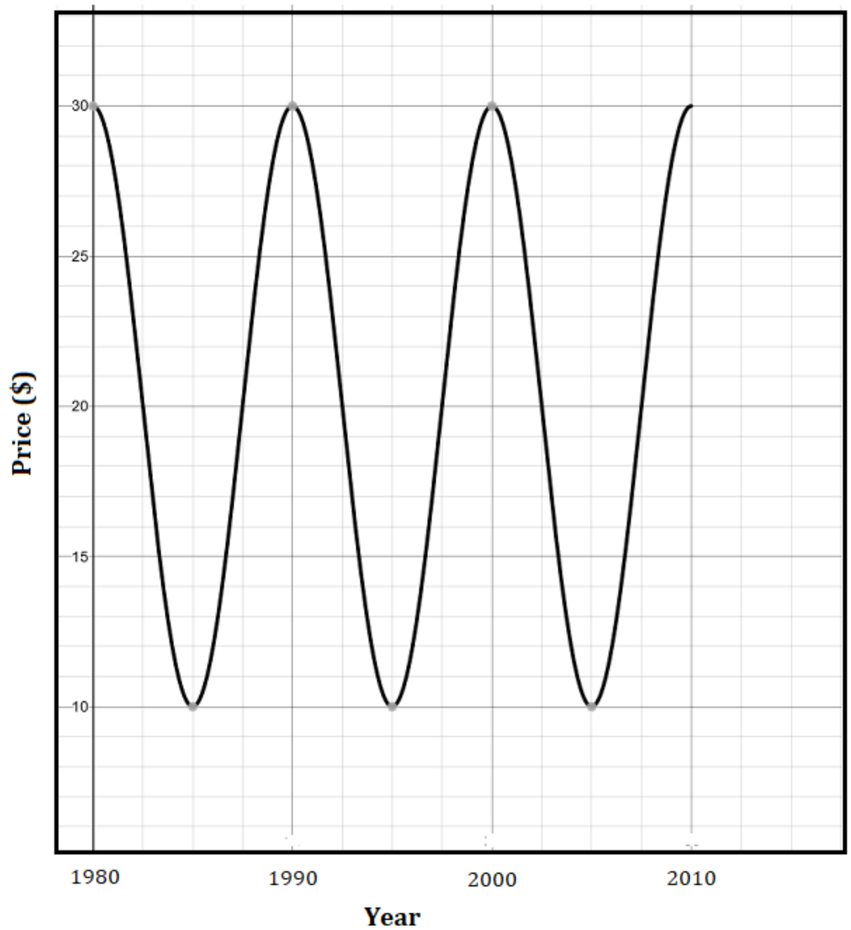
1. (4)	2. $30 + 27 = 57$ feet.
3. $\frac{2\pi}{\frac{\pi}{3}} = 6$ seconds	4. a) 1 cycle is $\frac{2\pi}{\frac{\pi}{5}} = 10$ secs, so a 40-sec ride would complete 4 revolutions. b) Radius is 20 (the amplitude). c) The base is $24 - 20 = 4$ feet off the ground.

5.



$d(t) = \sin t$

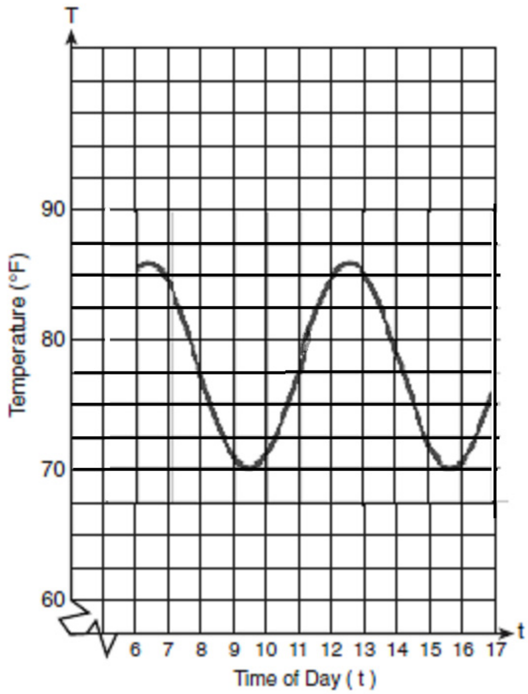
6.



The minimum value of the cosine function is –1.
$10(-1) + 20 = 10$, so the minimum price is $10.

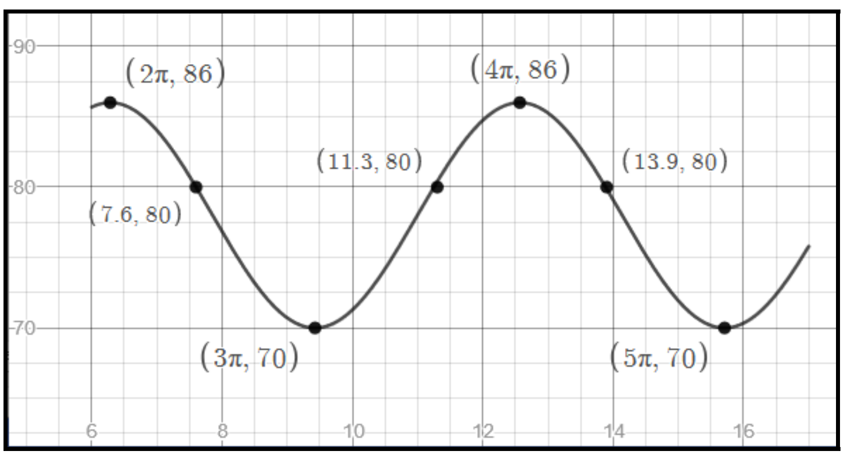
The price is $10 when
$10\cos\left(\frac{\pi}{5}t\right) + 20 = 10$
$10\cos\left(\frac{\pi}{5}t\right) = -10$
$\cos\left(\frac{\pi}{5}t\right) = -1$
$\cos^{-1}(-1) = \pi$,
so $\frac{\pi}{5}t = \pi$ or $t = 5$ (year 1985).
Each period is $\frac{2\pi}{\frac{\pi}{5}} = 10$, so the price cycles every 10 years.
The minimum price of $10 occurs in 1985, 1995, and 2005.

7.



$8\cos t + 78 = 80$
$8\cos t = 2$
$\cos t = \frac{1}{4}$
$\cos^{-1}\frac{1}{4} \approx 1.3$ (left of the interval)

Since the period is 2π, the graph intersects
$y = 80$ at:
$x = 2\pi + 1.3 \approx 7.6$
$x = 4\pi - 1.3 \approx 11.3$
$x = 4\pi + 1.3 \approx 13.9$
(See below)



Chapter 13. Properties of Functions

13.1 Compare Functions

1. (3)	2. (2) All of the others have a maximum of 1
3. $g(x)$ and $j(x)$. $\log 0$ and $\frac{1}{0}$ are both undefined. Both graphs have asymptotes at $x = 0$.	4. $h(x)$ and $j(x)$. Neither e^x nor $\frac{1}{x}$ can equal 0 for any x. Both graphs have asymptotes at $y = 0$.
5. $b(0) = -4$ $c(0) = -2$ $a(0) = 0$ $d(0) = 1$ $f(0) = 2$	6. $f(x)$ has 0 (roots are imaginary), $d(x)$ has 1 at $x = -9$, $b(x)$ has 2 at $x = \{-2, 2\}$ $c(x)$ has 3 at $x = \{-2, -1, 1\}$ $a(x)$ has an infinite number

13.2 Even and Odd Functions

1. even (all even powers of x including the constant)	2. neither (odd powers of x except the constant)
3. odd	4. neither
5. even	6. odd
7. (2)	8. (1)

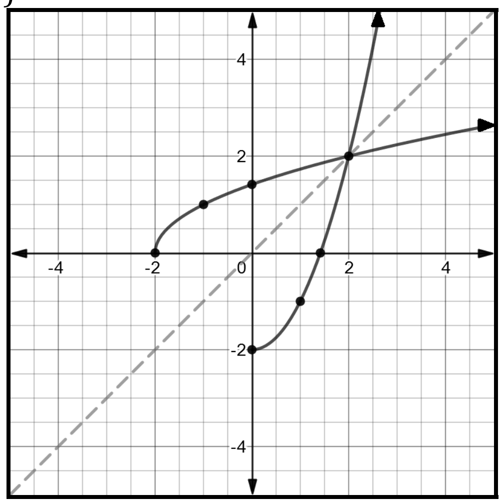
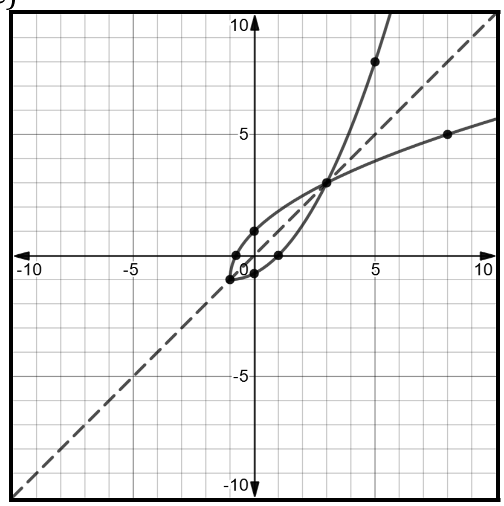
13.3 Algebraically Determine Even or Odd [CC]

1. even $f(-x) = (-x)^4 - 3(-x)^2 + 7 =$ $x^4 - 3x^2 + 7 = f(x)$	2. neither $f(-x) = (-x)^5 - 3(-x)^3 + 7 =$ $-x^5 + 3x^3 + 7$
3. odd $f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x)$	4. even $f(-x) = \left\|\frac{1}{-x}\right\| = \frac{1}{x} = f(x)$
5. neither $f(-x) = 2^{-x} - 1 = \frac{1}{2^x} - 1$	6. odd $f(-x) = \frac{(-x)^2 + 4}{(-x)^3 - (-x)} = \frac{x^2 + 4}{-x^3 + x} =$ $\frac{x^2 + 4}{-(x^3 - x)} = -\left(\frac{x^2 + 4}{x^3 - x}\right) = -f(x)$

13.4 Inverse Functions

1. (2)

2. $x = 5y + 2$ $x - 2 = 5y$ $\frac{x-2}{5} = y$ $f^{-1}(x) = \frac{x-2}{5}$	3. $x = \frac{2y+5}{3}$ $3x = 2y + 5$ $3x - 5 = 2y$ $\frac{3x-5}{2} = y$ $f^{-1}(x) = \frac{3x-5}{2}$
4. $x = -\frac{2}{3}y$ $-\frac{3}{2}x = y$ $f^{-1}(x) = -\frac{3}{2}x$	5. $x = \frac{1}{3}y + 2$ $x - 2 = \frac{1}{3}y$ $3(x-2) = y$ $3x - 6 = y$ $f^{-1}(x) = 3x - 6$
6. $x = y^2 - 5$ $x + 5 = y^2$ $\pm\sqrt{x+5} = y$ $f^{-1}(x) = \pm\sqrt{x+5}$	7. $x = (y-2)^3$ $\sqrt[3]{x} = y - 2$ $\sqrt[3]{x} + 2 = y$ $f^{-1}(x) = \sqrt[3]{x} + 2$
8. $x = 3^y + 1$ $x - 1 = 3^y$ $\log_3(x-1) = \log_3 3^y$ $\log_3(x-1) = y$ $f^{-1}(x) = \log_3(x-1)$	9. $x = \log(y-2) + 3$ $x - 3 = \log(y-2)$ $10^{(x-3)} = y - 2$ $10^{(x-3)} + 2 = y$ $f^{-1}(x) = 10^{(x-3)} + 2$
10. $x = \frac{y}{y+2}$ $x(y+2) = y$ $xy + 2x = y$ $2x = y - xy$ $2x = y(1-x)$ $\frac{2x}{1-x} = y$ $f^{-1}(x) = \frac{2x}{1-x}$	11. $x = \sin y - 1$ $x + 1 = \sin y$ $\arcsin(x+1) = y$ $p^{-1}(x) = \arcsin(x+1)$ Note: parentheses are important here.

12. a) $x = \sqrt{2}$ b) $x = y^2 - 2$ $x + 2 = y^2$ $\sqrt{x+2} = y$ $f^{-1}(x) = \sqrt{x+2}$ c) 	13. a) $2\sqrt{x+1} - 1 = 0$ $\sqrt{x+1} = \frac{1}{2}$ $x + 1 = \frac{1}{4}$ $x = -\frac{3}{4}$ b) $x = 2\sqrt{y+1} - 1$ $\frac{x+1}{2} = \sqrt{y+1}$ $y = \left(\frac{x+1}{2}\right)^2 - 1$ $f^{-1}(x) = \left(\frac{x+1}{2}\right)^2 - 1$ c) 

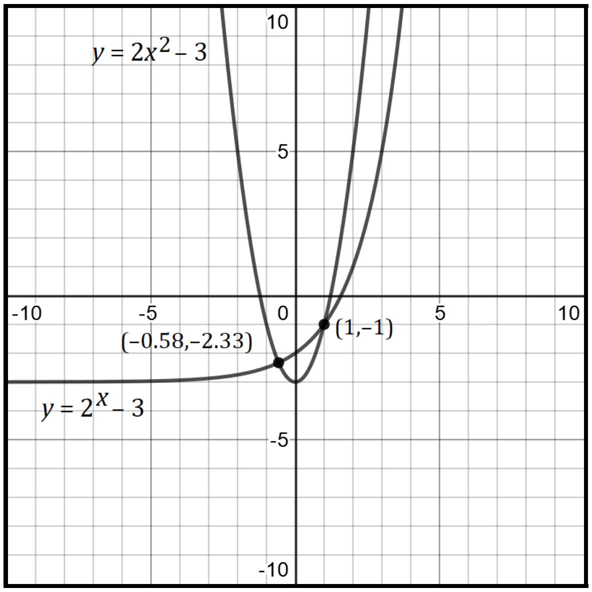
13.5 Average Rate of Change

1. a) $\frac{139-79}{80-60} = \frac{60}{20} = 3$	b) $\frac{49-139}{90-80} = \frac{-90}{10} = -9$	c) $\frac{49-79}{90-60} = \frac{-30}{30} = -1$
2. a) $\frac{5.06-3.91}{1999-1987} = \frac{1.15}{12} \approx 0.096$	b) $\frac{7.50-5.06}{2009-1999} = \frac{2.44}{10} \approx 0.244$	

3. $f(5) = 5^2 + 2 = 27$ $f(15) = 15^2 + 2 = 227$ $R = \frac{227-27}{15-5} = \frac{200}{10} = 20$	4. $f(-3) = (-3)^2 + 10(-3) + 16 = -5$ $f(3) = 3^2 + 10(3) + 16 = 55$ $R = \frac{55-(-5)}{3-(-3)} = \frac{60}{6} = 10$
5. $f(-1) = (-1)^4 + 2(-1)^3 = -1$ $f(1) = 1^4 + 2(1^3) = 3$ $R = \frac{3-(-1)}{1-(-1)} = \frac{4}{2} = 2$	6. $f\left(\frac{\pi}{2}\right) = 2 \cdot 1 = 2$ $f\left(\frac{3\pi}{2}\right) = 2 \cdot (-1) = -2$ $R = \frac{-2-2}{\frac{3\pi}{2} - \frac{\pi}{2}} = -\frac{4}{\pi}$

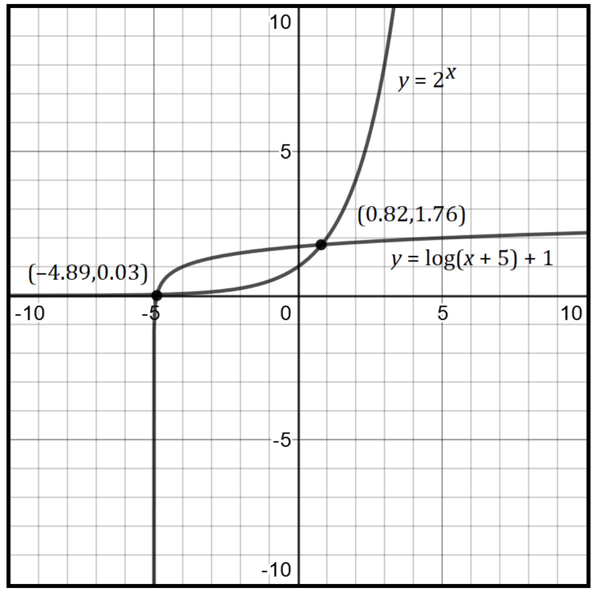
13.6 Solutions to Equation of Two Functions

1.



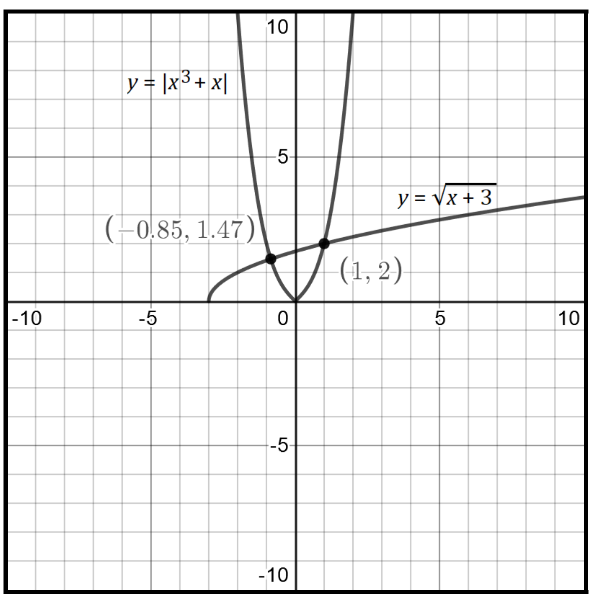
Real solutions: $x = \{-0.58, 1\}$

2.



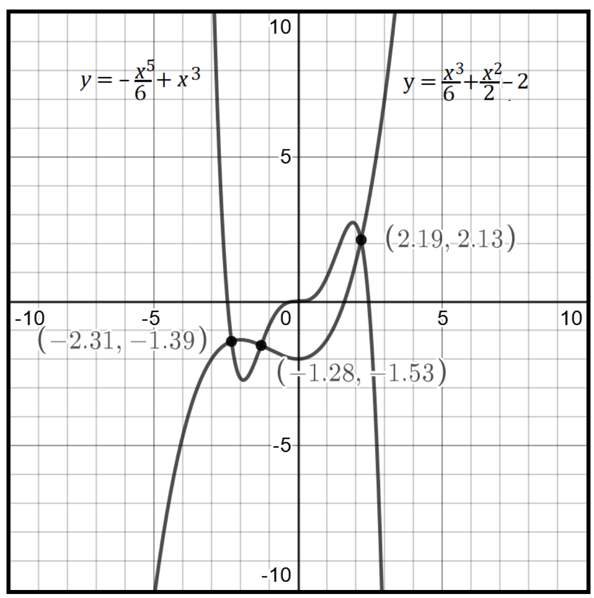
Real solutions: $x = \{-4.89, 0.82\}$

3.



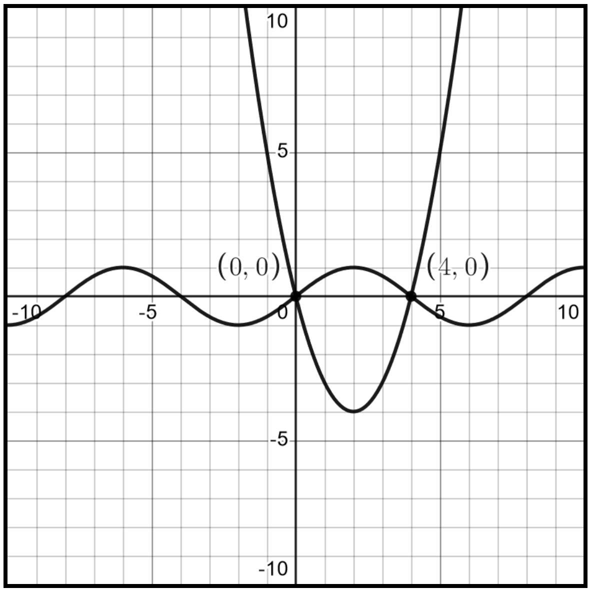
Real solutions: $x = \{-0.85, 1\}$

4.

