Answer Key

Algebra II
Course Workbook

2023-24 Edition
Donny Brusca

ISBN 979-8378331246

www.CourseWorkBooks.com

Table of Contents

Chapter 1. Linear Functions

1.1 Linear Systems in Three Variables [CC]

1. The first variable, *x*, is already isolated, so find the others by substitution:

$5(3)+4y=-9$ Eq. 2
$15+4y=-9$
$4y=-24$
$y=-6$

$-(3)+4(-6)-2z=-25$ Eq. 3
$-27-2z=-25$
$-2z=2$
$z=-1$
$x=3, y=-6, z=-1$

2. $x+3y+z=10$ Eq. 1
$\underline{-x-y-z=-2}$ Eq. 2 × –1
$2y=8$
$y=4$

We have already isolated y, so substitute:
$(4)-2z=2$ Eq. 3
$-2z=-2$
$z=1$

$x+(4)+(1)=2$ Eq. 2
$x=-3$
$x=-3, y=4, z=1$

3. $2x-4y+5z=-33$ Eq. 1
$\underline{-2x+2y-3z=19}$ Eq. 3
$-2y+2z=-14$
$-y+z=-7$ Result A

$4x-y=-5$ Eq. 2
$\underline{-4x+4y-6z=38}$ Eq. 3 × 2
$3y-6z=33$ Result B

$-3y+3z=-21$ Result A × 3
$\underline{3y-6z=33}$ Result B
$-3z=12$
$z=-4$

$-y+(-4)=-7$ Result A
$y=3$

$4x-(3)=-5$ Eq. 2
$x=-\frac{1}{2}$
$x=-\frac{1}{2}, y=3, z=-4$

4. $x-2y+3z=7$ Eq. 1
$\underline{4x+2y+2z=8}$ Eq. 2 × 2
$5x \quad +5z=15$
$x+z=3$ Result A

$x-2y+3z=7$ Eq. 1
$\underline{-3x+2y-2z=-10}$ Eq. 3
$-2x \quad +z=-3$ Result B

$2x+2z=6$ Result A × 2
$\underline{-2x+z=-3}$ Result B
$3z=3$
$z=1$

$x+(1)=3$ Result A
$x=2$

$2(2)+y+(1)=4$ Eq. 2
$y=-1$

$x=2, y=-1, z=1$

5. $ax^2 + bx + c = y$
$a(-1)^2 + b(-1) + c = 9$
$a(2)^2 + b(2) + c = 3$
$a(5)^2 + b(5) + c = 15$

$a - b + c = 9$ Eq. 1
$4a + 2b + c = 3$ Eq. 2
$25a + 5b + c = 15$ Eq. 3

$a - b + c = 9$ Eq. 1
$\underline{-4a - 2b - c = -3}$ Eq. 2 × –1
$-3a - 3b = 6$
$a + b = -2$ Result A

$a - b + c = 9$ Eq. 1
$\underline{-25a - 5b - c = -15}$ Eq. 3 × –1
$-24a - 6b = -6$
$4a + b = 1$ Result B

$a + b = -2$ Result A
$\underline{-4a - b = -1}$ Result B × –1
$-3a = -3$
$a = 1$

$(1) + b = -2$ Result A
$b = -3$

$(1) - (-3) + c = 9$ Eq. 1
$c = 5$

$y = x^2 - 3x + 5$

6. $ax^2 + bx + c = y$
$a(-1)^2 + b(-1) + c = -2$
$a(1)^2 + b(1) + c = 0$
$a(2)^2 + b(2) + c = 7$

$a - b + c = -2$ Eq. 1
$a + b + c = 0$ Eq. 2
$4a + 2b + c = 7$ Eq. 3

$a - b + c = -2$ Eq. 1
$\underline{-a - b - c = 0}$ Eq. 2 × –1
$-2b = -2$
$b = 1$

$a - (1) + c = -2$ Eq. 1
$a + c = -1$ Result A

$4a + 2(1) + c = 7$ Eq. 3
$4a + c = 5$ Result B

$a + c = -1$ Result A
$\underline{-4a - c = -5}$ Result B × –1
$-3a = -6$
$a = 2$