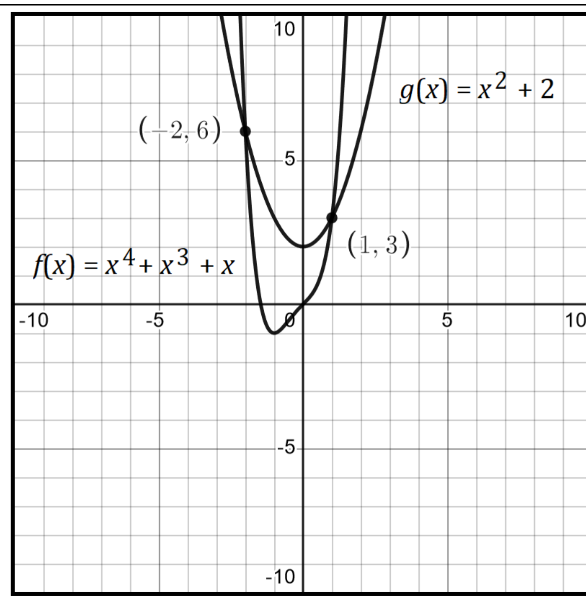


Real solutions: $x = \{-2.31, -1.28, 2.19\}$

5. $x = \{0, 4\}$



6. a) Graph shown to the right.
   Real solutions: $x = \{-2, 1\}$



   b) $x^4 + x^3 + x = x^2 + 2$

   $x^4 + x^3 - x^2 + x - 2 = 0$

   We know –2 and 1 are roots, so

$\boxed{-2}$	1	1	−1	1	−2
		−2	2	−2	2
	1	−1	1	−1	0

   $(x + 2)(x^3 - x^2 + x - 1) = 0$

$\boxed{1}$	1	−1	1	−1
		1	0	1
	1	0	1	0

   $(x + 2)(x - 1)(x^2 + 1) = 0$

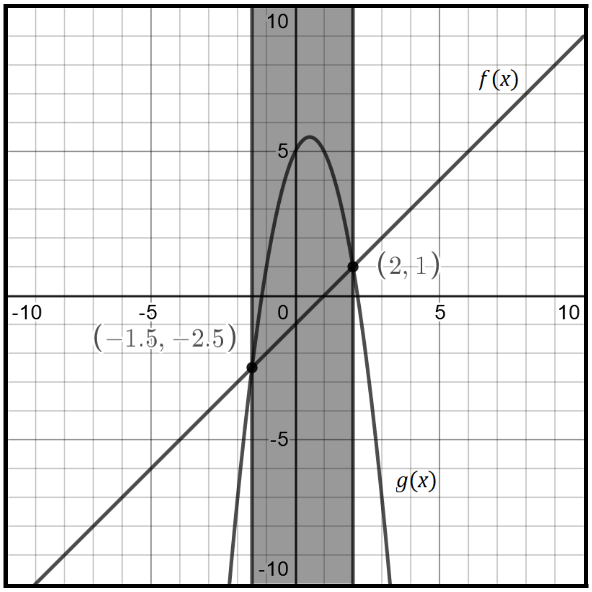
   This leaves us $x^2 + 1 = 0$,

   which gives us $x = \pm i$.

   Solutions are $x = \{-2, 1, -i, i\}$

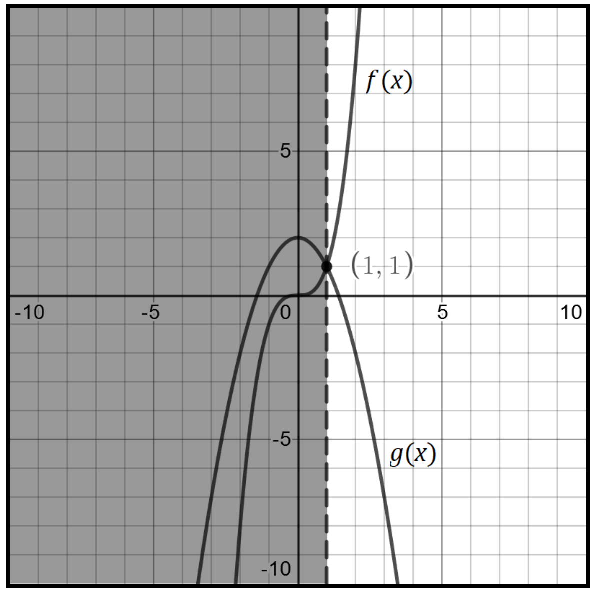
13.7 Solutions to Inequality of Two Functions [NG]

1.



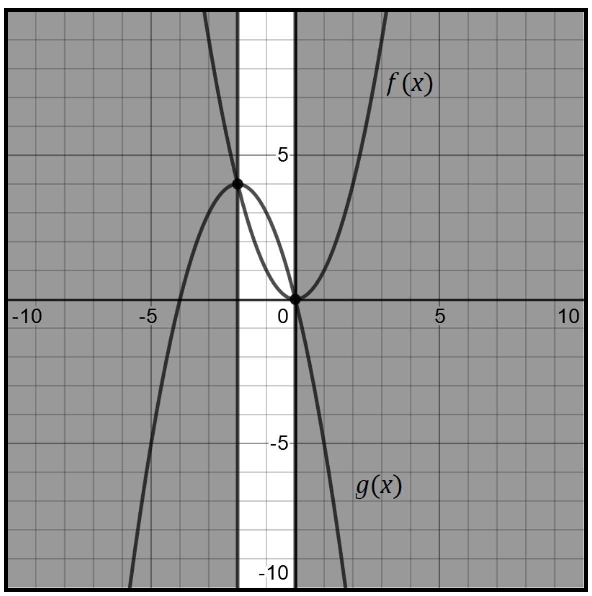
Solution: $-1.5 \leq x \leq 2$

2.



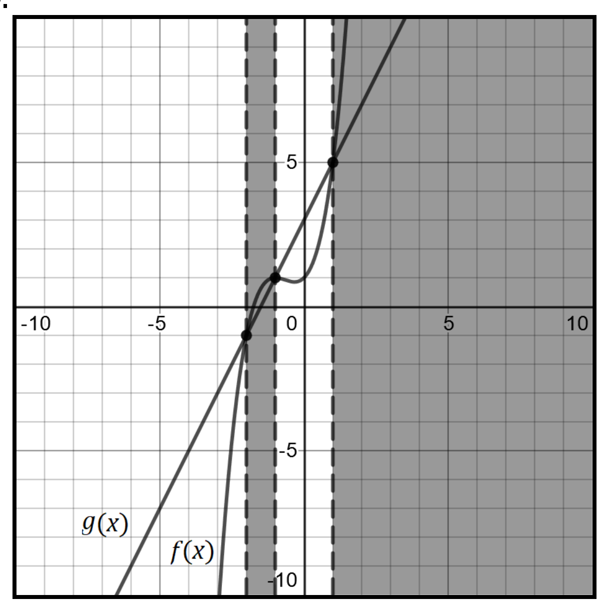
Solution: $x < 1$

3.



Solution: $x \leq -2$ or $x \geq 0$

4.



Solution: $-2 < x < -1$ or $x > 1$

14.1 Arithmetic Sequences

1. $a_1 = 32$ and $d = 5$ $a_n = a_1 + (n-1)d$ $a_n = 32 + (n-1) \cdot 5$ $a_n = 32 + 5n - 5$ $a_n = 5n + 27$	2. $a_1 = 24$ and $d = -7$ $a_n = a_1 + (n-1)d$ $a_n = 24 + (n-1)(-7)$ $a_n = 24 - 7n + 7$ $a_n = -7n + 31$
3. $a_1 = -1$ and $d = \frac{1}{2}$ $a_n = a_1 + (n-1)d$ $a_n = -1 + (n-1) \cdot \frac{1}{2}$ $a_n = -1 + \frac{1}{2}n - \frac{1}{2}$ $a_n = \frac{1}{2}n - \frac{3}{2}$	4. $a_n = a_1 + (n-1)d$ $a_8 = 21 + (8-1) \cdot 9 = 84$
5. $a_n = a_1 + (n-1)d$ $a_{12} = 16 + (12-1) \cdot 11 = 137$	6. $a_n = a_1 + (n-1)d$ $a_9 = 35 + (9-1) \cdot (-5) = -5$
7. $a_n = a_1 + (n-1)d$ $a_{27} = 5 + (27-1) \cdot 3 = 83$	8. $a_n = a_1 + (n-1)d$ $a_{20} = -8 + (20-1) \cdot 6 = 106$
9. $(6, 10)$ and $(21, 55)$ $d = \frac{55-10}{21-6} = \frac{45}{15} = 3$ $a_n = a_1 + (n-1)d$ $10 = a_1 + (6-1) \cdot 3$ $10 = a_1 + 15$ $a_1 = -5$ $a_n = a_1 + (n-1)d$ $a_n = -5 + (n-1) \cdot 3$ $a_n = -5 + 3n - 3$ $a_n = 3n - 8$	10. $(4, -23)$ and $(22, 49)$ $d = \frac{49-(-23)}{22-4} = \frac{72}{18} = 4$ $a_n = a_1 + (n-1)d$ $-23 = a_1 + (4-1) \cdot 4$ $-23 = a_1 + 12$ $a_1 = -35$ $a_n = a_1 + (n-1)d$ $a_n = -35 + (n-1) \cdot 4$ $a_n = -35 + 4n - 4$ $a_n = 4n - 39$
11. $(3, 29)$ and $(15, 53)$ a) $d = \frac{53-29}{15-3} = \frac{24}{12} = 2$ $a_n = a_1 + (n-1)d$ $29 = a_1 + (3-1) \cdot 2$ $29 = a_1 + 4$ $a_1 = 25$ $a_n = 25 + (n-1) \cdot 2$ $a_n = 2n + 23$	b) $a_{10} = 2(10) + 23 = 43$ There are 43 seats in the tenth row.

14.2 Geometric Sequences

1. $a_n = a_1 r^{n-1}$ $a_n = 6(3)^{n-1}$ $a_n = \frac{6(3)^n}{3}$ $a_n = 2(3)^n$	2. $a_n = a_1 r^{n-1}$ $a_n = 12(0.5)^{n-1}$ $a_n = \frac{12(0.5)^n}{0.5}$ $a_n = 24(0.5)^n$
3. $a_n = a_1 r^{n-1}$ $a_n = -2(-4)^{n-1}$ $a_n = \frac{-2(-4)^n}{-4}$ $a_n = 0.5(-4)^n$	4. $a_n = a_1 r^{n-1}$ $a_n = -1(-2)^{n-1}$ $a_n = \frac{-1(-2)^n}{-2}$ $a_n = 0.5(-2)^n$
5. $a_n = 4(2.5)^{n-1}$ $a_n = \frac{4(2.5)^n}{2.5}$ $a_n = 1.6(2.5)^n$	6. $a_n = a_1 r^{n-1}$ $a_9 = 16(2)^{9-1} = 16(2)^8 = 4{,}096$
7. $a_n = a_1 r^{n-1}$ $a_5 = 12(-3)^4 = 972$	8. $a_n = a_1 r^{n-1}$ $a_7 = 8(1.5)^6 = 91.125$
9. $a_n = a_1 r^{n-1}$ $a_{15} = 5(-2)^{14} = 81{,}920$	10. $a_n = a_1 r^{n-1}$ $a_7 = 6\left(-\frac{1}{2}\right)^6 = 6\left(\frac{1}{2^6}\right) = \frac{6}{64} = \frac{3}{32}$ (Using the calculator, $a_7 = 0.09375$.)
11. a) $r^{6-3} = \frac{3645}{135}$ $r^3 = 27$ $r = \sqrt[3]{27} = 3$ $a_6 r^4 = a_{10}$ $a_{10} = 3645(3)^4 = 295{,}245$ b) $a_n = a_1 r^{n-1}$ $135 = a_1(3)^{3-1}$ $135 = a_1(9)$ $a_1 = 15$ $a_n = 15(3)^{n-1} = \frac{15(3)^n}{3} = 5(3)^n$	12. a) $r^{10-5} = \frac{112{,}640}{3{,}520}$ $r^5 = 32$ $r = \sqrt[5]{32} = 2$ $a_5 r^3 = a_8$ $a_8 = 3520(2)^3 = 28{,}160$ b) $a_n = a_1 r^{n-1}$ $3520 = a_1(2)^{5-1}$ $3520 = a_1(16)$ $a_1 = 220$ $a_n = 220(2)^{n-1} = \frac{220(2)^n}{2} = 110(2)^n$

14.3 Recursively Defines Sequences

1. $a_1 = 6$ $a_n = a_{n-1} + 4$	2. $a_1 = 8$ $a_n = 3a_{n-1}$
3. $a_1 = 16$ $a_n = -0.5a_{n-1}$	4. $a_1 = 3$ $a_2 = 2(3) + 1 = 7$ $a_3 = 2(7) + 1 = 15$ $a_4 = 2(15) + 1 = 31$ $3, 7, 15, 31$
5. $a_1 = 3$ $a_2 = 2(3) - 4 = 2$ $a_3 = 2(2) - 4 = 0$ $a_4 = 2(0) - 4 = -4$ $3, 2, 0, -4$	6. $a_2 = 3 + 2 = 5$ $a_3 = 5 + 3 = 8$ $a_4 = 8 + 4 = 12$ $3, 5, 8, 12$ Neither; there is no common difference nor common ratio.
7. $a_2 = 3(2) + 2 = 8$ $a_3 = 3(8) + 3 = 27$ $a_4 = 3(27) + 4 = 85$ $2, 8, 27, 85$	8. $a_2 = 2(-3) - 2 = -8$ $a_3 = 2(-8) - 3 = -19$ $a_4 = 2(-19) - 4 = -42$ $-3, -8, -19, -42$
9. $a_1 = 2$ $a_2 = 3$ $a_n = a_{n-2} \cdot a_{n-1}$ for $n > 2$	10. a) $a_1 = 40$ $a_2 = 8$ $a_n = \frac{1}{2}(a_{n-2} + a_{n-1})$, for $n > 2$ b) $40, 8, 24, 16, 20, 18, 19, \ldots$ First odd term is a_7.

14.4 Sigma Notation

1. $3(2) + 3(3) + 3(4) + 3(5) =$ $6 + 9 + 12 + 15 = 42$	2. $(2^2 - 1) + (3^2 - 1) + (4^2 - 1) + (5^2 - 1)$ $= 3 + 8 + 15 + 24 = 50$
3. $-2(1) + 100 - 2(2) + 100 - 2(3) +$ $100 - 2(4) + 100 - 2(5) + 100 =$ $-30 + 500 = 470$	4. $(2^3 + 2) + (2^4 + 2) + (2^5 + 2) =$ $10 + 18 + 34 = 62$
5. $(1^2 + 1) + (2^2 + 2) + (3^2 + 3) +$ $(4^2 + 4) + (5^2 + 5) =$ $2 + 6 + 12 + 20 + 30 = 70$	6. $(2 - 1)^2 + (2 - 2)^2 + (2 - 3)^2 =$ $1 + 0 + 1 = 2$
7. $3(2)^0 + 3(2)^1 + 3(2)^2 =$ $3 + 6 + 12 = 21$	8. $(-3^2 + 3) + (-4^2 + 4) + (-5^2 + 5) =$ $-6 - 12 - 20 = -38$
9. $(-1^4 - 1) + (-2^4 - 2) + (-3^4 - 3) =$ $-2 - 18 - 84 = -104$	10. $(2 \cdot 1 + 1)^0 + (2 \cdot 2 + 1)^1 +$ $(2 \cdot 3 + 1)^2 = 1 + 5 + 49 = 55$
11. (3)	12. (4)
13. (2)	14. (4)

15. $13567(0) + 294 = 294$ $13567(1) + 294 = 13861$ $13567(2) + 294 = 27428$ Sum is \$41,583

14.5 Arithmetic Series

1. $\sum_{k=1}^{5}(2k+1)$ $S_5 = \frac{5(3+11)}{2} = 35$	2. $\sum_{k=1}^{7}(4k+3)$ $S_7 = \frac{7(7+31)}{2} = 133$
3. $\sum_{k=1}^{6}(4k+21)$ $S_6 = \frac{6(25+45)}{2} = 210$	4. $\sum_{k=1}^{n}(2k+3)$ $2n + 3 = 43$ $n = 20$ $\sum_{k=1}^{20}(2k+3)$
5. $\sum_{k=1}^{n}(2k-1)$ $2n - 1 = 39$ $n = 20$ $\sum_{k=1}^{20}(2k-1)$	6. $\sum_{k=1}^{n}(12k-1)$ $12n - 1 = 119$ $n = 10$ $\sum_{k=1}^{10}(12k-1) = 650$
7. $\sum_{k=1}^{n}7k$ $7n = 105$ $n = 15$ $\sum_{k=1}^{15}7k = 840$	8. $\sum_{k=1}^{n}(8k+10)$ $8n + 10 = 122$ $n = 14$ $\sum_{k=1}^{14}(8k+10) = 980$
9. $S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$ $S_{19} = \frac{19(3 + 3 + (19-1)\cdot 7)}{2} = 1{,}254$	10. $S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$ $S_{20} = \frac{20(5 + 5 + (20-1)\cdot 9)}{2} = 1{,}810$
11. $S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$ $S_{30} = \frac{30(15 + 15 + (30-1)\cdot 2)}{2} = 1{,}320$	12. $S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$ $S_{21} = \frac{21(18 + 18 + (21-1)\cdot 2)}{2} = 798$

14.6 Geometric Series

1. $\sum_{k=1}^{5} 7(3)^{k-1}$	2. $\sum_{n=1}^{6} (-2)^{n-1}$
3. $\sum_{k=1}^{4} 200\left(\frac{1}{2}\right)^{k-1}$	4. $S_7 = \frac{9-9(4)^7}{1-4} = 49{,}149$ $\sum_{k=1}^{7} 9(4)^{k-1} = 49{,}149$
5. $S_8 = \frac{3-3(-4)^8}{1-(-4)} = -39{,}321$	6. $\sum_{k=1}^{n} 10(2)^{k-1}$ $10(2)^{n-1} = 5120$ $2^{n-1} = 512$ $\log 2^{n-1} = \log 512$ $(n-1)\log 2 = \log 512$ $n-1 = \frac{\log 512}{\log 2} = 9$ $n = 10$ $\sum_{k=1}^{10} 10(2)^{k-1}$
7. $\sum_{k=1}^{n} \frac{1}{2}(2)^{k-1}$ $\frac{1}{2}(2)^{n-1} = 1024$ $2^{n-1} = 2048$ $\log 2^{n-1} = \log 2048$ $n-1 = \frac{\log 2048}{\log 2} = 11$ $n = 12$ $\sum_{k=1}^{12} \frac{1}{2}(2)^{k-1}$	8. $\sum_{k=1}^{n} 6(5)^{k-1}$ $6(5)^{n-1} = 93{,}750$ $5^{n-1} = 15{,}625$ $\log 5^{n-1} = \log 15{,}625$ $n-1 = \frac{\log 15{,}625}{\log 5} = 6$ $n = 7$ $\sum_{k=1}^{7} 6(5)^{k-1} = 117{,}186$
9. $a_1 = 9$ and $r = -3$ a) $\sum_{n=1}^{7} 9(-3)^{n-1} = 4{,}923$ b) $S_7 = \frac{9-9(-3)^7}{1-(-3)} = 4{,}923$	
10. (1)	

Chapter 15. Probability

15.1 Theoretical and Empirical Probability

1. $\frac{1}{4}$	2. $\frac{6}{10}=\frac{3}{5}$
3. $\frac{6}{22}=\frac{3}{11}$	4. $\frac{3}{6}=\frac{1}{2}$
5. $\frac{1}{6}$	6. $\frac{23}{29}$
7. $\frac{6}{20}=\frac{3}{10}$	8. $\frac{13}{52}=\frac{1}{4}$
9. $\frac{5}{8}$	10. $P(red)=\frac{30}{90}$ $P(white)=\frac{31}{90}$ $P(blue)=\frac{29}{90}$ White is the most likely to be picked.
11. $\frac{2{,}000}{80{,}000}=\frac{1}{40}$	12. $\frac{8}{20}=\frac{2}{5}$
13. The trials in this case are 100 products per month for 10 months, or 1,000. The empirical probability of a faulty bulb is $\frac{20}{1000}=\frac{1}{50}$.	14. $20\%=\frac{2}{10}$. There are 10 numbers from 0 to 9, so any two numbers (such as 0 and 1) can represent the event occurring.

15.2 Probability Involving And or Or

1. $\frac{6}{11}$	2. $\frac{4}{5}$
3. $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$	4. $P(pen\ or\ red)=$ $P(pen)+P(red)-P(red\ pen)=$ $\frac{6}{14}+\frac{9}{14}-\frac{4}{14}=\frac{11}{14}$

5. a) $P(A \text{ and } B) = P(A \cap B) =$

$P(\{5,8\}) = \frac{2}{10} = \frac{1}{5}$

b) $P(A \text{ or } B) = P(A \cup B) =$

$P(\{2,3,4,5,7,8,9\}) = \frac{7}{10}$

6. $P(A) = 0.05$, $P(B) = 0.08$, and $P(A \text{ and } B) = 0.004$

a) not mutually exclusive because $P(A \text{ and } B) \neq 0$

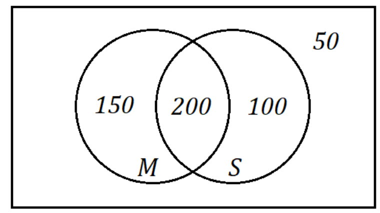
b) $P(A \text{ or } B) = 0.05 + 0.08 - 0.004 = 0.126$

7. $P(G \text{ or } A)$

$= P(G) + P(A) - P(G \text{ and } A)$

$= \frac{11}{20} + \frac{9}{20} - \frac{5}{20} = \frac{15}{20} = \frac{3}{4}$

8.



$\frac{50}{500} = \frac{1}{10}$

9. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) + P(A \text{ and } B) = P(A) + P(B)$ *[add $P(A \text{ and } B)$ to both sides]*

$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$ *[subtract $P(A \text{ or } B)$ from both sides]*

15.3 Two-Way Frequency Tables

1. a) $\frac{15}{113} \approx 13.3\%$ of the students are undecided.

b) $\frac{31}{60} \approx 51.7\%$ of the 9th graders are watching.

2. Given data in bold below.

	Coca-Cola	Sprite	Total
Table	16	**14**	30
Garbage	34	8	**42**
Total	**50**	22	**72**

3. a) $P(F) = \frac{72}{240} = \frac{3}{10}$ *[from the Total row]*

b) $P(C) = \frac{80}{240} = \frac{1}{3}$ *[from the Total column]*

c) $P(F|C) = \frac{24}{80} = \frac{3}{10}$ *[from the first row]*

d) $P(C|F) = \frac{24}{72} = \frac{1}{3}$ *[from the first column]*

e) $P(C \text{ and } F) = \frac{24}{240} = \frac{1}{10}$ *[from the one cell and the grand total]*

f) $P(F|C) = P(F) = \frac{3}{10}$ and $P(C|F) = P(C) = \frac{1}{3}$, so they appear to be independent.

4. It is helpful to calculate the totals first:

	Dogs	Cats	Rabbits	Total
Girls	53	72	25	150
Boys	62	28	40	130
Total	115	100	65	280

a) $P(G|R) = \frac{25}{65} = \frac{5}{13}$.

b) $P(R|G) = \frac{25}{150} = \frac{1}{6}$.

c) $P(B|D\ or\ C) = \frac{62+28}{115+100} = \frac{90}{215} = \frac{18}{43}$ *[from the first two columns]*

15.4 Series of Events [CC]

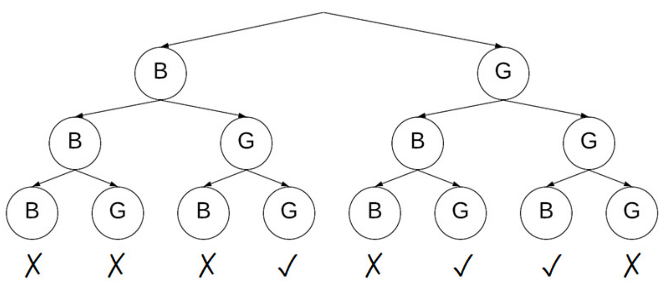
1. $\frac{1}{3} \times \frac{3}{7} = \frac{1}{7}$

2. $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

3. $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

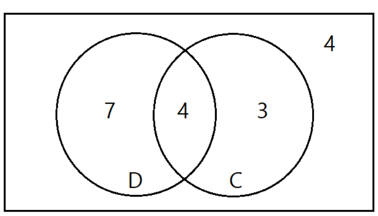
4. $0.95 \times 0.93 \times 0.98 \approx 87\%$

5.



a) $\frac{3}{8}$ (see check marks above) b) $\frac{7}{8}$ (all except the first leaf)

6.



a) 3 b) 4

7. $P(at\ least\ one\ blue) =$

$1 - P(red\ or\ white\ on\ all\ 5\ picks) =$

$1 - \left(\frac{2}{3}\right)^5 = 1 - \frac{32}{243} = \frac{211}{243} \approx 87\%$

8. $\frac{1}{20} \times \frac{1}{19} = \frac{1}{380}$

9. $\frac{5}{7} \times \frac{2}{6} = \frac{5}{21}$	10. $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$
11. $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$	12. $P(M\|S) = \frac{P(S\ and\ M)}{P(S)} = \frac{\frac{3}{10}}{\frac{3}{5}} = \frac{15}{30} = \frac{1}{2}$
13. $P(H_1\ and\ H_2) = P(H_1) \cdot P(H_2\|H_1) =$ $\frac{13}{52} \times \frac{12}{51} = \frac{156}{2652} = \frac{1}{17}$	14. $P(same\ suit) =$ $P(2Hs\ or\ 2Ds\ or\ 2Cs\ or\ 2Ss) =$ $\frac{1}{17} + \frac{1}{17} + \frac{1}{17} + \frac{1}{17} = \frac{4}{17}$

15. a) $\frac{10}{25} \times \frac{10}{25} \times \frac{10}{25} \times \frac{10}{25} \times \frac{10}{25} = \left(\frac{10}{25}\right)^5 = \frac{32}{3125}$

b) $\frac{10}{25} \times \frac{9}{24} \times \frac{8}{23} \times \frac{7}{22} \times \frac{6}{21} = \frac{6}{1265}$

16. Let A = the patient has arthritis and H = the patient has hay fever.
We want to find $P(A|H)$.
$P(A) = 0.10$, $P(H) = 0.05$, and $P(H|A) = 0.07$

$$P(A|H) = \frac{P(A\ and\ H)}{P(H)} = \frac{P(A) \times P(H|A)}{P(H)} = \frac{(0.10)(0.07)}{(0.05)} = 0.14$$

Chapter 16. Statistics

16.1 Data Collection

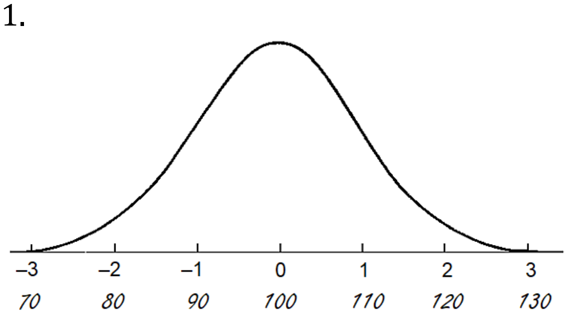
1. (4)	2. (2)
3. (2)	
4. a) The control group of plants would receive the normal level of CO2 (300 ppm). There should be two experimental groups, one which is exposed to 400 ppm and one which is exposed to 500 ppm. b) The independent variable is level of CO_2 exposure. The dependent variable is the rate of photosynthesis.	

16.2 Bias

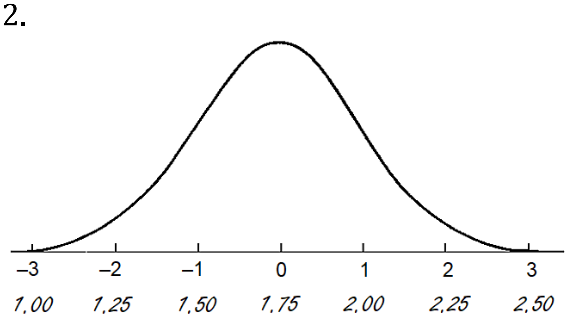
1. (3) Seniors or physics students may be biased by aspects of class scheduling specific to their groups. Selecting only students from the cafeteria would omit students who have already chosen not to eat there.	
2. (4) Allowing subjects to self-select their participation can lead to bias. Honors calculus students may tend to spend more (or less) time on homework due to the nature of their courses. Surveying only teenagers at a movie theater would omit other age groups as well as people who don't like to go to movie theaters.	
3. (4) People who attend a football game are more likely to prefer an increase in the sports budget since they are sports fans.	
4. (2)	5. (1)

16.3 Normal Distribution

1.



2.



3. $74 + 6 = 80$	4. $85 - 2(4) = 77$

5. The interval from 115 to 125 is 1 standard deviation from the mean, which is about 68% of the data.	6. 95% of the data is within 2 standard deviations from the mean, so this is the interval between $66-2(4)=58$ and $66+2(4)=74$ inches.
7. $\frac{1}{2}(80-50)=15$	8. $\frac{1}{4}(92-78)=3.5$
9. $\frac{1}{4}(69-63)=1.5$	10. $SD=\frac{1}{3}(81-57)=8$ $57+8=65$ Mean = 65 (check: $81-2(8)=65$ ✓)
11. From $56-2(5)$ to $56+2(5)$, or between 46 and 66.	12. Interval is within 1 SD of the mean, representing about 68% of the homes. $75\times 68\%=51$ homes.
13. a) 50% b) 68% c) 2.5%	14. a) $\frac{1}{2}(68\%)+\frac{1}{2}(95\%)=81.5\%$ b) $\frac{1}{2}(100\%-99.7\%)=0.15\%$ c) $0.15\%\times 424=0.636\approx 1$ student d) $\frac{1}{2}(68\%)+\frac{1}{2}(99.7\%)=83.85\%$ $83.85\%\times 424=355.5\approx 356$ students

16.4 Areas Under Normal Curves

1. normalcdf(60, 73, 65, 5) ≈ 0.787	2. normalcdf(620, 1E99, 500, 100) ≈ 0.115
3. normalcdf(54.3, 63.5, 54.3, 4.6) ≈ 48%	4. normalcdf(74, 82, 80, 4) ≈ 0.62
5. normalcdf(80, 100, 72, 9) ≈ 19%	6. normalcdf(12.5, 1E99, 11, 1.5) ≈ 0.16
7. normalcdf(3, 1E99, 2.75, 0.42) ≈ 0.28	8. a) normalcdf(90, 1E99, 75, 8) ≈ 3.04% b) normalcdf(80, 90, 75, 8) ≈ 23.56% c) normalcdf(−1E99, 60, 75, 8) ≈ 3.04%
9. normalcdf (42, 1E99, 35, 2.8) ≈ 0.62% $0.62\%\times 3000\approx 19$	10. normalcdf(550, 1E99, 510, 110) ≈ .358 $0.358\times 1000=358$

16.5 Plausible Outcomes

1. $5.6-2(0.2)$ and $5.6+2(0.2)$, or between 5.2 and 6.0. Yes, 5.9 is within the margin of error.
2. $CI=11.3095\pm 2(0.7625)$, so the interval is approximately 9.78 to 12.83. The claim is plausible because 10 is within this interval.

16.6 Difference in Means (CC)

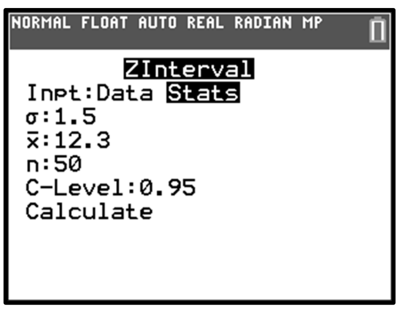
1. a) $3.2 - 2.5 = 0.7$
   b) According to the graph, at least 18 of the 100 rerandomized groups (18%) showed a mean difference of 1.0 or higher. Therefore, a mean difference of 0.7 is certainly within the 95% interval and is not statistically significant. There is no strong evidence of the success of the drug despite an observed difference between the effects on the groups in the sample.

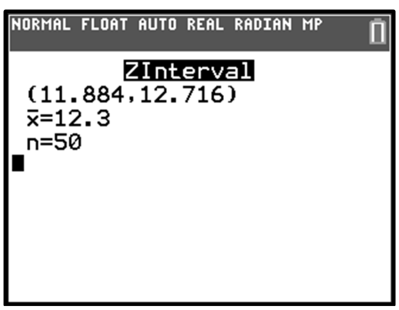
16.7 Estimate Population Parameters

1. $ME = 2\left(\frac{s}{\sqrt{n}}\right) = 2\left(\frac{12}{\sqrt{200}}\right) \approx 1.7$

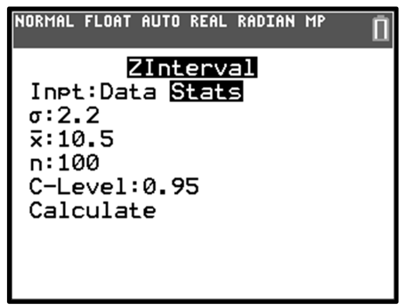
2. $ME = 2\left(\frac{s}{\sqrt{n}}\right) = 2\left(\frac{0.125}{\sqrt{50}}\right) \approx 0.04$

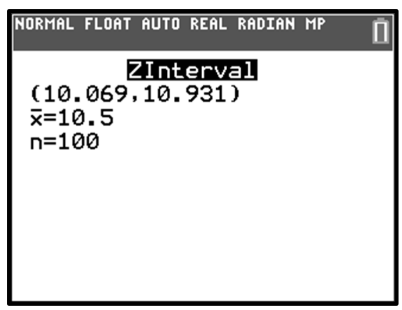
3. $ME = 2\left(\frac{s}{\sqrt{n}}\right) = 2\left(\frac{1.5}{\sqrt{50}}\right) \approx 0.42$
   $12.3 \pm 0.42 \rightarrow (11.9, 12.7)$





4. $ME = 2\left(\frac{s}{\sqrt{n}}\right) = 2\left(\frac{2.2}{\sqrt{100}}\right) = 0.44$
   $10.5 \pm 0.44 \rightarrow (10.1, 10.9)$





5. $ME = 2\left(\frac{s}{\sqrt{n}}\right) = 2\left(\frac{10.2}{\sqrt{100}}\right) = 2.04$
   $44.25 \pm 2.04 \rightarrow (42, 46)$

6. $ME = 2\left(\frac{s}{\sqrt{n}}\right) = 2(\frac{2.5}{\sqrt{49}}) \approx 0.71$
   $12 \pm 0.71 \rightarrow (11.3, 12.7)$

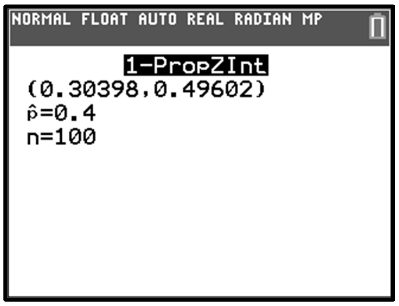
7. a) $\hat{p} = \frac{245}{350} = 0.7$
   b) $\hat{q} = 1 - \hat{p} = 0.3$
   c) $\sqrt{\frac{(0.7)(0.3)}{350}} \approx 0.024$

8. a) $\hat{p} = \frac{290}{500} = 0.58$
   b) $\hat{q} = 1 - \hat{p} = 0.42$
   c) $\sqrt{\frac{(0.58)(0.42)}{500}} \approx 0.022$

9. $ME = 2\sqrt{\frac{(0.4)(0.6)}{100}} \approx 0.098$

$0.4 - 0.098 < p < 0.4 + 0.098$

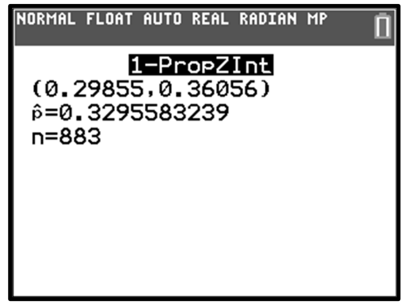
Population proportion should fall within the interval $(0.30, 0.50)$.



10. $\hat{p} = \frac{291}{883} \approx 0.330$

$ME = 2\sqrt{\frac{(0.33)(0.67)}{883}} \approx 0.032$

$0.330 \pm 0.031 \rightarrow (0.30, 0.36)$



11. a) $\hat{p} = \frac{180}{300} = 0.6 = 60\%$

$ME = 2\sqrt{\frac{(0.60)(0.40)}{300}} \approx 0.057$

$0.60 \pm 0.057 \rightarrow (0.54, 0.66)$

b) $\frac{1150}{1800} \approx 0.64$

Yes, this is plausible because 0.64 falls within the 95% confidence interval.

Regents Questions

Chapter 1. Linear Functions

1.1 Linear Systems in Three Variables [CC]

1. CC AUG '16 [23] Ans: 2
2. CC JAN '18 [3] Ans: 4
3. CC JUN '19 [23] Ans: 2
4. CC JAN '20 [18] Ans: 1
5. CC JUN '22 [8] Ans: 2
6. CC JUN '23 [11] Ans: 3
7. CC SPR '15 [10]
   $7x + 7z = 35$ (1) + (2)
   $x + z = 5$
   $4x + 10z = 62$ (2) + (3)
   $-4(x + z = 5)$
   $\underline{4x + 10z = 62}$
   $6z = 42$
   $z = 7$
   $x + (7) = 5; x = -2$
   $(-2) + 3y + 5(7) = 45;$
   $3y = 12;\ y = 4$
8. CC JUN '17 [33]
   $4y - 4z = 12$ (1) + (3)
   $y - z = 3$
   $2y + 4z = 0$ –2(1) + (2)
   $y + 2z = 0$
   $y = -2z$
   $(-2z) - z = 3;\ z = -1$
   $y - (-1) = 3;\ y = 2$
   $x + (2) + (-1) = 1;\ x = 0$
9. CC AUG '18 [33]
   $7y + 7z = 14$ 2(1) + (3)
   $y - z = 2$
   $y = z + 2$
   $-7y + 21z = 28$ 4(2) + (3)
   $y - 3z = -4$
   $y = 3z - 4$
   $z + 2 = 3z - 4$
   $2z = 6;\ z = 3$
   $y = (3) + 2;\ y = 5$
   $-4x + (5) + (3) = 16;$
   $-4x = 8;\ x = -2$
10. CC JAN '19 [33]
   Rewrite (3) as
   $-a + 6b + 2c = 14$
   $2b + 5c = 21$ (1) – (2)
   $8b + 3c = 16$ (2) + (3)
   $4(2b + 5c = 21)$
   $\underline{8b + 3c = 16}$
   $17c = 68$
   $c = 4$
   $2b + 5(4) = 21;\ 2b = 1;\ b = \frac{1}{2}$
   $a + 4\left(\frac{1}{2}\right) + 6(4) = 23;\ a + 26 = 23;$
   $a = -3$
11. CC AUG '23 [35]
   $8x + z = -6$ (1) + 2(2)
   $6x + 9z = 12$ (1) – 4(3)
   $2x + 3z = 4$
   $-3(8x + z = -6)$
   $\underline{2x + 3z = 4}$
   $-22x = 22$
   $x = -1$
   $8(-1) + z = -6; z = 2$
   $-(-1) + y - 3(2) = 0; y = 5$

Chapter 2. Irrational Expressions

2.1 Operations with Square Roots [CC]

There are no Regents exam questions on this topic.

2.2 Rationalize Monomial Denominators [CC]

There are no Regents exam questions on this topic.

2.3 Rationalize Binomial Denominators

There are no Regents exam questions on this topic.

Chapter 3. Quadratic Functions

3.1 Factor a Trinomial by Grouping

There are no Regents exam questions on this topic.

3.2 Solve Quadratics with a≠1

1. CC AUG ‘22 [32]
   $2x^2 - 7x + 4 = 11 - 2x$
   $2x^2 - 5x - 7 = 0$
   $2x^2 + 2x - 7x - 7 = 0$
   $2x(x+1) - 7(x+1) = 0$
   $(2x-7)(x+1) = 0$
   $x = \left\{-1, \frac{7}{2}\right\}$
   $y = 11 - 2(-1) = 13$
   $y = 11 - 2\left(\frac{7}{2}\right) = 4$
   $(-1,13)$ and $\left(\frac{7}{2}, 4\right)$

3.3 Graphs of Quadratic Functions

1. CC JAN ‘19 [22] Ans: 4
2. CC AUG ‘23 [5] Ans: 4

3.4 Vertex Form and Transformations

1. CC JUN ‘18 [8] Ans: 1

3.5 Focus and Directrix [CC]

1. CC SPR ‘15 [2] Ans: 4
2. CC AUG ‘16 [19] Ans: 4
3. CC JUN ‘17 [17] Ans: 4
4. CC AUG ‘17 [6] Ans: 2
5. CC JAN ‘18 [16] Ans: 1
6. CC JUN ‘18 [21] Ans: 4
7. CC AUG ‘18 [23] Ans: 4
8. CC JAN ‘19 [14] Ans: 3
9. CC JUN ‘22 [13] Ans: 1
10. CC AUG ‘22 [12] Ans: 3
11. CC JAN ‘23 [22] Ans: 4
12. CC JUN ‘23 [23] Ans: 2
13. CC JAN ’24 [9] Ans: 1
14. CC JUN ‘16 [30]
    The vertex is $(4,-3)$ and $p = 12 \div 4 = 3$. The x-coordinates of the focus and the vertex are the same. Since $p = 3$, the focus is 3 units up from the vertex, so the coordinates of the focus are (4,0).

15. CC JUN '19 [35]

The x-coordinate of the vertex is $\frac{-1-5}{2} = -3$, so the vertex is $(4, -3)$.

$p = y_{focus} - y_{vertex} = -1 + 3 = 2$.

$y = \frac{1}{8}(x-4)^2 - 3$

16. CC JAN '20 [28]

The vertex is (3,6), which is 5 units up from the focus, so the directrix is 5 units above the vertex at $y = 11$.

17. CC AUG '23 [30]

p is the difference of the y-values between the focus and vertex:

$p = y_{focus} - y_{vertex} = 8 - 7 = 1$

p is also the distance between the vertex and directrix:

$p = y_{vertex} - y_{directrix}$

By substitution, $1 = 7 - y$, so the directrix is $y = 6$.