$(2) - (1) + c = -2$ Eq. 1
$c = -3$

$y = 2x^2 + x - 3$

Chapter 2. Irrational Expressions

2.1 Operations with Square Roots [CC]

1. $5\sqrt{3}+\sqrt{3}=6\sqrt{3}$	2. $3\sqrt{3}+2\sqrt{3}=5\sqrt{3}$
3. $4\sqrt{3}-2\sqrt{3}=2\sqrt{3}$	4. $5\sqrt{6}+2\sqrt{6}=7\sqrt{6}$
5. $3\sqrt{2}+2\sqrt{2}=5\sqrt{2}$	6. $5\sqrt{2}-4\sqrt{2}=\sqrt{2}$
7. $6\sqrt{2}-3\sqrt{2}=3\sqrt{2}$	8. $10\sqrt{2}-\sqrt{2}=9\sqrt{2}$
9. $5\sqrt{7}+6\sqrt{7}=11\sqrt{7}$	10. $30\sqrt{2}+6\sqrt{2}=36\sqrt{2}$
11. $5-2\sqrt{3}+3\sqrt{3}+6=11+\sqrt{3}$	12. $y\sqrt{3}-4\sqrt{2}-3y\sqrt{3}=-2y\sqrt{3}-4\sqrt{2}$
13. $\sqrt{90}=3\sqrt{10}$	14. $\sqrt{3600}-\sqrt{144}=60-12=48$
15. $6\sqrt{100}-21\sqrt{20}=60-42\sqrt{5}$	16. $3\sqrt{98}+12\sqrt{392}=21\sqrt{2}+168\sqrt{2}=$ $189\sqrt{2}$
17. $9-5=4$	18. $5\sqrt{2}-20+2-4\sqrt{2}=-18+\sqrt{2}$
19. $10\sqrt{18x^7}=10\sqrt{9x^6}\sqrt{2x}=30x^3\sqrt{2x}$	20. $15\sqrt{2x^6y^2}=15x^3y\sqrt{2}$
21. $9x-18\sqrt{x}+9$	22. $c^2=\left(x+\sqrt{2}\right)^2+\left(x-\sqrt{2}\right)^2$ $c^2=x^2+2x\sqrt{2}+2+x^2-2x\sqrt{2}+2$ $c^2=2x^2+4$ $c=\sqrt{2x^2+4}$
23. $\sqrt{13}$	24. $\frac{\sqrt{28}}{2}=\frac{2\sqrt{7}}{2}=\sqrt{7}$
25. $5\sqrt{50}=25\sqrt{2}$	26. $\frac{15\sqrt{3}+3\sqrt{3}}{3}=\frac{18\sqrt{3}}{3}=6\sqrt{3}$
27. $\frac{4\sqrt{3}-15\sqrt{3}+10\sqrt{3}}{\sqrt{3}}=\frac{-\sqrt{3}}{\sqrt{3}}=-1$	28. $\frac{3\sqrt{3}+5\sqrt{3}}{2\sqrt{3}}=\frac{8\sqrt{3}}{2\sqrt{3}}=4$
29. $\frac{16\sqrt{21}}{2\sqrt{7}}-5\sqrt{12}=8\sqrt{3}-10\sqrt{3}=-2\sqrt{3}$	30. $\sqrt{18x^4y^3}=3x^2y\sqrt{2y}$

2.2 Rationalize Monomial Denominators [CC]

1. $\frac{3}{\sqrt{7}}\left(\frac{\sqrt{7}}{\sqrt{7}}\right)=\frac{3\sqrt{7}}{7}$	2. $\frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{2\sqrt{2}}{2}=\sqrt{2}$
3. $\frac{3\sqrt{5}}{2\sqrt{10}}\left(\frac{\sqrt{10}}{\sqrt{10}}\right)=\frac{3\sqrt{50}}{2\cdot 10}=\frac{15\sqrt{2}}{20}=\frac{3\sqrt{2}}{4}$	4. $\frac{3-\sqrt{8}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{3\sqrt{3}-\sqrt{24}}{3}=\frac{3\sqrt{3}-2\sqrt{6}}{3}$
5. $\frac{2}{\sqrt{3}}\times\frac{\sqrt{2}}{5}=\frac{2\sqrt{2}}{5\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{2\sqrt{6}}{15}$	6. $\frac{\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}}{\sqrt{5}}=\frac{\frac{2}{\sqrt{5}}}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)=\frac{2}{5}$