Chapter 4. Imaginary Numbers

4.1 Set of Complex Numbers

There are no Regents exam questions on this topic.

4.2 Operations with Complex Numbers

1. CC JUN '16 [3] Ans: 2
2. CC JUN '17 [4] Ans: 2
3. CC AUG '17 [2] Ans: 3
4. CC JUN '18 [5] Ans: 3
5. CC AUG '18 [15] Ans: 3
6. CC JAN '19 [11] Ans: 1
7. CC JUN '19 [15] Ans: 1
8. CC JAN '20 [22] Ans: 1
9. CC JUN '22 [23] Ans: 4
10. CC AUG '22 [2] Ans: 4
11. CC JAN '23 [8] Ans: 3
12. CC JUN '23 [7] Ans: 3
13. CC SPR '15 [6]
    $(4-3i)(5+2yi-5+2yi)$
    $(4-3i)(4yi)$
    $16yi-12yi^2$
    $12y+16yi$
14. CC AUG '16 [27]
    $xi(-6i)^2 = xi(36i^2) = -36xi$
15. CC JAN '17 [25]
    $(1-i)(1-i)(1-i) =$
    $(1-2i+i^2)(1-i) =$
    $-2i(1-i) = -2i+2i^2 = -2-2i$
16. CC JAN '18 [25]
    $i^2 = -1$, not 1.
    $6+10i-4(-1) = 10+10i$
17. CC AUG '19 [27]
    $-\frac{1}{2}i^3\left(\sqrt{-9}-4\right)-3i^2 =$
    $\frac{1}{2}i(3i-4)+3 =$
    $-\frac{3}{2}-2i+3 = \frac{3}{2}-2i$
18. CC AUG '23 [29]
    $(-5xi-4i)^2$
    $25x^2i^2+40xi^2+16i^2$
    $-25x^2-40x-16$
19. CC JAN '24 [31]
    $4x^2i^6-12xyi^3+9y^2$
    $-4x^2+12xyi+9y^2$

4.3 Imaginary Roots

1. CC FALL '15 [4] Ans: 3
2. CC JUN '16 [12] Ans: 1
3. CC AUG '16 [1] Ans: 4
4. CC JAN '17 [11] Ans: 4
5. CC JUN '17 [7] Ans: 4
6. CC AUG '17 [3] Ans: 3
7. CC AUG '18 [9] Ans: 3
8. CC JAN '19 [5] Ans: 2
9. CC JAN '19 [9] Ans: 4
10. CC JUN '19 [12] Ans: 4
11. CC JAN '20 [20] Ans: 2
12. CC AUG '22 [8] Ans: 1
13. CC JAN '23 [10] Ans: 4
14. CC JAN '23 [24] Ans: 2
15. CC JUN '23 [12] Ans: 1
16. CC AUG '23 [8] Ans: 2
17. CC JAN '24 [2] Ans: 2
18. CC JAN '24 [16] Ans: 3
19. CC JUN '18 [27]
    $x = \frac{-5 \pm \sqrt{5^2-4(2)(8)}}{2(2)} = -\frac{5}{4} \pm \frac{i\sqrt{39}}{4}$

20. CC JUN '22 [25]
Yes, because the discriminant $b^2 - 4ac = (-4)^2 - 4(1)(13) = -36$ is negative.

21. CC AUG '23 [27]
$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(8)}}{2(3)} = -\frac{5}{6} \pm \frac{i\sqrt{71}}{6}$

Chapter 5. Circles

5.1 Equations of Circles

1. CC JUN '16 [19] Ans: 4

5.2 Circle-Linear Systems

1. CC AUG '17 [19] Ans: 1
2. CC AUG '19 [16] Ans: 3
3. CC JAN '24 [7] Ans: 4
4. CC JUN '16 [33]
   $y = -x + 5$
   $(x-3)^2 + (-x+5+2)^2 = 16$
   $x^2 - 6x + 9 + x^2 - 14x + 49 = 16$
   $2x^2 - 20x + 42 = 0$
   $x^2 - 10x + 21 = 0$
   $(x-3)(x-7) = 0$
   $x = \{3, 7\}$
   $y = -(3) + 5 = 2;$
   $y = -(7) + 5 = -2$
   $(3,2)$ and $(7,-2)$
5. CC AUG '18 [31]
   $x^2 + (x-28)^2 = 400$
   $x^2 + x^2 - 56x + 784 = 400$
   $2x^2 - 56x + 384 = 0$
   $x^2 - 28x + 192 = 0$
   $(x-12)(x-16) = 0$
   $x = \{12, 16\}$
   $y = (12) - 28 = -16;$
   $y = (16) - 28 = -12$
   $(12,-16)$ and $(16,-12)$
6. CC JAN '20 [35]
   $y = -x + 1$
   $(x-2)^2 + (-x+1-3)^2 = 16$
   $(x-2)^2 + (-x-2)^2 = 16$
   $x^2 - 4x + 4 + x^2 + 4x + 4 = 16$
   $2x^2 + 8 = 16$
   $2x^2 = 8$
   $x^2 = 4$
   $x = \pm 2$
   $y = -(-2) + 1 = 3;$
   $y = -2 + 1 = -1$
   $(-2,3)$ and $(2,-1)$
7. CC JUN '22 [36]
   $y = 2x - 5$
   $x^2 + (2x-5)^2 = 25$
   $x^2 + 4x^2 - 20x + 25 = 25$
   $5x^2 - 20x = 0$
   $5x(x-4) = 0$
   $x = \{0, 4\}$
   $y = 2(0) - 5 = -5;$
   $y = 2(4) - 5 = 3$
   $(0,-5)$ and $(4,3)$
8. CC JUN '23 [35]
   $(x-2)^2 - (-2x+7-3)^2 = 20$
   $(x-2)^2 + (-2x+4)^2 = 20$
   $x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$
   $5x^2 - 20x = 0$
   $5x(x-4) = 0$
   $x = \{0, 4\}$
   $y = -2(0) + 7 = 7$
   $y = -2(4) + 7 = -1$
   $(0,7)$ and $(4,-1)$

Chapter 6. Polynomial Functions

6.1 Operations with Functions

1. CC JUN '16 [8] Ans: 4
2. CC JAN '17 [10] Ans: 3
3. CC JUN '17 [9] Ans: 2
4. CC JUN '18 [13] Ans: 1
5. CC AUG '18 [3] Ans: 4
6. CC JUN '22 [10] Ans: 3
7. CC AUG '23 [22] Ans: 3
8. CC JAN '18 [33]
$(2x^2 + x - 3)(x - 1) -$
$[(2x^2 + x - 3) + (x - 1)] =$
$(2x^3 - 2x^2 + x^2 - x - 3x + 3) -$
$(2x^2 + 2x - 4) =$
$2x^3 - 3x^2 - 6x + 7$
9. CC JAN '23 [30]
$(x^3 + 2x - 1)(x^2 + 7) - 3(x^4 - 5x) =$
$x^5 + 7x^3 + 2x^3 + 14x - x^2 - 7 -$
$3x^4 + 15x =$
$x^5 - 3x^4 + 9x^3 - x^2 + 29x - 7$

6.2 Long Division

1. CC FALL '15 [3] Ans: 1
2. CC JUN '16 [14] Ans: 2
3. CC AUG '17 [13] Ans: 1
4. CC JAN '18 [9] Ans: 4
5. CC AUG '19 [10] Ans: 1
6. CC JUN '22 [18] Ans: 2
7. CC JUN '23 [13] Ans: 1
8. CC JUN '18 [29]

$$\begin{array}{r|l} & 2a^2 + 5a + 2 \\ \hline 3a - 2 & 6a^3 + 11a^2 - 4a - 9 \\ & -(6a^3 - 4a^2) \\ \hline & 15a^2 - 4a \\ & -(15a^2 - 10a) \\ \hline & 6a - 9 \\ & -(6a - 4) \\ \hline & -5 \end{array}$$

$$2a^2 + 5a + 2 - \frac{5}{3a - 2}$$

9. CC JUN '19 [30]
$p(x) = (x - 1)(x^2 + 7) + 5 =$
$x^3 - x^2 + 7x - 2$

6.3 Synthetic Division

1. CC AUG '16 [11] Ans: 2
2. CC AUG '18 [5] Ans: 3
3. CC JAN '20 [7] Ans: 3
4. CC JUN '22 [3] Ans: 1
5. CC AUG '22 [17] Ans: 2
6. CC JAN '23 [5] Ans: 1
7. CC AUG '23 [2] Ans: 2
8. CC JAN '24 [8] Ans: 2

9. CC JAN ‘17 [32]

[2]	3	7	−20
		6	26
	3	13	| 6

$3x + 13 + \frac{6}{x-2}$

10. CC JAN ‘19 [34]

[−2]	1	2	4	−10
		−2	0	−8
	1	0	4	| −18

$x^3 + 4 - \frac{18}{x+2}$

11. CC AUG ‘22 [35]

[4]	3	−4	2	−1
		12	32	136
	3	8	34	| 135

$3x^2 + 8x + 34 + \frac{135}{x-4}$

$x = 4$ is not a root because $r(x) \neq 0$.

6.4 Remainder Theorem

1. CC AUG ‘16 [21] Ans: 3
2. CC JAN ‘17 [20] Ans: 2
3. CC JUN ‘17 [11] Ans: 1
4. CC AUG ‘17 [20] Ans: 2
5. CC JAN ‘18 [19] Ans: 4
6. CC JUN ‘18 [12] Ans: 3
7. CC JUN ‘19 [7] Ans: 4
8. CC JUN ‘22 [6] Ans: 2
9. CC JAN ‘23 [7] Ans: 2
10. CC JUN ‘23 [3] Ans: 4
11. CC AUG ‘23 [20] Ans: 1
12. CC SPR ‘15 [7]
By the Factor Theorem, $x - a$ is a factor of $f(x)$ only when
$f(a) = 0$.
$f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 0$

13. CC FALL ‘15 [15]
$0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15$
$0 = -750 + 25b + 260 + 15$
$475 = 25b$
$b = 19$
$z(x) = 6x^3 + 19x^2 - 52x + 15$
By the Factor Theorem, since $z(-5) = 0$, then $x + 5$ is a factor of $z(x)$.

[−5]	6	19	−52	15
		−30	55	−15
	6	−11	3	| 0

$(x + 5)(6x^2 - 11x + 3) = 0$
$(x + 5)(3x - 1)(2x - 3) = 0$
$x = \left\{-5, \frac{1}{3}, \frac{3}{2}\right\}$

14. CC JUN ‘16 [27]
By the Factor Theorem, $x - a$ is a factor of $f(x)$ only when
$f(a) = 0$.
$2(5)^3 - 4(5)^2 - 7(5) - 10 = 105$, so
$x - 5$ is not a factor.

15. CC JUN ‘17 [25]
$r(2) = (2)^3 - 4(2)^2 + 4(2) - 6 = -6$
By the Factor Theorem, $x - a$ is a factor of $f(x)$ only when $f(a) = 0$.
Since $r(2) = -6$, $x - 2$ is not a factor of $r(x)$.

16. CC AUG ‘18 [34]
$j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48$
$j(-1) = 0$
By the Factor Theorem, since $j(-1) = 0$, then $x + 1$ is a factor of $j(x)$.

$\boxed{-1}$	2	−1	−35	16	48
		−2	3	32	−48
	2	−3	−32	48	0

Factor by grouping:
$2x^3 - 3x^2 - 32x + 48$
$x^2(2x - 3) - 16(2x - 3)$
$(x^2 - 16)(2x - 3)$
$(x + 4)(x - 4)(2x - 3)$
$(x + 1)(x + 4)(x - 4)(2x - 3) = 0$
$x = \left\{-1, -4, 4, \frac{3}{2}\right\}$

17. CC AUG ‘19 [29]
$P(-2) = 60$ and $Q(-2) = 0$; $(x + 2)$ is a factor of $Q(x)$ by the Factor Theorem since $Q(-2) = 0$.

18. CC JAN ‘10 [26]
$m(3) = 3^3 - 3^2 - 5(3) - 3 = 0$; $x - 3$ is a factor of $m(x)$ by the Factor Theorem since $m(3) = 0$.

19. CC JUN ‘23 [28]
$g(3) = 0$
$(3)^3 + a(3)^2 - 5(3) + 6 = 0$
$9a + 18 = 0$
$a = -\frac{18}{9}$
$a = -2$

20. CC JAN ’24 [26]
$2(-4)^3 + 10(-4)^2 + 4(-4) - 16 = 0$;
$x + 4$ is a factor by the Factor Theorem since evaluating the expression for $x = -4$ results in 0.

6.5 Factor Polynomials

1. CC FALL ‘15 [5] Ans: 4
2. CC AUG ‘16 [5] Ans: 3
3. CC AUG ‘16 [15] Ans: 3
4. CC JAN ‘17 [3] Ans: 4
5. CC AUG ‘17 [15] Ans: 1
6. CC AUG ‘18 [14] Ans: 4
7. CC JAN ‘19 [3] Ans: 1
8. CC JUN ‘19 [11] Ans: 2
9. CC AUG ‘19 [1] Ans: 2
10. CC AUG ‘19 [4] Ans: 2
11. CC JAN ‘20 [6] Ans: 3
12. CC JAN ‘23 [1] Ans: 1
13. CC JUN ‘23 [2] Ans: 3
14. CC JUN ‘23 [10] Ans: 2
15. CC FALL ‘15 [12]
The expression is of the form $u^2 - 5u - 6$ or $(u - 6)(u + 1)$.
Let $u = 4x^2 + 5x$.
$(4x^2 + 5x - 6)(4x^2 + 5x + 1)$
$(4x - 3)(x + 2)(4x + 1)(x + 1)$

16. CC JUN ‘17 [27]
$x^2(4x - 1) + 4(4x - 1)$
$(x^2 + 4)(4x - 1)$

17. CC JAN ‘18 [28]
$3x^3 + x^2 + 3xy + y$
$x^2(3x + 1) + y(3x + 1)$
$(x^2 + y)(3x + 1)$

18. CC AUG ‘18 [25]
$(x^2 - 6)(x^2 + 2)$

19. CC JUN ‘22 [28]
$-x(2x^3 - x^2 - 18x + 9)$
$-x[x^2(2x - 1) - 9(2x - 1)]$
$-x(x^2 - 9)(2x - 1)$
$-x(x + 3)(x - 3)(2x - 1)$

20. CC AUG ‘22 [26]
$x^2(x - 2) - 9(x - 2)$
$(x^2 - 9)(x - 2)$
$(x + 3)(x - 3)(x - 2)$

21. CC JAN ‘23 [31]
$(x^2 - 4)(x^2 - 1)$
$(x + 2)(x - 2)(x + 1)(x - 1)$

22. CC AUG '23 [25]
$x^2(2x-3)-9(2x-3)$
$(x^2-9)(2x-3)$
$(x+3)(x-3)(2x-3)$

23. CC JAN '24 [25]
$x^2(x+4)-9(x+4)$
$(x^2-9)(x+4)$
$(x+3)(x-3)(x+4)$

6.6 Find Roots by Factoring

1. CC JUN '16 [6] Ans: 1
2. CC AUG '17 [8] Ans: 4
3. CC AUG '18 [21] Ans: 4
4. CC AUG '19 [21] Ans: 4
5. CC JAN '20 [19] Ans: 4
6. CC AUG '22 [6] Ans: 2
7. CC JAN '24 [3] Ans: 2
8. CC JUN '19 [33]
$(4x^2+9)(2x+3)(2x-3)$;
No, because $4x^2+9=0$ leads to roots of $\pm\frac{3}{2}i$
9. CC JAN '23 [25]
$3(x^3+4x^2-x-4)=0$
$3\left(x^2(x+4)-(x+4)\right)=0$
$3(x^2-1)(x+4)=0$
$\{-4,-1,1\}$

6.7 Root Theorems

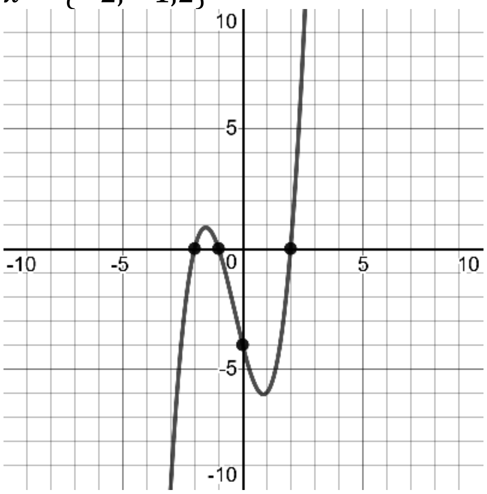
There are no Regents exam questions on this topic.

6.8 Properties of Polynomial Graphs

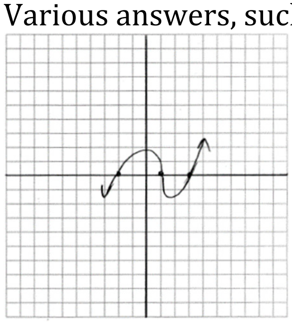
1. CC SPR '15 [1] Ans: 1
2. CC JUN '16 [4] Ans: 3
3. CC JUN '16 [20] Ans: 2
4. CC JUN '17 [1] Ans: 1
5. CC AUG '17 [12] Ans: 4
6. CC JAN '18 [17] Ans: 3
7. CC JUN '18 [16] Ans: 2
8. CC AUG '18 [4] Ans: 1
9. CC JAN '19 [8] Ans: 1
10. CC JAN '19 [19] Ans: 1
11. CC JUN '19 [21] Ans: 4
12. CC AUG '19 [8] Ans: 2
13. CC JAN '20 [5] Ans: 3
14. CC JAN '23 [16] Ans: 2
15. CC JAN '23 [18] Ans: 2
16. CC AUG '23 [18] Ans: 4
17. CC AUG '23 [24] Ans: 2
18. CC JAN '24 [5] Ans: 1
19. CC JAN '24 [14] Ans: 2

6.9 Graph Polynomial Functions

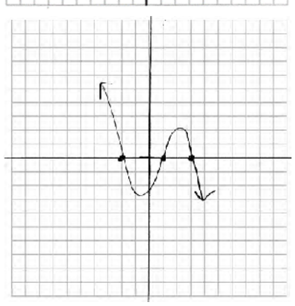
1. CC AUG '16 [33]
   $0 = x^2(x+1) - 4(x+1)$
   $0 = (x^2 - 4)(x+1)$
   $0 = (x+2)(x-2)(x+1)$
   $x = \{-2, -1, 2\}$



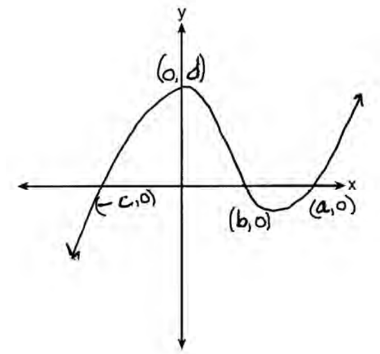
2. CC JAN '17 [29]
   Various answers, such as



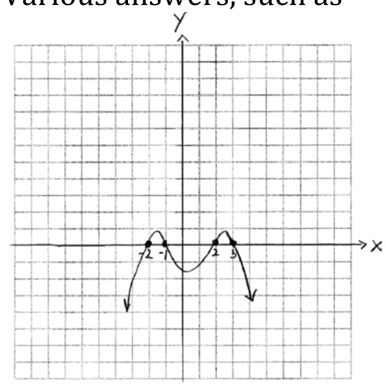
or



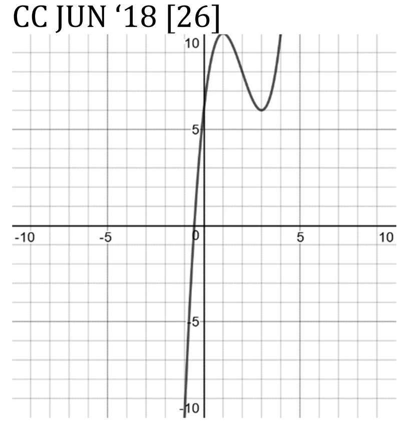
3. CC AUG '17 [32]
   Various answers, such as



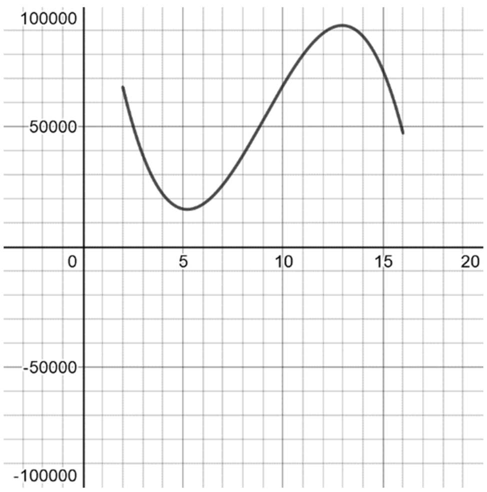
4. CC JAN '18 [31]
   Various answers, such as



5. CC JUN '18 [26]

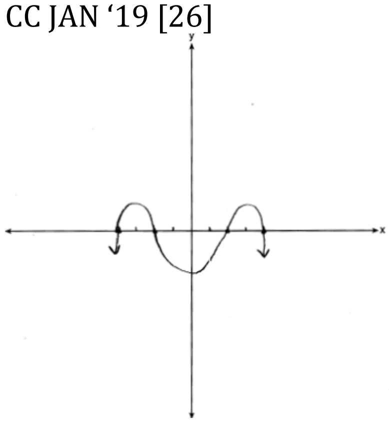


6. CC AUG '18 [37]
$P(x) = R(x) - C(x) = -330x^3 + 9{,}000x^2 - 67{,}000x + 167{,}000$

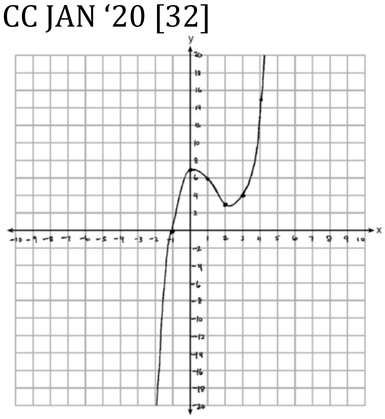


Least profitable at year 5 because there is a minimum at $P(5)$. Most profitable at year 13 because there is a maximum at $P(13)$.