2.3 Rationalize Binomial Denominators

1. $\frac{\sqrt{5}}{7-\sqrt{5}}\left(\frac{7+\sqrt{5}}{7+\sqrt{5}}\right)=\frac{7\sqrt{5}+5}{49-5}=\frac{7\sqrt{5}+5}{44}$	2. $\frac{1}{3-\sqrt{7}}\left(\frac{3+\sqrt{7}}{3+\sqrt{7}}\right)=\frac{3+\sqrt{7}}{9-7}=\frac{3+\sqrt{7}}{2}$
3. $\frac{5}{4-\sqrt{11}}\left(\frac{4+\sqrt{11}}{4+\sqrt{11}}\right)=\frac{20+5\sqrt{11}}{16-11}=$ $\frac{20+5\sqrt{11}}{5}=4+\sqrt{11}$	4. $\frac{4}{5-\sqrt{13}}\left(\frac{5+\sqrt{13}}{5+\sqrt{13}}\right)=\frac{20+4\sqrt{13}}{25-13}=$ $\frac{20+4\sqrt{13}}{12}=\frac{5+\sqrt{13}}{3}$
5. $\frac{\sqrt{2}}{\sqrt{14}+4}\left(\frac{\sqrt{14}-4}{\sqrt{14}-4}\right)=\frac{\sqrt{28}-4\sqrt{2}}{14-16}=$ $\frac{2\sqrt{7}-4\sqrt{2}}{-2}=-\sqrt{7}+2\sqrt{2}$	6. $\frac{\sqrt{3}}{\sqrt{3}+5}\left(\frac{\sqrt{3}-5}{\sqrt{3}-5}\right)=\frac{3-5\sqrt{3}}{3-25}=\frac{3-5\sqrt{3}}{-22}=$ $-\frac{3-5\sqrt{3}}{22}$
7. $\frac{2-\sqrt{2}}{2+\sqrt{2}}\left(\frac{2-\sqrt{2}}{2-\sqrt{2}}\right)=\frac{4-4\sqrt{2}+2}{4-2}=\frac{6-4\sqrt{2}}{2}$ $=3-2\sqrt{2}$	8. $\frac{\sqrt{3}+5}{\sqrt{3}-5}\left(\frac{\sqrt{3}+5}{\sqrt{3}+5}\right)=\frac{3+10\sqrt{3}+25}{3-25}=$ $\frac{28+10\sqrt{3}}{-22}=-\frac{14+5\sqrt{3}}{11}$
9. $\frac{\sqrt{6}+8}{\sqrt{2}+\sqrt{3}}\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}\right)=$ $\frac{\sqrt{12}-\sqrt{18}+8\sqrt{2}-8\sqrt{3}}{2-3}=$ $\frac{2\sqrt{3}-3\sqrt{2}+8\sqrt{2}-8\sqrt{3}}{-1}=$ $-\left(5\sqrt{2}-6\sqrt{3}\right)=6\sqrt{3}-5\sqrt{2}$	10. $\frac{\sqrt{xy}}{\sqrt{x}-\sqrt{y}}\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)=\frac{x\sqrt{y}+y\sqrt{x}}{x-y}$

11. $A = lw$

$$2 = \left(\sqrt{5} - 1\right)w$$

$$w = \frac{2}{\sqrt{5} - 1}$$

$$w = \frac{2}{\sqrt{5} - 1}\left(\frac{\sqrt{5} + 1}{\sqrt{5} + 1}\right) = \frac{2\sqrt{5} + 2}{4} = \frac{\sqrt{5} + 1}{2}$$

12. $A = \frac{1}{2}bh$

$$8 + 12\sqrt{2} = \tfrac{1}{2}\left(6 + 2\sqrt{2}\right)h$$

$$8 + 12\sqrt{2} = \left(3 + \sqrt{2}\right)h$$

$$h = \frac{8 + 12\sqrt{2}}{3 + \sqrt{2}}$$

$$h = \frac{8 + 12\sqrt{2}}{3 + \sqrt{2}}\left(\frac{3 - \sqrt{2}}{3 - \sqrt{2}}\right) = \frac{24 + 28\sqrt{2} - 24}{9 - 2} =$$

$$\frac{28\sqrt{2}}{7} = 4\sqrt{2}$$

Chapter 3. Quadratic Functions

3.1 Factor a Trinomial by Grouping [NG]

1. $6x^2 + x - 2 =$ $6x^2 - 3x + 4x - 2 =$ $3x(2x - 1) + 2(2x - 1) =$ $(3x + 2)(2x - 1)$	2. $12x^2 + 5x - 2 =$ $12x^2 - 3x + 8x - 2 =$ $3x(4x - 1) + 2(4x - 1) =$ $(3x + 2)(4x - 1)$
3. $12x^2 - 29x + 15 =$ $12x^2 - 9x - 20x + 15 =$ $3x(4x - 3) - 5(4x - 3) =$ $(3x - 5)(4x - 3)$	4. $6x^2 - 11x + 4 =$ $6x^2 - 3x - 8x + 4 =$ $3x(2x - 1) - 4(2x - 1) =$ $(3x - 4)(2x - 1)$
5. $15x^2 + 14x - 8 =$ $15x^2 + 20x - 6x - 8 =$ $5x(3x + 4) - 2(3x + 4) =$ $(5x - 2)(3x + 4)$	6. $-10x^2 - 29x - 10 =$ $-10x^2 - 25x - 4x - 10 =$ $-5x(2x + 5) - 2(2x + 5) =$ $(-5x - 2)(2x + 5)$
7. $4x^2 + 12x + 9 =$ $4x^2 + 6x + 6x + 9 =$ $2x(2x + 3) + 3(2x + 3) =$ $(2x + 3)(2x + 3)$ The square root is $2x + 3$.	8. First, write in standard form: $6x^2 - 57x - 30 =$ $6x^2 - 60x + 3x - 30 =$ $6x(x - 10) + 3(x - 10) =$ $(6x + 3)(x - 10)$
9. $2x^2 + 4x - 3x - 6 =$ $2x(x + 2) - 3(x + 2) =$ $(2x - 3)(x + 2)$ So, $(2x - 3)$	

3.2 Solve Quadratics with $a \neq 1$ [NG]

1. $10x^2 + 9x + 2 = 0$
$n^2 + 9n + 20 = 0$
$(n+5)(n+4) = 0$
$n = -5$ or $n = -4$
$x = -\frac{5}{10}$ or $x = -\frac{4}{10}$
$\left\{-\frac{1}{2}, -\frac{2}{5}\right\}$

2. $2x^2 - 3x - 2 = 0$
$n^2 - 3n - 4 = 0$
$(n+1)(n-4) = 0$
$n = -1$ or $n = 4$
$x = -\frac{1}{2}$ or $x = \frac{4}{2}$
$\left\{-\frac{1}{2}, 2\right\}$

3. $12x^2 + 29x + 15 = 0$
$n^2 + 29n + 180 = 0$
$(n+20)(n+9) = 0$
$n = -20$ or $n = -9$
$x = -\frac{20}{12}$ or $x = -\frac{9}{12}$
$\left\{-\frac{5}{3}, -\frac{3}{4}\right\}$

4. $4x^2 + 109x + 225 = 0$
$n^2 + 109n + 900 = 0$
$(n+100)(n+9) = 0$
$n = -100$ or $n = -9$
$x = -\frac{100}{4}$ or $x = -\frac{9}{4}$
$\left\{-25, -\frac{9}{4}\right\}$

5. $ac = 20$ and $b = 9$, so use 5 and 4
$10x^2 + 5x + 4x + 2 = 0$
$5x(2x+1) + 2(2x+1) = 0$
$(5x+2)(2x+1) = 0$
$5x + 2 = 0$ $\quad 2x + 1 = 0$
$x = -\frac{2}{5}$ $\quad x = -\frac{1}{2}$
$\left\{-\frac{1}{2}, -\frac{2}{5}\right\}$

6. $ac = 24$ and $b = -14$; use -12 and -2
$3x^2 - 12x - 2x + 8 = 0$
$3x(x-4) - 2(x-4) = 0$
$(3x-2)(x-4) = 0$
$3x - 2 = 0$ $\quad x - 4 = 0$
$x = \frac{2}{3}$ $\quad x = 4$
$\left\{\frac{2}{3}, 4\right\}$

7. $ac = -4$ and $b = -3$, so use -4 and 1
$2x^2 - 4x + x - 2 = 0$
$2x(x-2) + 1(x-2) = 0$
$(2x+1)(x-2) = 0$
$2x + 1 = 0$ $\quad x - 2 = 0$
$x = -\frac{1}{2}$ $\quad x = 2$
$\left\{-\frac{1}{2}, 2\right\}$