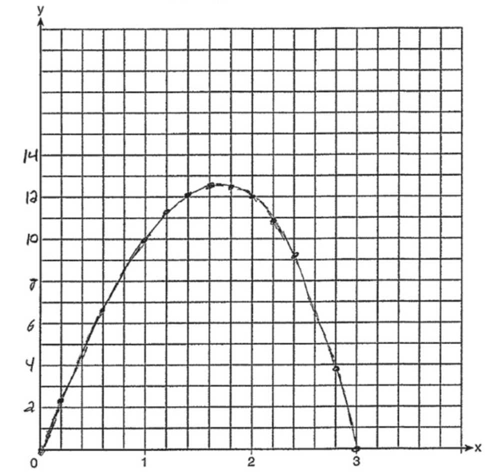
7. CC JAN '19 [26]



8. CC JAN '20 [32]

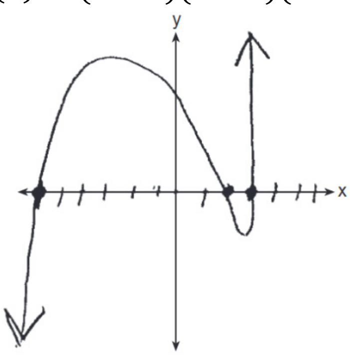


9. CC AUG '22 [34]

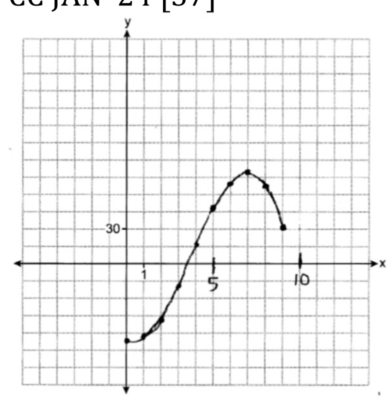


maximum is 12.6 cu. in.

10. CC JUN '23 [33]
$p(x) = (x - 2)(x - 3)(x + 6)$



11. CC JAN '24 [37]



(7,78); the maximum profit is $78,000 when 7000 sweatshirts are sold;
$x = 3{,}549$ sweatshirts, when $p(x)$ is first greater than 0.

6.10 Polynomial Transformations

1. CC AUG '18 [17] Ans: 4

2. CC JAN '18 [36]
$f(x) = x^2(x+4)(x-3)$
$g(x) = f(x+2) = (x+2)^2(x+6)(x-1)$

6.11 Systems of Polynomial Functions

1. CC JUN '16 [22] Ans: 4
2. CC AUG '16 [6] Ans: 3
3. CC JAN '17 [5] Ans: 4
4. CC JUN '18 [1] Ans: 2
5. CC JUN '19 [3] Ans: 3
6. CC FALL '15 [7]
$-2x + 1 = -2x^2 + 3x + 1$
$2x^2 - 5x = 0$
$x(2x-5) = 0$
$x = \left\{0, \frac{5}{2}\right\}$
7. CC AUG '19 [36]
$x^2 - 6x = -17$
$x^2 - 6x + 9 = -17 + 9$
$(x-3)^2 = -8$
$x - 3 = \pm 2i\sqrt{2}$
$x = 3 \pm 2i\sqrt{2}$
A real solution would appear as a point of intersection on the graph; since the parabola and line do not intersect, the solutions are imaginary.
8. CC AUG '23 [26]
$x^2 + 8x - 5 = 8x - 4$
$x^2 - 1 = 0$
$(x+1)(x-1) = 0$
$x = \{-1, 1\}$

6.12 Polynomial Identities [CC]

1. CC AUG '16 [20] Ans: 4
2. CC JAN '18 [6] Ans: 2
3. CC JUN '18 [22] Ans: 4
4. CC JUN '19 [2] Ans: 4
5. CC JAN '20 [3] Ans: 3
6. CC AUG '22 [19] Ans: 1
7. CC JAN '23 [11] Ans: 2
8. CC JUN '23 [22] Ans: 4
9. CC JAN '24 [17] Ans: 4
10. CC FALL '15 [11]
Let x and $x+1$ represent the integers.
$(x+1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$
$2x$ is an even integer, so $2x+1$ is an odd integer.
11. CC JUN '16 [31]
$1 + \frac{1}{x^3+8} = \frac{x^3+8}{x^3+8} + \frac{1}{x^3+8} = \frac{x^3+9}{x^3+8}$
12. CC JAN '17 [33]
(1) $2x^3 - 10x^2 + 11x - 7 = (x-4)(2x^2 + hx + 3) + k$
(2) $2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k$
(3) $-2x^2 + 8x + 5 = hx^2 - 4hx + k$
(4) So, $h = -2$ and $k = 5$
13. CC AUG '17 [27]
$(x^2 - y^2)^2 + (2xy)^2$
$= x^4 - 2x^2y^2 + y^4 + (2xy)^2$
$= x^4 - 2x^2y^2 + y^4 + 4x^2y^2$
$= x^4 + 2x^2y^2 + y^4$
$= (x^2 + y^2)^2$
14. CC JAN '19 [27]
$(a+b)^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab(a+b)$
No. Erin's shortcut only works if $3ab(a+b) = 0$; that is, only if $a = 0$, $b = 0$, or $a = -b$.

Chapter 7. Radicals and Rational Exponents

7.1 nth Roots

There are no Regents exam questions on this topic.

7.2 Operations with Radicals

1. CC AUG '22 [13] Ans: 1

7.3 Solve Equations with Radicals

1. CC JUN '16 [5] Ans: 3
2. CC AUG '17 [4] Ans: 2
3. CC JAN '18 [2] Ans: 3
4. CC AUG '18 [7] Ans: 3
5. CC JUN '19 [19] Ans: 2
6. CC JAN '20 [10] Ans: 2
7. CC AUG '23 [15] Ans: 3
8. CC SPR '15 [8]
   $\sqrt{x-5} = -x+7$
   $x-5 = x^2-14x+49$
   $x^2-15x+54=0$
   $(x-6)(x-9)=0$
   $x=\{6,\not{9}\}$
   Reject $x=9$ because
   $\sqrt{9-5}+9 \neq 7.$
   $x=6$
9. CC AUG '16 [35]
   $\left(\sqrt{2x-7}\right)^2 = (5-x)^2$
   $2x-7 = 25-10x+x^2$
   $x^2-12x+32=0$
   $(x-4)(x-8)=0$
   $x=\{4,\not{8}\}$
   Reject $x=8$ because
   $\sqrt{2(8)-7}+8 \neq 5.$
   $x=4$
10. CC JAN '17 [37]
    $0=\sqrt{t}-2t+6$
    $\sqrt{t}=2t-6$
    $\left(\sqrt{t}\right)^2=(2t-6)^2$
    $t=4t^2-24t+36$
    $4t^2-25t+36=0$
    $(4t-9)(t-4)=0$
    $t=\left\{\not{\tfrac{9}{4}},4\right\}$
    Reject $t=\frac{9}{4}$ because $2\left(\frac{9}{4}\right)-6<0.$
    $\sqrt{1}-2(1)+6=5$
    $\sqrt{3}-2(3)+6=\sqrt{3}$
    $5-\sqrt{3}\approx 3.268$
    327 mph
11. CC JUN '17 [30]
    $\sqrt{x-4}=-x+6$
    $x-4=x^2-12x+36$
    $x^2-13x+40=0$
    $(x-5)(x-8)=0$
    $x=\{5,\not{8}\}$
    8 is extraneous because
    $\sqrt{8-4}\neq -8+6$
12. CC JUN '18 [33]
    $\sqrt{6-2x}+x=2x+30-9$
    $\sqrt{6-2x}=x+21$
    $6-2x=x^2+42x+441$
    $x^2+44x+435=0$
    $(x+29)(x+15)=0$
    $x=\{\not{-29},-15\}$
    –29 is extraneous because
    $\sqrt{6-2(-29)}-29\neq 2(-14)-9.$

13. CC JAN '19 [36]
$3\sqrt{x} - 2x = -5$
$3\sqrt{x} = 2x - 5$
$9x = 4x^2 - 20x + 25$
$4x^2 - 29x + 25 = 0$
$(4x - 25)(x - 1) = 0$
$x = \left\{\frac{25}{4}, \cancel{1}\right\}$
1 is extraneous because
$3\sqrt{1} - 2(1) \neq -5$

14. CC AUG '19 [37]
$B = 1.69\sqrt{30 + 4.45} - 3.49 \approx 6$, which is a steady breeze;
$15 = 1.69\sqrt{s + 4.45} - 3.49$
$18.49 = 1.69\sqrt{s + 4.45}$
$\frac{18.49}{1.69} = \sqrt{s + 4.45}$
$\left(\frac{18.49}{1.69}\right)^2 = s + 4.45$
$s = \left(\frac{18.49}{1.69}\right)^2 - 4.45 \approx 115$
B values of 9.5 to 10.49 would round to a 10
$9.5 = 1.69\sqrt{s + 4.45} - 3.49$ solves to $s \approx 55$ and
$10.49 = 1.69\sqrt{s + 4.45} - 3.49$ solves to $s \approx 64$, so the range is 55 to 64 mph

15. CC JUN '22 [34]
$t = 2\pi\sqrt{\frac{67}{9.81}} \approx 16.4;$
$9.6 = 2\pi\sqrt{\frac{L}{9.81}}$
$\frac{9.6}{2\pi} = \sqrt{\frac{L}{9.81}}$
$1.528 \approx \frac{\sqrt{L}}{3.132}$
$4.785 \approx \sqrt{L}$
$L \approx 22.9$

16. CC AUG '22 [27]
$4x + 1 = (11 - x)^2$
$4x + 1 = 121 - 22x + x^2$
$x^2 - 26x + 120 = 0$
$(x - 6)(x - 20) = 0$
$x = \{6, \cancel{20}\}$
20 is extraneous

17. CC JAN '23 [33]
$\sqrt{49 - 10x} = 2x - 5$
$49 - 10x = 4x^2 - 20x + 25$
$4x^2 - 10x - 24 = 0$
$2x^2 - 5x - 12 = 0$
$(2x + 3)(x - 4) = 0$
$x = \left\{\cancel{-\frac{3}{2}}, 4\right\}$
$-\frac{3}{2}$ is extraneous

18. CC JUN '23 [26]
$3x + 7 = (x - 1)^2$
$3x + 7 = x^2 - 2x + 1$
$x^2 - 5x - 6 = 0$
$(x - 6)(x + 1) = 0$
$x = \{\cancel{-1}, 6\}$
–1 is extraneous

19. CC JAN '24 [34]
$2x - 6 = 2\sqrt{x - 1}$
$x - 3 = \sqrt{x - 1}$
$(x - 3)^2 = x - 1$
$x^2 - 6x + 9 = x - 1$
$x^2 - 7x + 10 = 0$
$(x - 2)(x - 5) = 0$
$x = \{\cancel{2}, 5\}$
2 is extraneous

7.4 Graphs of Radical Functions

There are no Regents exam questions on this topic.

7.5 Negative Exponents

There are no Regents exam questions on this topic.

7.6 Rational Exponents

1. CC JUN '16 [1] Ans: 4
2. CC JAN '17 [7] Ans: 2
3. CC JUN '17 [16] Ans: 4
4. CC AUG '17 [23] Ans: 4
5. CC JAN '18 [11] Ans: 4
6. CC JUN '18 [20] Ans: 2
7. CC AUG '18 [12] Ans: 3
8. CC JUN '19 [8] Ans: 1
9. CC AUG '19 [14] Ans: 4
10. CC JAN '20 [1] Ans: 1
11. CC JUN '22 [1] Ans: 1
12. CC JAN '23 [12] Ans: 3
13. CC JUN '23 [6] Ans: 2
14. CC JUN '23 [20] Ans: 4
15. CC JAN '24 [13] Ans: 3
16. CC SPR '15 [5]
$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} = x^y;\ x^{\frac{4}{3}} = x^y;\ y = \frac{4}{3}$
17. CC AUG '16 [26]
$\left(3^{\frac{1}{5}}\right)^2 = \left(\sqrt[5]{3}\right)^2 = \sqrt[5]{9}$
18. CC JAN '17 [30]
$\left(x^{\frac{5}{3}}\right)^{\frac{6}{5}} = \left(y^{\frac{5}{6}}\right)^{\frac{6}{5}}$
$x^2 = y$
19. CC JUN '17 [31]
$\left(x^{\frac{1}{3}}\right)\left(x^{\frac{1}{2}}\right) = \left(x^{\frac{2}{6}}\right)\left(x^{\frac{3}{6}}\right) = x^{\frac{5}{6}}$
20. CC AUG '17 [25]
$(-8)^{\frac{4}{3}} = \left(\sqrt[3]{-8}\right)^4 = (-2)^4 = 16$
21. CC JAN '18 [32]
The denominator of the rational exponent represents the index of a root, and the 4th root of 81 is 3 and 3^3 is 27.
22. CC AUG '18 [26]
$\frac{2x^{\frac{3}{2}}}{2x^{\frac{2}{2}}} = x^{\frac{1}{2}} = \sqrt{x}$
23. CC JAN '19 [25]
$\frac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}} = \frac{x^{\frac{2}{3}}y^{\frac{5}{3}}}{x^{\frac{3}{4}}y} = \frac{x^{\frac{8}{12}}y^{\frac{5}{3}}}{x^{\frac{9}{12}}y^{\frac{3}{3}}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$
24. CC JUN '19 [29]
No. $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right) = \left(x^{\frac{2}{7}}\right)\left(x^{\frac{3}{5}}\right) =$
$x^{\frac{31}{35}} = \sqrt[35]{x^{31}}$.
25. CC AUG '19 [26]
The denominator of the rational exponent represents the index of a root, and the numerator of the rational exponent represents the power of the base.
$9^{\frac{5}{2}} = \left(\sqrt{9}\right)^5 = 243$.
26. CC JAN '20 [25]
Yes. $\left(p^2n^{\frac{1}{2}}\right)^8\sqrt{p^5n^4} = p^{16}n^4p^{\frac{5}{2}}n^2 =$
$p^{18\frac{1}{2}}n^6 = p^{18}n^6\sqrt{p}$
27. CC JUN '22 [30]
$\sqrt[3]{81} = \sqrt[3]{3^4} = 3^{\frac{4}{3}}$, so $a = \frac{4}{3}$.
28. CC AUG '22 [28]
$\frac{17}{8} - \frac{10}{8} = \frac{7}{8}$
$\left(y^{\frac{7}{8}}\right)^{-4} = y^{-\frac{7}{2}}$, so $n = -\frac{7}{2}$.
29. CC AUG '23 [31]
$\frac{x \cdot x^{\frac{3}{2}}}{x^{\frac{5}{3}}} = \frac{x^{\frac{5}{2}}}{x^{\frac{5}{3}}} = x^{\frac{5}{6}}$
30. CC JAN '24 [28]
$\frac{y^4}{y^{\frac{2}{3}}} = y^{\frac{10}{3}}$, so $n = \frac{10}{3}$.

Chapter 8. Rational Functions

8.1 Undefined Expressions

1. CC AUG ‘17 [1] Ans: 1

8.2 Simplify Rational Expressions

1. CC JUN ‘17 [23] Ans: 4
2. CC JAN ‘18 [18] Ans: 3
3. CC JUN ‘18 [3] Ans: 3
4. CC JAN ‘19 [21] Ans: 4
5. CC JAN ‘20 [23] Ans: 1
6. CC AUG ‘22 [15] Ans: 2
7. CC AUG ‘23 [21] Ans: 4
8. CC JAN ’24 [12] Ans: 1
9. CC JUN ‘23 [31]

$$\frac{x^2(2x+1)-9(2x+1)}{x(3-x)}$$
$$\frac{(x^2-9)(2x+1)}{x(3-x)}$$
$$\frac{(x+3)\cancel{(x-3)}(2x+1)}{-x\cancel{(x-3)}}$$
$$-\frac{(x+3)(2x+1)}{x}$$

8.3 Multiply and Divide Rational Expressions

There are no Regents exam questions on this topic.

8.4 Add and Subtract Rational Expressions

1. CC AUG ‘19 [7] Ans: 2

8.5 Simplify Complex Fractions

There are no Regents exam questions on this topic.

8.6 Solve Rational Equations

1. CC FALL ‘15 [1] Ans: 4
2. CC AUG ‘16 [17] Ans: 3
3. CC JAN ‘17 [17] Ans: 1
4. CC JUN ‘17 [19] Ans: 1
5. JAN’18 [12] Ans: 1
6. CC JUN ‘18 [9] Ans: 4
7. CC JAN ‘19 [15] Ans: 3
8. CC AUG ‘19 [15] Ans: 4
9. CC AUG ‘22 [18] Ans: 3
10. CC JAN ‘23 [9] Ans: 3
11. CC JUN ‘23 [19] Ans: 1
12. CC JAN ’24 [22] Ans: 3
13. CC JUN ‘16 [25]

$$3x\left[\frac{1}{x}-\frac{1}{3}=-\frac{1}{3x}\right]$$
$$3-x=-1$$
$$x=4$$

14. CC AUG '17 [33]
$(p+3)(p-5)\left[\frac{3p}{p-5}-\frac{2}{p+3}=\frac{p}{p+3}\right]$
$3p(p+3)-2(p-5)=p(p-5)$
$3p^2+9p-2p+10=p^2-5p$
$2p^2+12p+10=0$
$p^2+6p+5=0$
$(p+5)(p+1)=0$
$\{-5,-1\}$

15. CC AUG '18 [29]
$6(x+3)\left[\frac{-3}{x+3}+\frac{1}{2}=\frac{x}{6}-\frac{1}{2}\right]$
$-18+3(x+3)=$
$x(x+3)-3(x+3)$
$-18+3x+9=x^2+3x-3x-9$
$x^2-3x=0$
$x(x-3)=0$
$\{0,3\}$

16. CC JUN '19 [26]
$4x(x+1)\left[\frac{7}{2x}-\frac{2}{x+1}=\frac{1}{4}\right]$
$14(x+1)-8x=x(x+1)$
$14x+14-8x=x^2+x$
$x^2-5x-14=0$
$(x+2)(x-7)=0$
$\{-2,7\}$

17. CC JUN '22 [27]
$n^2\left[\frac{2}{n^2}+\frac{3}{n}=\frac{4}{n^2}\right]$
$2+3n=4$
$3n=2$
$n=\frac{2}{3}$

18. CC AUG '23 [34]
$(x-6)(x-2)\left[\frac{1}{x-6}+\frac{x}{x-2}=\frac{4}{(x-6)(x-2)}\right]$
$x-2+x^2-6x=4$
$x^2-5x-6=0$
$(x+1)(x-6)=0$
$\{-1, 6\}$

8.7 Model Rational Expressions and Equations

1. CC JUN '16 [2] Ans: 3
2. CC JUN '17 [22] Ans: 3
3. CC JUN '18 [24] Ans: 3
4. CC JUN '19 [16] Ans: 1
5. CC AUG '22 [22] Ans: 2
6. CC JAN '18 [27]
$\frac{1}{8}+\frac{1}{6}=\frac{1}{t_b}$
$24t_b\left[\frac{1}{8}+\frac{1}{6}=\frac{1}{t_b}\right]$
$3t_b+4t_b=24$
$7t_b=24$
$t_b=\frac{24}{7}\approx 3.4$

7. CC JAN '20 [37]
$n(0)=\frac{1}{5}+\frac{18}{15}=1.4$; $a(0)=\frac{9}{3}=3$; the antibiotic has a greater amount at $t=0$.
$\frac{t+1}{t+5}+\frac{18}{t^2+8t+15}=\frac{9}{t+3}$
$(t+3)(t+5)\left[\frac{t+1}{t+5}+\frac{18}{t^2+8t+15}=\frac{9}{t+3}\right]$
$(t+3)(t+1)+18=9(t+5)$
$t^2+4t+3+18=9t+45$
$t^2-5t-24=0$
$(t-8)(t+3)=0$; (reject $t=-3$)
drugs have the same amount at $t=8$ hours.

8.8 Graphs of Rational Functions

There are no Regents exam questions on this topic.

Chapter 9. Exponential Functions

9.1 Solve Simple Exponential Equations

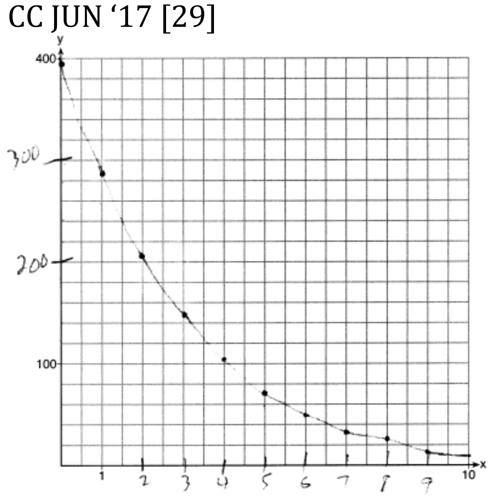
1. CC JAN '23 [26]
   $a^{x+1} = a^{\frac{2}{3}}$
   $x + 1 = \frac{2}{3}$
   $x = -\frac{1}{3}$

9.2 Rewrite Exponential Expressions

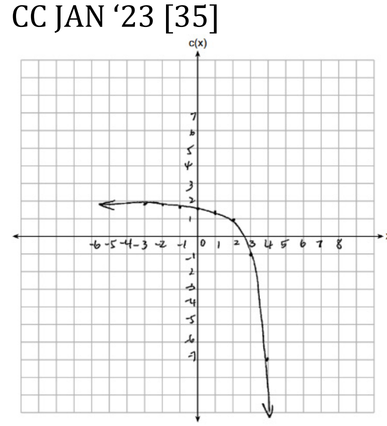
1. CC AUG '17 [10] Ans: 3
2. CC JAN '18 [8] Ans: 4
3. CC AUG '22 [24] Ans: 1

9.3 Graphs of Exponential Functions

1. CC JUN '18 [2] Ans: 2
2. CC JUN '19 [6] Ans: 3
3. CC JUN '19 [20] Ans: 1
4. CC JUN '22 [5] Ans: 3
5. CC AUG '22 [14] Ans: 3
6. CC JUN '17 [29]



7. CC JAN '20 [27]
   $q(x)$ is a translation of $p(x)$ by 3 units to the right and 4 units up.
8. CC JAN '23 [35]



As $x \to \infty, c(x) \to -\infty$
As $x \to -\infty, c(x) \to 2$

9.4 Exponential Regression

1. CC JAN '17 [13] Ans: 3
2. CC JUN '18 [4] Ans: 2
3. CC JUN '23 [14] Ans: 3
4. CC JAN '18 [26]
   $D = 1.223(2.652)^A$
5. CC JUN '22 [32]
   $F(t) = 169.136(0.971)^t$

6. CC JAN '23 [29]
$y = 2.459(1.616)^x$

9.5 Exponential Growth or Decay

1. CC JUN '16 [15] Ans: 4
2. CC AUG '16 [13] Ans: 1
3. CC JAN '17 [2] Ans: 1
4. CC JAN '18 [5] Ans: 4
5. CC JAN '18 [23] Ans: 4
6. CC JAN '19 [20] Ans: 3
7. CC AUG '19 [3] Ans: 1
8. CC JAN '20 [2] Ans: 3
9. CC JUN '22 [11] Ans: 2
10. CC AUG '22 [11] Ans: 1
11. CC JAN '23 [3] Ans: 4
12. CC JAN '23 [4] Ans: 2
13. CC AUG '23 [9] Ans: 1
14. CC AUG '23 [14] Ans: 3
15. CC FALL '15 [17]
$A(t) = 100(0.5)^{\frac{t}{63}}$
t is time in years, and $A(t)$ is the amount of titanium-44 left after t years.
$\frac{A(10) - A(0)}{10 - 0} = \frac{89.58132 - 100}{10} =$
-1.041868
The estimated mass at $t = 40$ is
$100 - 40(-1.041868) \approx 58.3$.
The actual mass is
$A(40) = 100(0.5)^{\frac{40}{63}} \approx 64.3976$.
The estimated mass is less than the actual mass.
16. CC JAN '20 [31]
$B(t) = 100(2)^{\frac{t}{30}}$

9.6 Rate Conversion

1. CC SPR '15 [4] Ans: 2
2. CC JUN '16 [21] Ans: 3
3. CC JUN '17 [13] Ans: 3
4. CC JUN '18 [23] Ans: 4
5. CC AUG '18 [8] Ans: 2
6. CC JAN '19 [6] Ans: 3
7. CC JUN '19 [24] Ans: 4
8. CC AUG '19 [24] Ans: 1
9. CC JUN '22 [24] Ans: 1
10. CC JAN '23 [23] Ans: 1
11. CC AUG '23 [11] Ans: 2
12. CC JAN '24 [24] Ans: 1

9.7 Continuous Growth or Decay

1. CC JUN '17 [18] Ans: 2
2. CC JAN '24 [15] Ans: 4
3. CC AUG '19 [33]
$N(t) = 950e^{0.0475t}$; the base of e is used for continuous growth;
36 hours = 1.5 days; $N(1.5) \approx 1020$.
4. CC JUN '23 [27]
$P(t)$ is increasing; $r = 0.0532 > 0$; as long as $r > 0$, then $e^r > 1$ and the function is increasing.

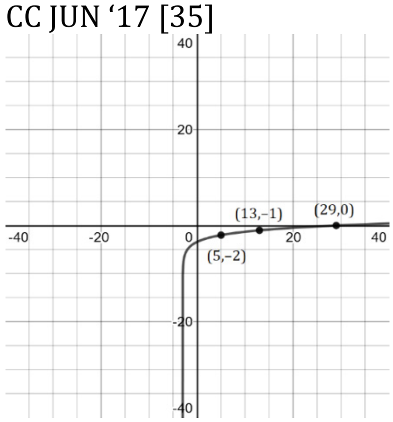
Chapter 10. Logarithms

10.1 General and Common Logarithms

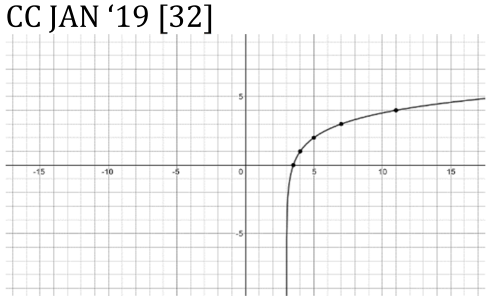
1. CC JAN ‘17 [15] Ans: 3
2. CC JAN ’24 [4] Ans: 3

10.2 Graphs of Log Functions

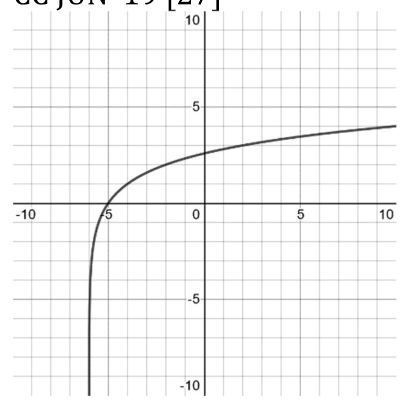
1. CC JUN ‘16 [18] Ans: 1
2. CC JUN ‘18 [19] Ans: 4
3. CC AUG ‘18 [16] Ans: 2
4. CC JAN ‘19 [2] Ans: 1
5. CC JUN ‘22 [15] Ans: 4
6. CC AUG ‘22 [7] Ans: 4
7. CC JUN ‘23 [8] Ans: 1
8. CC AUG ‘16 [30]
   $0 = \log_{10}(x - 4)$
   $10^0 = x - 4$
   $1 = x - 4$
   $x = 5$
   The x-intercept of $h(x)$ is (2,0).
   f has the larger value.
9. CC JUN ‘17 [35]



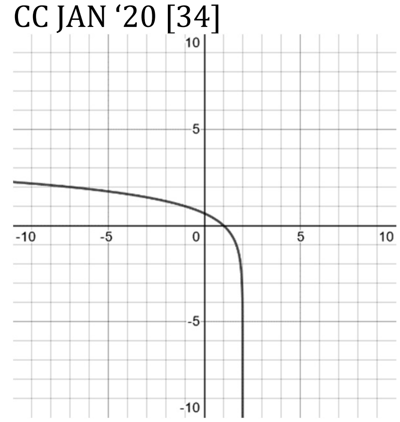
   as $x \to -3, y \to -\infty$, and
   as $x \to \infty, y \to \infty$.
10. CC JAN ‘19 [32]



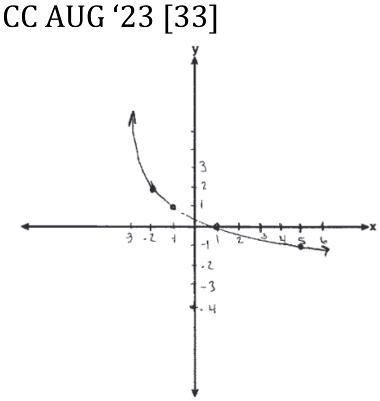
11. CC JUN ‘19 [27]



12. CC JAN ‘20 [34]



   Domain is $(-\infty, 2)$;
   asymptote is $x = 2$.
13. CC AUG ‘23 [33]



As $x \to -3, y \to \infty$; As $x \to \infty, y \to -\infty$

10.3 Properties of Logarithms

There are no Regents exam questions on this topic.

10.4 Use Logarithms to Solve Equations

1. CC JUN '18 [14] Ans: 1
2. CC JAN '20 [9] Ans: 4
3. CC AUG '23 [6] Ans: 1
4. CC AUG '23 [17] Ans: 1
5. CC AUG '16 [34]

$7 = 20(0.5)^{\frac{t}{8.02}}$

$0.35 = (0.5)^{\frac{t}{8.02}}$

$\log 0.35 = \frac{t \log 0.5}{8.02}$

$t = \frac{8.02 \log 0.35}{\log 0.5} \approx 12$

6. CC JUN '17 [37]

(a) $100 = 140\left(\frac{1}{2}\right)^{\frac{5}{h}}$

(b) $\log\left(\frac{100}{140}\right) = \log\left(\frac{1}{2}\right)^{\frac{5}{h}}$

$\log\frac{5}{7} = \frac{5}{h}\log\frac{1}{2}$

$h = \frac{5\log\frac{1}{2}}{\log\frac{5}{7}} \approx 10.3002$

(c) $40 = 140\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$

$\log\frac{2}{7} = \log\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$

$\log\frac{2}{7} = \frac{t\log\frac{1}{2}}{10.3002}$

$t = \frac{10.3002\log\frac{2}{7}}{\log\frac{1}{2}} \approx 18.6$

7. CC AUG '17 [36]

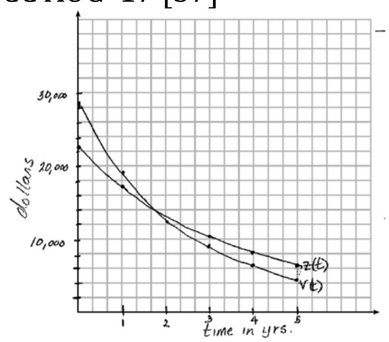
(a) $y = 4.168(3.981)^x$

(b) $100 = 4.168(3.981)^x$

$\log\frac{100}{4.168} = x\log 3.981$

$x = \frac{\log\frac{100}{4.168}}{\log 3.981} \approx 2.25$

8. CC AUG '17 [37]



$28482.698(0.684)^t =$
$22151.327(0.778)^t$

$\frac{28482.698}{22151.327} = \left(\frac{0.778}{0.684}\right)^t$

$\log\frac{28482.698}{22151.327} = t\log\left(\frac{0.778}{0.684}\right)$

$V(t) = Z(t)$ when $t \approx 1.95$.

At 1.95 years, the value of the car equals the loan balance. Zach can cancel the policy after 6 years because at that point the value of the car is less than $3,000.

9. CC JUN '19 [34]

$s(t) = 200(0.5)^{\frac{t}{15}}$

$\frac{1}{10} = (0.5)^{\frac{t}{15}}$

$\log\frac{1}{10} = \log(0.5)^{\frac{t}{15}}$

$-1 = \frac{t}{15}\log(0.5)$

$t = -\frac{15}{\log(0.5)} \approx 50$

10. CC JAN '20 [36]

$y = 101.523(0.883)^x$;

$29 = 101.523(0.883)^x$

$.28565 = (0.883)^x$

$\log(.28565) = x\log(0.883)$

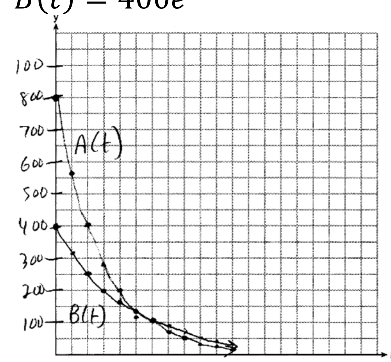
$x \approx 10.07$ km

11. CC AUG '22 [33]
a) $p(t) = 11{,}000(2)^{\frac{t}{20}}$
b) $1{,}000{,}000 = 11{,}000(2)^{\frac{t}{20}}$
$\frac{1000}{11} = 2^{\frac{t}{20}}$
$\log\left(\frac{1000}{11}\right) = \log 2^{\frac{t}{20}}$
$\log\left(\frac{1000}{11}\right) = \frac{t}{20}\log 2$
$t = \frac{20\log\left(\frac{1000}{11}\right)}{\log 2} \approx 130.13$

12. CC JAN '23 [37]
$T = (400 - 75)e^{-0.0735t} + 75;$
$325e^{-0.0735(5)} + 75 \approx 300;$
$270 = (450 - 75)e^{-8r} + 75$
$0.52 = e^{-8r}$
$\ln 0.52 = \ln e^{-8r}$
$\ln 0.52 = -8r$
$r \approx 0.0817;$
$325e^{-0.0735t} + 75 = 375e^{-0.0817t} + 75$
$e^{-0.0735t} = \frac{375}{325}e^{-0.0817t}$
$\ln e^{-0.0735t} = \ln\frac{375}{325} + \ln e^{-0.0817t}$
$-0.0735t = \ln\frac{375}{325} - 0.0817t$
$0.0082t = \ln\frac{375}{325}$
$t \approx 17$

10.5 Natural Logarithms

1. CC JAN '17 [23] Ans: 1
2. CC JUN '17 [2] Ans: 1
3. CC JAN '18 [13] Ans: 3
4. CC JUN '18 [18] Ans: 1
5. CC AUG '18 [1] Ans: 4
6. CC AUG '18 [19] Ans: 3
7. CC AUG '19 [18] Ans: 4
8. CC JUN '22 [7] Ans: 1
9. CC AUG '22 [9] Ans: 4
10. CC JAN '23 [2] Ans: 2
11. CC JUN '23 [15] Ans: 3
12. CC FALL '15 [13]
(a) $100 = 325 + (68 - 325)e^{-2k}$
$-225 = -257e^{-2k}$
$\ln\frac{225}{257} = -2k$
$k = \frac{\ln\frac{225}{257}}{-2} \approx 0.066$
(b) $T(7) = 325 - 257e^{-0.066(7)} \approx 163$
13. CC JUN '16 [32]
$135{,}000 = 100{,}000e^{5r}$
$1.35 = e^{5r}$
$\ln 1.35 = 5r$
$r = \frac{\ln 1.35}{5} \approx 0.06$ or 6%
14. CC JUN '16 [37]
(a) $A(t) = 800e^{-0.347t}$
$B(t) = 400e^{-0.231t}$



(b) $800e^{-0.347t} = 400e^{-0.231t}$
$\frac{800}{400} = \frac{e^{-0.231t}}{e^{-0.347t}}$
$2 = e^{0.116t}$
$\ln 2 = 0.116t$
$t \approx 6$
(c) $0.15 = e^{-0.347t}$
$\ln 0.15 = \ln e^{-0.347t}$
$\ln 0.15 = -0.347t$
$t = \frac{\ln 0.15}{-0.347} \approx 5.5$
15. CC JAN '17 [28]
$\frac{\ln\frac{1}{2}}{1590}$ is negative, so $M(t)$ represents decay.

16. CC JAN ‘18 [29]

$20e^{0.05t} = 30e^{0.03t}$

$\frac{2}{3}e^{0.05t} = e^{0.03t}$

$\frac{2}{3} = \frac{e^{0.03t}}{e^{0.05t}}$

$\frac{2}{3} = e^{-0.02t}$

$\ln\frac{2}{3} = -0.02t$

$t = \frac{\ln\frac{2}{3}}{-0.02} \approx 20.3 \text{ months}$

17. CC JUN ‘23 [30]

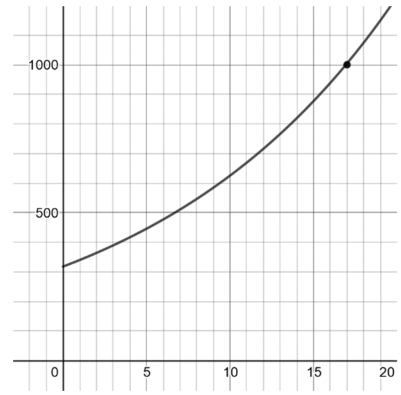
$e^{0.49x} = 7.5$

$\ln e^{0.49x} = \ln 7.5$

$0.49x = \ln 7.5$

$x = \frac{\ln 7.5}{0.49} \approx 4.112$

11.1 Periodic Compound Interest

1. CC AUG '16 [22] Ans: 4
2. CC AUG '19 [17] Ans: 4
3. CC JAN '20 [4] Ans: 1
4. CC AUG '16 [37]
(a) $A(n) = 5{,}000(1.045)^n$
$B(n) = 5{,}000\left(1 + \frac{0.046}{4}\right)^{4n}$
(b) $B(6) - A(6) \approx$
$6578.87 - 6511.30 \approx 67.57$
$10{,}000 = 5{,}000\left(1 + \frac{0.046}{4}\right)^{4n}$
(c) $2 = 1.0115^{4n}$
$\log 2 = 4n \log 1.0115$
$n = \frac{\log 2}{4 \log 1.0115} \approx 15.2$ yrs
5. CC JUN '18 [35]
$C(t) = 63{,}000\left(1 + \frac{0.0255}{12}\right)^{12t}$
$63{,}000\left(1 + \frac{0.0255}{12}\right)^{12t} = 100{,}000$
$(1.002125)^{12t} = \frac{100}{63}$
$12t \log(1.002125) = \log \frac{100}{63}$
$t = \frac{\log \frac{100}{63}}{12 \log 1.002125} \approx 18.14$
6. CC JAN '19 [37]
(a) $A(t) = 318{,}000(1.07)^t$
(b)


(c) $1{,}000{,}000 = 318{,}000(1.07)^t$
$\frac{1000}{318} = 1.07^t$
$\log \frac{1000}{318} = t \log 1.07$
$t = \frac{\log \frac{1000}{318}}{\log 1.07} \approx 17$ yrs
(c) The graph nearly intersects the point (17,1000000)

11.2 Continuous Compound Interest

1. CC JAN '18 [10] Ans: 1
2. CC JAN '19 [18] Ans: 4
3. CC JUN '19 [17] Ans: 2
4. CC AUG '18 [35]
$2 = e^{0.0375t}$
$\ln 2 = 0.0375t$
$t = \frac{\ln 2}{0.0375} \approx 18.5$
5. CC AUG '23 [37]
$A(t) = 8000\left(1 + \frac{0.042}{4}\right)^{4t}$
$B(t) = 8000e^{0.039t}$
$A(18) \approx 16{,}970.900$
$B(18) \approx 16{,}142.274$
$A(18) - B(18) \approx 828.63$
$24{,}000 = 8000e^{0.039t}$
$\ln 3 = \ln e^{0.039t}$
$\ln 3 = 0.039t$
$t = \frac{\ln 3}{0.039} \approx 28.2$

11.3 Regular Contributions

1. CC JUN '17 [24] Ans: 2
2. CC JUN '23 [24] Ans: 2
3. CC JAN '24 [23] Ans: 4

11.4 Evaluate Loan Formulas

1. CC JUN '22 [9] Ans: 3
2. CC AUG '23 [16] Ans: 4
3. CC JAN '24 [20] Ans: 2
4. CC SPR '15 [9]

$720 = \frac{120{,}000\left(\frac{0.048}{12}\right)\left(1+\frac{0.048}{12}\right)^n}{\left(1+\frac{0.048}{12}\right)^n - 1}$

$720 = \frac{480(1.004)^n}{(1.004)^n - 1}$

$1.5 = \frac{(1.004)^n}{(1.004)^n - 1}$

$1.5(1.004)^n - 1.5 = (1.004)^n$

$0.5(1.004)^n = 1.5$

$1.004^n = 3$

$n \log 1.004 = \log 3$

$n = \frac{\log 3}{\log 1.004} \approx 275.2$ months

$\frac{275.2}{12} \approx 23$ years

5. CC JAN '17 [36]

(a) $20{,}000 = PMT\left(\frac{1-1.00625^{-60}}{0.00625}\right)$

$PMT = 20{,}000\left(\frac{0.00625}{1-1.00625^{-60}}\right)$

$PMT \approx \$\,400.76$

(b) $21{,}000 - x = 300\left(\frac{1-1.00625^{-60}}{0.00625}\right)$

$x = -300\left(\frac{1-1.00625^{-60}}{0.00625}\right) + 21{,}000$

$x \approx \$\,6{,}028$

6. CC JUN '17 [34]

$N = 12 \times 15 = 180$

(a) $M = 172{,}600 \cdot \frac{0.00305(1.00305)^{180}}{1.00305^{180} - 1}$

$M \approx \$\,1{,}247$

(b) $1100 = (172{,}600 - x) \cdot \frac{0.00305(1.00305)^{180}}{1.00305^{180} - 1}$

$1100\left(\frac{1.00305^{180} - 1}{0.00305(1.00305)^{180}}\right) = 172{,}600 - x$

$x = -1100\left(\frac{1.00305^{180} - 1}{0.00305(1.00305)^{180}}\right) + 172{,}600$

$x \approx \$\,20{,}407$

7. CC JUN '18 [31]

$M = \frac{137{,}250\left(\frac{0.036}{12}\right)\left(1+\frac{0.036}{12}\right)^{360}}{\left(1+\frac{0.036}{12}\right)^{360} - 1} \approx \$624$

Chapter 12. Trigonometric Functions

12.1 Trigonometric Ratios

There are no Regents exam questions on this topic.

12.2 Radians

There are no Regents exam questions on this topic.

12.3 Unit Circle

1. CC AUG '16 [16] Ans: 1
2. CC AUG '17 [7] Ans: 4
3. CC JAN '18 [15] Ans: 1
4. CC JUN '22 [19] Ans: 2
5. CC AUG '22 [5] Ans: 4
6. CC JAN '17 [27]
   $\csc\theta = \frac{1}{\sin\theta}$, and $\sin\theta$ on a unit circle represents the y value of a point on the unit circle.
   Since $y = \sin\theta$. $\csc\theta = \frac{1}{y}$.
7. CC JAN '23 [32]
   $\pi < \theta < 2\pi$ in Quadrants III and IV.
   $\cos\theta$ is positive in Quadrants I and IV.
   Therefore, θ must be in Quadrant IV.
   $\tan\theta$ is negative in Quadrant IV.

12.4 Solve Simple Trigonometric Equations

There are no Regents exam questions on this topic.

12.5 Circles of Any Radius

1. CC SPR '15 [3] Ans: 1
2. CC JUN '16 [17] Ans: 1
3. CC AUG '19 [5] Ans: 1
4. CC JAN '23 [20] Ans: 3
5. CC JUN '23 [4] Ans: 2
6. CC AUG '23 [12] Ans: 3
7. CC JAN '24 [21] Ans: 1
8. CC JUN '18 [32]
   $r = \sqrt{2^2 + 1^2} = \sqrt{5}$
   $\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{5}}$
9. CC JUN '19 [28]
   $r = \sqrt{7^2 + 24^2} = \sqrt{625}$ =25
   $\cos\theta = -\frac{x}{r} = -\frac{24}{25}$

12.6 Pythagorean Identity

1. CC JAN '17 [4] Ans: 1
2. CC JUN '17 [12] Ans: 2
3. CC AUG '18 [11] Ans: 2

4. CC AUG '16 [28]
$\sin^2\theta + (-0.7)^2 = 1$
$\sin^2\theta = 0.51$
$\sin\theta = \pm\sqrt{0.51}$
Since θ is in Quadrant II, $\sin\theta = \sqrt{0.51}$
and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{0.51}}{-0.7} \approx -1.02$

5. CC JAN '19 [31]
$\sin^2\theta + \cos^2\theta = 1$
$\left(\frac{4}{7}\right)^2 + t^2 = 1$
$\frac{16}{49} + t^2 = 1$
$t^2 = \frac{33}{49}$
$t = \pm\frac{\sqrt{33}}{7}$
Since θ is in Quadrant II, $t = -\frac{\sqrt{33}}{7}$

12.7 Simplify Trigonometric Expressions

1. CC AUG '22 [29]
$\cot A = \frac{\cos A}{\sin A}$
$-3 = \frac{\frac{3}{\sqrt{10}}}{\sin A}$
$\sin A = \frac{\frac{3}{\sqrt{10}}}{-3}$
$\sin A = \left(\frac{3}{\sqrt{10}}\right)\left(-\frac{1}{3}\right) = -\frac{1}{\sqrt{10}}$ or $-\frac{\sqrt{10}}{10}$

12.8 Graphs of Parent Trig Functions

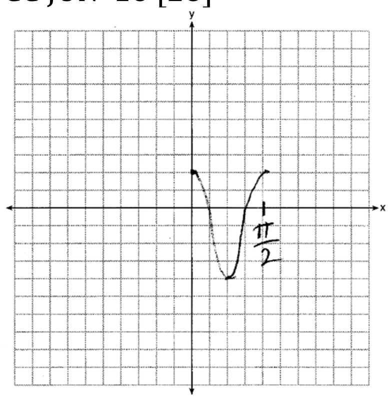
1. CC JAN '20 [16] Ans: 4

12.9 Trigonometric Transformations

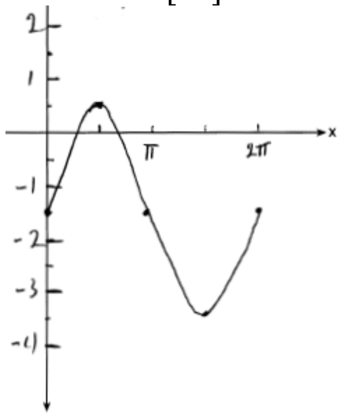
1. CC FALL '15 [6] Ans: 4
2. CC JAN '17 [1] Ans: 2
3. CC JAN '17 [22] Ans: 3
4. CC JUN '17 [6] Ans: 4
5. CC JUN '17 [8] Ans: 1
6. CC AUG '17 [5] Ans: 3
7. CC AUG '17 [18] Ans: 4
8. CC AUG '18 [20] Ans: 1
9. CC JAN '19 [13] Ans: 1
10. CC JUN '22 [22] Ans: 2
11. CC AUG '22 [3] Ans: 2
12. CC AUG '22 [20] Ans: 4
13. CC JAN '18 [30]
q has the smaller minimum value for the domain $[-2,2]$.
h's minimum is $2(-1) + 1 = -1$ and q's minimum is -8.

12.10 Graph Trigonometric Functions

1. CC JUN '16 [28]

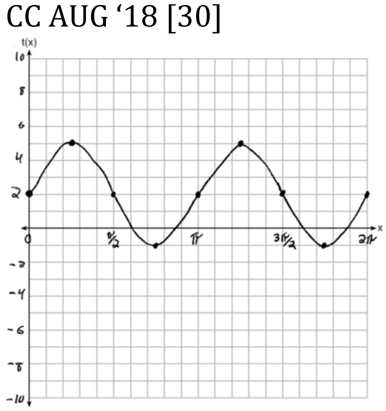


2. CC AUG '17 [35]

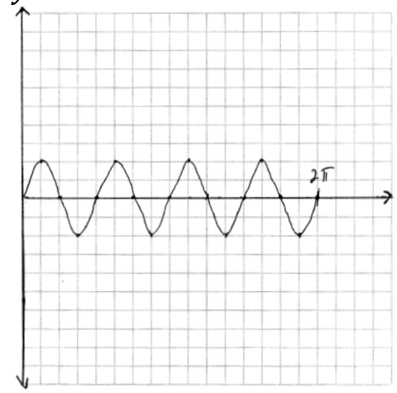


Part *a* sketch is shifted $\frac{\pi}{3}$ units right.

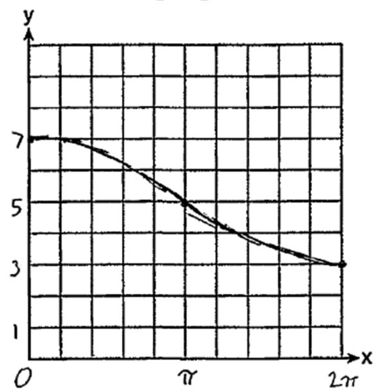
3. CC AUG '18 [30]



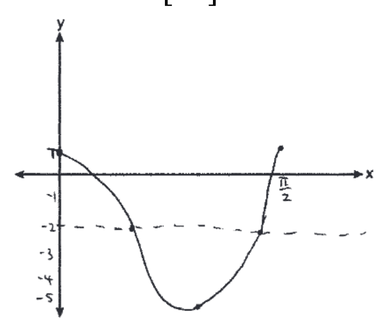
4. CC AUG '19 [34]
$y = 2\sin 4x$



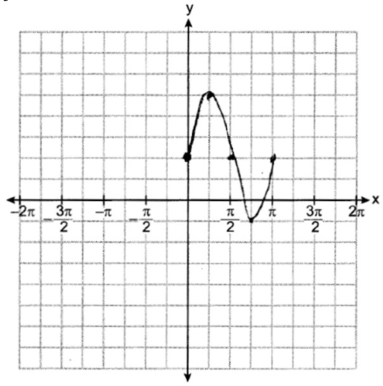
5. CC JUN '22 [31]



6. CC AUG '23 [28]



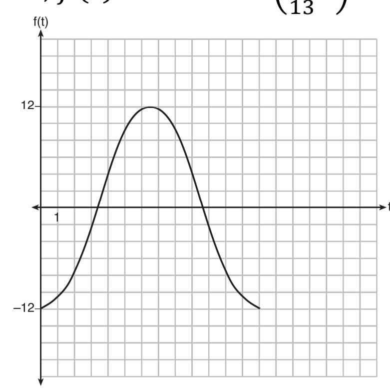
7. CC JAN '24 [36]
$y = 3\sin 2x + 2$



$0 < x < \frac{\pi}{4}$ or $\frac{3\pi}{4} < x < \pi$

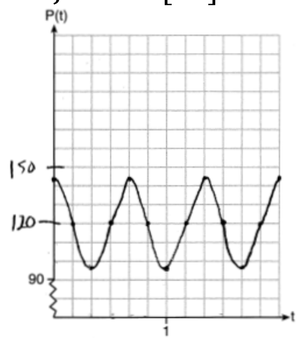
12.11 Model Trigonometric Functions

1. CC JUN '16 [13] Ans: 3
2. CC JUN '16 [24] Ans: 4
3. CC AUG '16 [10] Ans: 2
4. CC JUN '17 [15] Ans: 4
5. CC JAN '18 [4] Ans: 2
6. CC JUN '18 [10] Ans: 4
7. CC AUG '18 [22] Ans: 2
8. CC JAN '19 [7] Ans: 1
9. CC JUN '19 [22] Ans: 3
10. CC AUG '19 [12] Ans: 4
11. CC JUN '22 [20] Ans: 4
12. CC JAN '23 [13] Ans: 2
13. CC JUN '23 [5] Ans: 2
14. CC SPR '15 [14]
Since the water level has a minimum of −12 and a maximum of 12, the amplitude is 12. The value of A is −12 since at 8:30 it is low tide. The time from low to high tide is 6.5 hours, so the period of the function is 13 hours. $\frac{2\pi}{B} = 13$, so $B = \frac{2\pi}{13}$.
So, $f(t) = -12\cos\left(\frac{2\pi}{13}t\right)$.



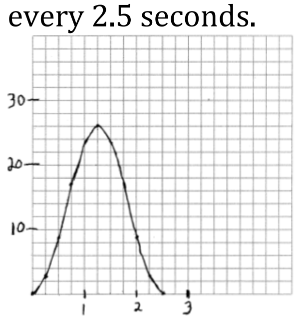
Since the function is increasing from $t = 13$ to $t = 19.5$ (which corresponds to 9:30 pm to 4:00 am), fishing at 10:30 pm is recommended.
15. CC AUG '16 [25]
Amplitude, because the height of the graph shows the volume of the air.
16. CC JUN '17 [28]
Period is $\frac{2}{3}$. The wheel rotates once every $\frac{2}{3}$ second.

17. CC JAN ‘18 [37]



The period of P is $\frac{2}{3}$, which means the patient’s blood pressure reaches a high every $\frac{2}{3}$ second and a low every $\frac{2}{3}$ second. The patient’s blood pressure is high because 144 over 96 is greater than 120 over 80.

18. CC JUN ‘19 [37]
Period is $\frac{2\pi}{0.8\pi} = 2.5$.
This means the wheel rotates once every 2.5 seconds.



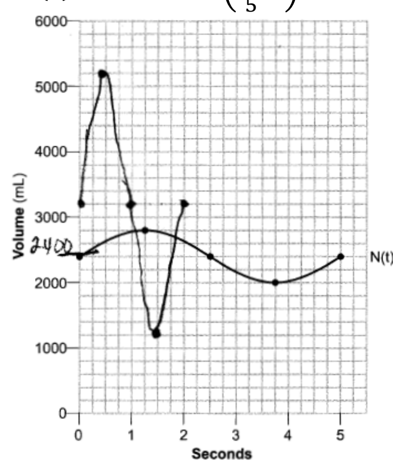
No, because the maximum of $f(t) = 26$.

19. CC AUG ‘19 [28]
$250(1) + 2450 = 2700$.
The maximum lung capacity of a person is 2700 mL.

20. CC AUG ‘19 [30]
$\frac{10.1+2}{2} = 6.05$; $\frac{2.5+0.1}{2} = 1.3$;
$6.05 - 1.3 = 4.75$

21. CC JAN ‘20 [30]
The period for A is $2(340 - 60) = 560$; for B, it is $2(400 - 180) = 440$; and for C, it is $2(380 - 60) = 640$. Therefore, C’s period is the longest.

22. CC JUN ‘23 [37]
$N(t) = 400\sin\left(\frac{2\pi}{5}t\right) + 2400$



4 times

Chapter 13. Properties of Functions

13.1 Compare Functions

1. CC JAN '17 [19] Ans: 2
2. CC JUN '23 [9] Ans: 4
3. CC AUG '23 [10] Ans: 4

13.2 Even and Odd Functions

1. CC FALL '15 [2] Ans: 3
2. CC AUG '16 [14] Ans: 1
3. CC JUN '18 [6] Ans: 2
4. CC AUG '19 [11] Ans: 2
5. CC JUN '23 [18] Ans: 1
6. CC AUG '23 [23] Ans: 2

13.3 Algebraically Determine Even or Odd [CC]

1. CC AUG '17 [31]
   $j(-x) = (-x)^4 - 3(-x)^2 - 4 = x^4 - 3x^2 - 4$
   Since $j(x) = j(-x)$, the function is even.

13.4 Inverse Functions

1. CC JUN '16 [16] Ans: 2
2. CC JAN '17 [8] Ans: 3
3. CC AUG '17 [14] Ans: 2
4. CC JAN '18 [21] Ans: 2
5. CC JUN '18 [15] Ans: 3
6. CC AUG '18 [6] Ans: 2
7. CC JAN '19 [17] Ans: 3
8. CC JUN '19 [9] Ans: 2
9. CC AUG '19 [23] Ans: 2
10. CC JUN '22 [17] Ans: 3
11. CC AUG '22 [23] Ans: 3
12. CC JAN '23 [15] Ans: 3
13. CC JUN '23 [21] Ans: 3
14. CC AUG '23 [4] Ans: 3
15. CC JAN '24 [19] Ans: 2
16. CC FALL '15 [9]
    $x = (y-3)^3 + 1$
    $x - 1 = (y-3)^3$
    $\sqrt[3]{x-1} = y - 3$
    $\sqrt[3]{x-1} + 3 = y$
    $f^{-1}(x) = \sqrt[3]{x-1} + 3$

13.5 Average Rate of Change

1. CC JAN '17 [21] Ans: 4
2. CC JAN '17 [24] Ans: 1
3. CC JUN '17 [21] Ans: 3
4. CC AUG '17 [9] Ans: 3
5. CC JUN '18 [7] Ans: 1
6. CC JUN '19 [4] Ans: 1
7. CC AUG '19 [22] Ans: 3
8. CC JAN '20 [12] Ans: 1
9. CC JUN '22 [12] Ans: 4
10. CC JUN '23 [1] Ans: 2
11. CC JUN '16 [36]
    $\frac{f(4) - f(-2)}{4 - (-2)} = \frac{80 - 1.25}{6} = 13.125$
    $\frac{g(4) - g(-2)}{4 - (-2)} = \frac{179 - (-49)}{6} = 38$
    $g(x)$ has a greater rate of change

12. CC AUG ‘16 [31]
$\frac{156.25-56.25}{70-50}=\frac{150}{20}=7.5$
Between 50-70 mph, each additional mph in speed requires 7.5 more feet to stop.

13. CC JUN ‘18 [36]
$\frac{h(2)-h(1)}{2-1}=-12$
Use the calculator's Calc → zero feature to find where $h(t)=0$, or:
$12\cos\left(\frac{\pi}{3}t\right)+8=0$
$\cos\left(\frac{\pi}{3}t\right)=-\frac{2}{3}$
Reference ∡ is $\cos^{-1}\left(\frac{2}{3}\right)\approx 0.8411$
cos is negative in Q2 and Q3, so
$\frac{\pi}{3}t=\pi-0.8411\approx 2.30$ and
$\frac{\pi}{3}t=\pi+0.8411\approx 3.98$
$\frac{\pi}{3}t=2.30$ gives us $t=\frac{3(2.30)}{\pi}\approx 2.2$
$\frac{\pi}{3}t=3.98$ gives us $t=\frac{3(3.98)}{\pi}\approx 3.8$
$h(t)=0$ at $t\approx\{2.2, 3.8\}$.

14. CC AUG ‘18 [27]
$\frac{p(8)-p(4)}{8-4}\approx 48.78$

15. CC JAN ‘19 [30]
$\frac{B(11)-B(8)}{11-8}\approx -10.1$
The average monthly high temperature decreases 10.1 degrees each month from August to November.

16. CC JUN ‘19 [26]
$\frac{13.9-9.4}{4-1}=1.5$
The average rate of change in the number of hours of daylight from January 1 to April 1 is 1.5.

17. CC AUG ‘22 [25]
$\frac{60-20}{4-2}=20$

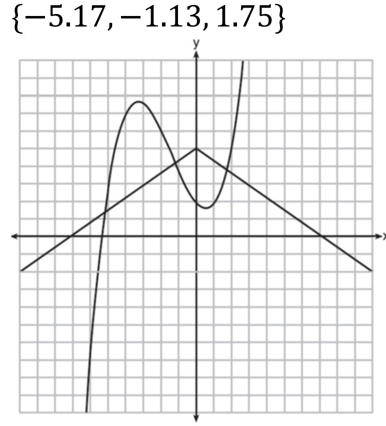
18. CC JAN ‘23 [36]
$\frac{B(10)-B(6)}{10-6}\approx -3.88°/\text{month}$
The average monthly high temperature decreases about 4° each month from June and October.

19. CC AUG ‘23 [32]
$\frac{P(10.5)-P(0)}{10.5-0}\approx 10.76$ fruit flies/day

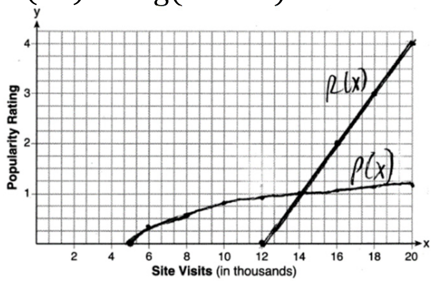
20. CC JAN ’24 [27]
$\frac{V(7)-V(2)}{7-2}\approx \$48/\text{year}$

13.6 Solutions to Equation of Two Functions

1. CC AUG ‘16 [3] Ans: 2
2. CC JAN ‘17 [12] Ans: 2
3. CC JAN ‘17 [16] Ans: 2
4. CC JUN ‘17 [5] Ans: 2
5. CC JAN ‘18 [14] Ans: 1
6. CC JAN ‘19 [24] Ans: 4
7. CC JUN ‘19 [14] Ans: 4
8. CC AUG ‘19 [20] Ans: 2
9. CC JAN ‘20 [21] Ans: 2
10. CC AUG ‘22 [10] Ans: 1
11. CC JAN ‘23 [17] Ans: 3
12. CC AUG ‘23 [19] Ans: 2
13. CC JAN ’24 [6] Ans: 3
14. CC FALL ‘15 [10]
$\{-5.17, -1.13, 1.75\}$



15. CC JUN '18 [37]
$P(16) = \log(16 - 4) \approx 1.1$



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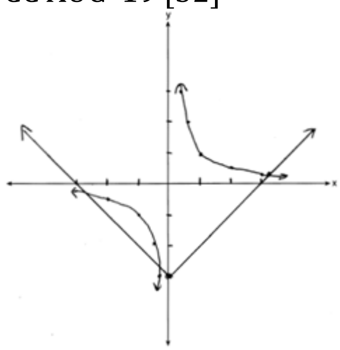
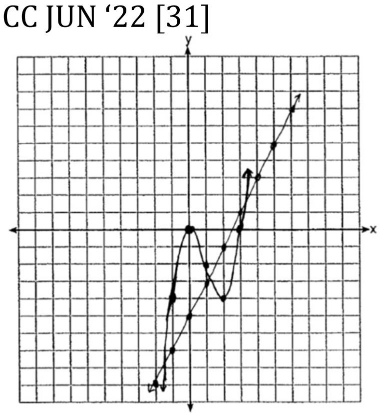
16. CC AUG '19 [32]



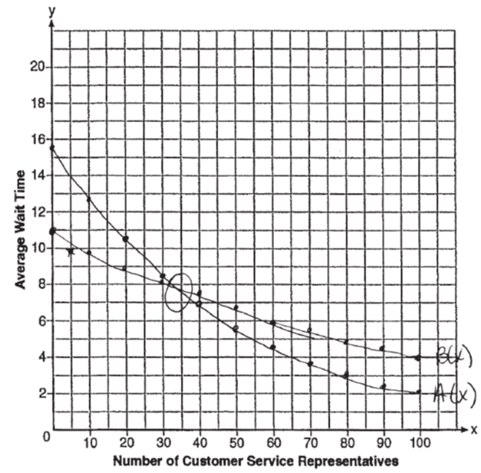
$x = 3.3$

17. CC JUN '22 [31]



3 solutions

18. CC AUG '22 [37]



$x \approx 35$
$4.0264 - 2.0812 \approx 2$ min, which is the difference in average wait time for companies with 100 representatives

19. CC JUN '23 [36]
$P(x) = 500(0.97)^x$
$F(x) = 200e^{0.02x}$
$x = 18$; In 18 years, there will be the same number of palm trees as flamingos.

20. CC JAN '24 [35]
$A(t) = 4000\left(1 + \frac{0.024}{12}\right)^{12t}$
$B(t) = 3500\left(1 + \frac{0.04}{4}\right)^{4t}$
8.4 years;
$A(t)$ and $B(t)$ intersect at $t \approx 8.4$.

13.7 Solutions to Inequality of Two Functions [NG]

There are no Regents exam questions on this topic.

Chapter 14. Sequences and Series

14.1 Arithmetic Sequences

1. CC JUN '17 [20] Ans: 3
2. CC AUG '17 [34]
   $\frac{6.25 - 2.25}{21 - 5} = \frac{4}{16} = \$0.25$ fine per day.
   $2.25 - 5(0.25) = \$1$ replacement fee.
   $a_n = 1.25 + (n-1)(0.25)$ or
   $a_n = 0.25n + 1.00$
   $a_{60} = \$16$

14.2 Geometric Sequences

1. CC JAN '17 [14] Ans: 1
2. CC JAN '19 [4] Ans: 2
3. CC JUN '19 [10] Ans: 3
4. CC JAN '20 [17] Ans: 1
5. CC JAN '24 [10] Ans: 3
6. CC AUG '17 [30]
   $a_{1966} = 1.25$ and $a_{2015} = 8.75$
   $r^{2015-1966} = \frac{8.75}{1.25}$
   $r^{49} = 7$
   $r = \sqrt[49]{7} \approx 1.04$
   4 % growth

14.3 Recursively Defines Sequences

1. CC JUN '16 [10] Ans: 4
2. CC JUN '16 [23] Ans: 3
3. CC AUG '16 [8] Ans: 1
4. CC AUG '16 [18] Ans: 3
5. CC AUG '16 [24] Ans: 4
6. CC AUG '17 [24] Ans: 3
7. CC JAN '18 [24] Ans: 3
8. CC AUG '18 [10] Ans: 4
9. CC JAN '19 [16] Ans: 4
10. CC AUG '19 [9] Ans: 3
11. CC JAN '20 [13] Ans: 4
12. CC JUN '22 [21] Ans: 2
13. CC AUG '22 [16] Ans: 4
14. CC JAN '23 [21] Ans: 2
15. CC SPR '15 [11]
    $a_1 = x + 1$
    $a_2 = x(x+1)$
    $a_3 = x^2(x+1)$
    $a_n = x^{n-1}(x+1)$
    $x^{n-1} = 0$ or $x + 1 = 0$
    $x = \{0, -1\}$
16. CC JAN '17 [34]
    Jillian's plan, because distance increases by one mile each week.
    $a_1 = 10$
    $a_n = a_{n-1} + 1$
    $a_n = n + 12$
17. CC AUG '17 [29] :
    $a_1 = 4$
    $a_n = 2a_{n-1} + 1$
    $a_8 = 639$

18. CC JUN '18 [30]
$a_1 = 3, a_2 = 7, a_3 = 15, a_4 = 31$
No, because there is no common ratio:
$\frac{7}{3} \neq \frac{15}{7}$

19. CC JUN '19 [31]
Common ratio is $\frac{9}{6} = 1.5$
$a_1 = 6$
$a_n = 1.5a_{n-1}$

20. CC AUG '19 [31]
$a_1 = 4$
$a_n = 3a_{n-1}$

21. CC JUN '22 [37]
1.5%; $P(t) = 92.2(1.015)^t$;
$300 = 92.2(1.015)^t$
$\frac{300}{92.2} = 1.015^t$
$\log\frac{300}{92.2} = t\log 1.015$
$t = \frac{\log\frac{300}{92.2}}{\log 1.015} \approx 79$ years

22. CC JUN '23 [29]
Common ratio is $\frac{63}{189} = \frac{1}{3}$
$a_1 = 189$
$a_n = \frac{1}{3}a_{n-1}$

23. CC JAN '24 [30]
$a_1 = 12$
$a_n = a_{n-1} + 6$

14.4 Sigma Notation

There are no Regents exam questions on this topic.

14.5 Arithmetic Series

There are no Regents exam questions on this topic.

14.6 Geometric Series

1. CC AUG '16 [9] Ans: 1
2. CC AUG '17 [21] Ans: 4
3. CC JAN '18 [22] Ans: 2
4. CC AUG '18 [13] Ans: 1
5. CC JUN '19 [5] Ans: 4
6. CC AUG '19 [2] Ans: 3
7. CC AUG '22 [21] Ans: 1
8. CC JAN '23 [6] Ans: 3
9. CC JUN '16 [34]
$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$
$S_{15} = \frac{33{,}000 - 33{,}000(1.04)^{15}}{1 - 1.04} \approx 660{,}778.39$
10. CC JAN '19 [29]
$S_{10} = \frac{15 - 15(1.03)^{10}}{1 - 1.03} \approx 171.958$
11. CC JAN '20 [29]
$S_n = \frac{300 - 300(1.2)^n}{1 - 1.2}$
$S_{10} = \frac{300 - 300(1.2)^{10}}{1 - 1.2} \approx 7787.6$
12. CC JAN '20 [33]
$a_n = 100(0.8)^{n-1}$
$S_{20} = \frac{100 - 100(0.8)^{20}}{1 - 0.8} \approx 494.2$
Sonja has $40 \times 12 = 480$ in of wire, so she does not have enough
13. CC JUN '22 [26]
$S_5 = \frac{6 - 6(0.8)^5}{1 - 0.8} \approx 20.17$

Chapter 15. Probability

15.1 Theoretical and Empirical Probability

1. CC JAN '20 [24] Ans: 3
2. CC AUG '17 [28]
   Since there are six flavors, each flavor can be assigned a number, 1-6. Use the simulation to see the number of times the same number is rolled 4 times in a row.

15.2 Probability Involving And or Or

1. CC JUN '22 [4] Ans: 2
2. CC AUG '23 [7] Ans: 4
3. CC JUN '16 [29]
   $P(S\ or\ M) = P(S) + P(M) - P(S\ and\ M)$
   $\frac{974}{1376} = \frac{649}{1376} + \frac{433}{1376} - x$
   $x = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376}$
4. CC AUG '19 [25]
   $P(W\ or\ E) = P(W) + P(E) - P(W\ and\ E)$
   $P(W\ or\ E) = \frac{165}{825} + \frac{66}{825} - \frac{33}{825} = \frac{198}{825}$

15.3 Two-Way Frequency Tables

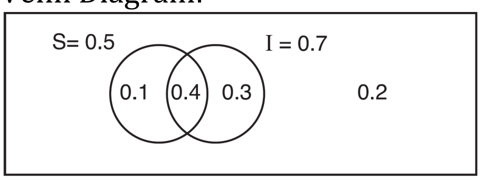
1. CC AUG '16 [7] Ans: 1
2. CC JAN '20 [11] Ans: 4
3. CC AUG '23 [3] Ans: 1
4. CC FALL '15 [8]
   Based on these data, the two events do not appear to be independent.
   $P(F) = \frac{106}{200} = 0.53$, while
   $P(F|T) = \frac{54}{90} = 0.6$,
   $P(F|R) = \frac{25}{65} = 0.39$, and
   $P(F|C) = \frac{27}{45} = 0.6$.
   The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.
5. CC JAN '17 [31]
   No, because $P(M|R) \neq P(M)$
   $\frac{70}{180} \neq \frac{230}{490}$ $\quad 0.38 \neq 0.47$
6. CC JUN '17 [32]
   A student is more likely to jog if both siblings jog.
   one jogs: $\frac{416}{2239} \approx 0.19$.
   both jog: $\frac{400}{1780} \approx 0.22$.
7. CC JUN '18 [25]
   $\frac{103}{110+103} = \frac{103}{213}$
8. CC JUN '19 [36]
   $P(F|L) = \frac{12}{27}$; $P(F) = \frac{22}{45}$;
   Since $P(F|L) \neq P(F)$, the events are not independent.
9. CC JUN '22 [29]
   $P(bl|gl) = \frac{0.14}{0.35} = 0.4$;
   $P(bl) = 0.14 + 0.26 = 0.4$;
   Yes, because $P(bl|gl) = P(bl)$
10. CC AUG '22 [31]
    $P(F|CR) = \frac{36}{78} \approx 0.462$;
    $P(F) = \frac{105}{215} \approx 0.488$;
    No, because $P(F|CR) \neq P(F)$

11. CC JUN '23 [35]
$\frac{1200}{1200+2016} \approx 0.373$
Yes, because the probability of selecting a supporter from all donors is the same:
$\frac{1600}{4288} \approx 0.373$

12. CC JAN '24 [33]
$\frac{45}{1500} = 0.03; \frac{3}{15} = 0.2$
Let M = allergic to milk and N = allergic to nuts. No, because $P(M) \neq P(M|N)$.

15.4 Series of Events (CC)

1. CC JUN '16 [11] Ans: 1
2. CC JUN '17 [14] Ans: 2
3. CC JUN '18 [11] Ans: 4
4. CC AUG '18 [18] Ans: 2
5. CC AUG '18 [24] Ans: 4
6. CC JAN '19 [12] Ans: 2
7. CC AUG '19 [13] Ans: 2
8. CC JAN '20 [8] Ans: 4
9. CC SPR '15 [13]
This scenario can be modeled with a Venn Diagram:



Since $P(\overline{S \text{ or } I}) = 0.2$, $P(S \text{ or } I) = 0.8$.
$P(S \text{ and } I) = P(S) + P(I) - P(S \text{ or } I) = 0.5 + 0.7 - 0.8 = 0.4$
If S and I are independent, $P(S \text{ and } I) = P(S) \cdot P(I)$.
However, $0.4 \neq (0.5)(0.7)$.
Therefore, salary and insurance have not been treated independently.

10. CC AUG '16 [32]
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$0.8 = 0.6 + 0.5 - P(A \text{ and } B)$
$P(A \text{ and } B) = 0.3$
A and B are independent since
$P(A \text{ and } B) = P(A) \times P(B)$
$0.3 = 0.6 \times 0.5$

11. CC JAN '17 [35]
$P(P|K) = \frac{P(P \text{ and } K)}{P(K)} = \frac{0.019}{0.023} \approx 82.6\%$
A key club member has an 82.6% probability of being enrolled in AP Physics.

12. CC AUG '17 [26]
$P(W|D) = \frac{P(W \text{ and } D)}{P(D)} = \frac{0.4}{0.5} \approx 0.8$

13. CC JAN '18 [34]
$\frac{47}{108} = \frac{1}{4} + \frac{116}{459} - P(M \text{ and } J)$
$P(M \text{ and } J) = \frac{31}{459}$
No, because $\frac{31}{459} \neq \frac{1}{4} \times \frac{116}{459}$

14. CC JAN '19 [28]
$P(A \text{ and } B) = P(A) \cdot P(B|A) = (0.8)(0.85) = 0.68$

15. CC JAN '23 [27]
$P(A \cap B) = \frac{1}{3} \cdot \frac{5}{12} = \frac{5}{36}$

Chapter 16. Statistics

16.1 Data Collection

1. CC JAN '17 [6] Ans: 3
2. CC AUG '17 [17] Ans: 2
3. CC AUG '18 [2] Ans: 2
4. CC JUN '19 [1] Ans: 3
5. CC AUG '19 [6] Ans: 4
6. CC JAN '20 [15] Ans: 3
7. CC JUN '22 [16] Ans: 4
8. CC AUG '22 [4] Ans: 2
9. CC JAN '24 [1] Ans: 3
10. CC JUN '16 [26]
    Randomly assign participants to two groups. One group uses the toothpaste with ingredient X and the other group uses the toothpaste without ingredient X.
11. CC JAN '17 [26]
    sample: pails of oranges; population: truckload of oranges. It is likely that about 5% of all the oranges are unsatisfactory.

16.2 Bias

1. CC AUG '16 [2] Ans: 1
2. CC JUN '17 [3] Ans: 3
3. CC JAN '18 [1] Ans: 4
4. CC JAN '19 [10] Ans: 2
5. CC JUN '22 [2] Ans: 3
6. CC AUG '22 [1] Ans: 3
7. CC JAN '23 [14] Ans: 4
8. CC AUG '23 [1] Ans: 4
9. CC JUN '18 [28]
   Self selection is a cause of bias because people with more free time are more likely to respond.
10. CC JUN '23 [25]
    Pick random names from a list of all students and ask them what method of payment they will use.

16.3 Normal Distribution

1. CC JUN '16 [9] Ans: 2
2. CC JAN '17 [18] Ans: 4
3. CC JAN '19 [1] Ans: 2
4. CC JAN '19 [23] Ans: 1
5. CC JUN '19 [18] Ans: 4
6. CC AUG '19 [19] Ans: 1
7. CC JUN '23 [16] Ans: 2

16.4 Areas Under Normal Curves

1. CC AUG '16 [4] Ans: 3
2. CC AUG '17 [11] Ans: 1
3. CC JAN '18 [7] Ans: 3
4. CC JUN '18 [17] Ans: 2
5. CC JUN '22 [14] Ans: 1
6. CC JUN '23 [16] Ans: 4
7. CC AUG '23 [13] Ans: 2

8. CC FALL '15 [16]
normalcdf(510, 540, 480, 24) ≈ 0.0994.
Use z-scores to compare the two sets of data. Joanne's score range corresponds to 1.25 to 2.5 SD above the mean:
$z = \frac{510-480}{24} = 1.25$
$z = \frac{540-480}{24} = 2.5$
Calculating equivalent scores,
$1.25 = \frac{x-510}{20}$
$x = 535$
$2.5 = \frac{x-510}{20}$
$x = 560$
Maria must score in the interval 535–560.

9. CC JUN '17 [26]
normalcdf(0, 8.25, 8, 0.5) ≈ 69%

10. CC AUG '18 [28]
normalcdf(200, 245, 225, 18) ≈ 0.784
$1200 \times 0.784 \approx 941$

11. CC AUG '22 [30]
normalcdf(4, 1e99, 3.39, 0.55) ≈ 0.1337
$0.1337 \times 9256 \approx 1237$

12. CC JAN '23 [28]
normalcdf(690, 900, 680, 120) ≈ 43%

13. CC JAN '24 [29]
normalcdf(67, 72, 64.7, 4.3) ≈ 25%

16.5 Plausible Outcomes

1. CC JUN '16 [7] Ans: 3
2. CC JUN '17 [10] Ans: 3
3. CC JAN '18 [20] Ans: 2
4. CC JUN '19 [13] Ans: 2
5. CC JAN '20 [14] Ans: 4
6. CC SPR '15 [12]
a) Yes. The margin of error = $2SD = 2(0.062) \approx 0.124$.
This indicates that 95% of the observations fall within ± 0.124 of 0.247. So, plausible values would fall into the interval (0.123, 0.371). 9 out of 50, or 0.18, falls within this interval.
b) The company has evidence that the population proportion could be at least 25%. As seen in the dot plot, it can be expected to obtain a sample proportion of 0.18 or less several times, even when the population proportion is 0.25, due to sampling variability. The results do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue.

7. CC JUN '16 [35]
$0.602 \pm 2(0.066) \approx (0.47, 0.73)$
Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

8. CC AUG '16 [29]
Using a 95% level of confidence, $\mu \pm 2$ standard deviations, or $226 \pm 2(38)$, sets the usual wait time as 150 to 302 seconds. 360 seconds is unusual.

9. CC JUN '17 [36]
$0.506 \pm 2(0.078) = (0.35, 0.66)$
The 32.5% value falls below (outside of) the 95% interval.

10. CC JAN '18 [35]
$138.905 \pm 2(7.95) = (123, 155)$
No, since 125 (50% of 250) falls within the 95% interval.

11. CC AUG '18 [32]
$2(0.042) \approx 0.08$
The percent of users making in-app purchases will be within 8 percentage points of 35%; that is, between 27% and 43%.

12. CC JAN '19 [35]
$29.101 \pm 2(0.934) = (27.23, 30.97)$
Yes, since 30 falls within the 95% interval.

13. CC JUN '19 [32]
$0.499 \pm 2(0.049) = (0.401, 0.597)$
No, since 0.43 falls within this interval, Robin's coin is likely not unfair.

14. CC AUG '19 [35]
$0.301 \pm 2(0.058) = (0.185, 0.417)$
$\frac{14}{60} \approx 0.23$, which is not unusual because it falls within this interval.

15. CC JUN '22 [35]
$0.651 \pm 2(0.034) = (0.58, 0.72)$
No, because $\frac{122}{200} = 0.61$ falls within this interval.

16. CC AUG '22 [36]
$0.819 \pm 2(0.053) = (0.713, 0.925)$
Because 0.7 does not fall within this interval.

17. CC JAN '23 [34]
$\frac{1}{10} = 0.10$; $P(2) = \frac{1}{5} = 0.20$; No, because 0.10 occurs 21% of the time, which is not unusual.

18. CC JUN '23 [32]
$0.852 \pm 2(0.029) = (0.794, 0.91)$
No, because 0.88 falls within this interval.

16.6 Difference in Means (CC)

1. CC JAN '17 [9] Ans: 2

2. CC FALL '15 [14] :
a) The mean difference between the students' final grades in group 1 and group 2 is –3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription.
b) If the observed difference –3.64 is the result of the assignment of students to groups alone, then a difference of –3.64 or less should be observed fairly regularly in the simulation output. However, a difference of –3 or less occurs in only about 2% of the results. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups' mean final grades.

3. CC AUG '16 [36]
Some of the students who did not drink energy drinks read faster than those who did drink energy drinks.
$17.7 - 19.1 = -1.4$. Differences of -1.4 and less occur $\frac{25}{232}$ or about 10% of the time, so the difference is not unusual.

4. CC JUN '18 [34]
$23 - 18 = 5$
$\bar{x} \pm 2\sigma = 0.030 \pm 2(1.548) = (-3.066, 3.126)$
Yes, 5 is outside of the interval of the middle 95%, so it is statistically significant.

5. CC AUG '18 [36]
John found the means of the scores of the two rooms and subtracted the means. The mean score for the classical room was 7 higher than the rap room $(82 - 75)$. Yes, only 4 of the 250 results (1.6%) had a mean difference of 7 or more, which is less than 5% and therefore statistically significant. It is likely the difference was due to the music.

6. CC AUG '23 [36]
$\bar{x} \pm 2\sigma = 0.01 \pm 0.38 = (-0.75, 0.77)$
No, since 0.6 falls within the 95% interval.

16.7 Estimate Population Parameters

1. CC AUG ‘16 [12] Ans: 2
2. CC AUG ‘17 [16] Ans: 2
3. CC AUG ‘17 [22] Ans: 1
4. CC JAN ‘23 [19] Ans: 4
5. CC JAN ’24 [11] Ans: 2
6. CC JAN ’24 [18] Ans: 3
7. CC JAN ’24 [32]
   $\frac{475}{1250} = 38\%$; About 38% of the high school juniors in the population will choose a four-year college.