8. $4x(x-1) = 15$
$4x^2 - 4x = 15$
$4x^2 - 4x - 15 = 0$
$ac = -60$ and $b = -4$; use 6 and -10
$4x^2 + 6x - 10x - 15 = 0$
$2x(2x+3) - 5(2x+3) = 0$
$(2x-5)(2x+3) = 0$
$2x - 5 = 0$ $\quad 2x + 3 = 0$
$x = \frac{5}{2}$ $\quad x = -\frac{3}{2}$
$\left\{-\frac{3}{2}, \frac{5}{2}\right\}$

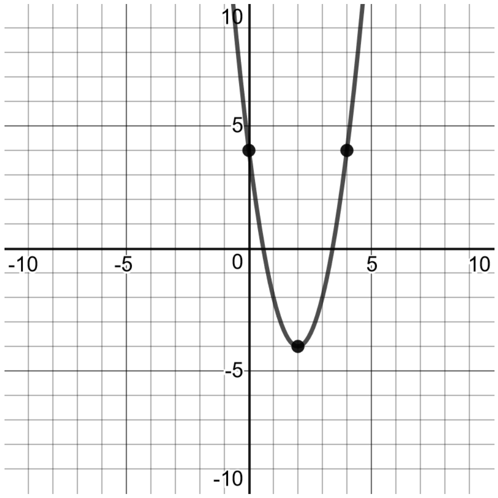
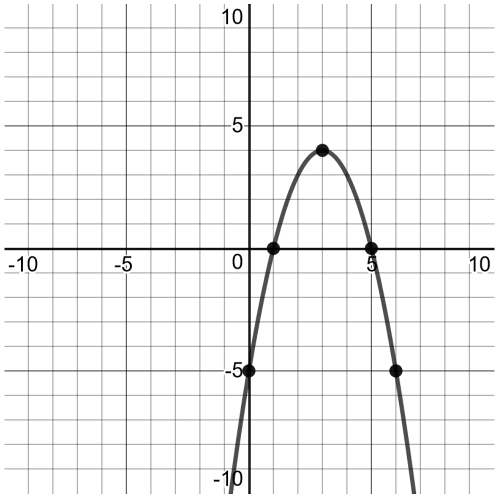
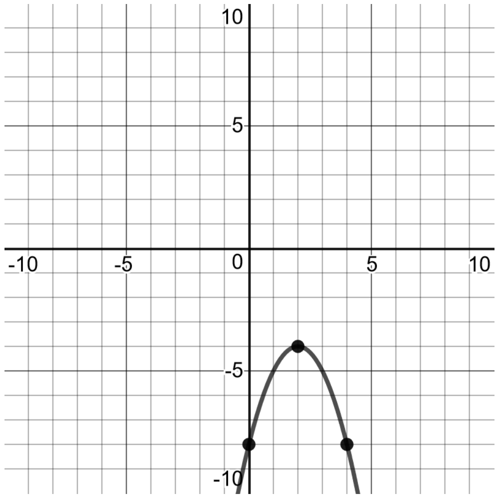
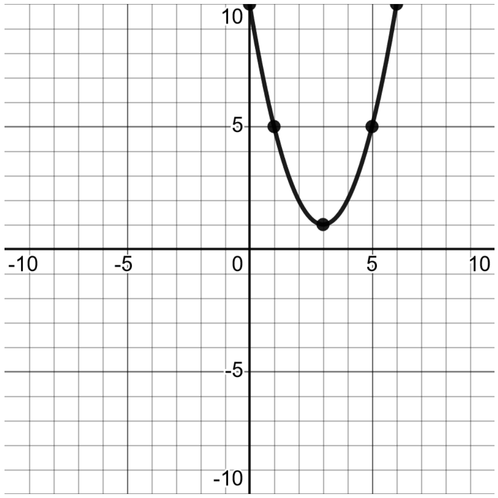
9. $4x^2 + 8x - 12 = 0$
$x^2 + 2x - 3 = 0$
$x^2 + 2x = 3$ $\quad \left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$
$x^2 + 2x + 1 = 3 + 1$
$(x+1)^2 = 4$
$x + 1 = \pm\sqrt{4} = \pm 2$
$x = -1 \pm 2$
$\{-3, 1\}$

10. $3x^2 - 18x - 21 = 0$
$x^2 - 6x - 7 = 0$
$x^2 - 6x = 7$ $\quad \left(\frac{b}{2}\right)^2 = \left(-\frac{6}{2}\right)^2 = 9$
$x^2 - 6x + 9 = 7 + 9$
$(x-3)^2 = 16$
$x - 3 = \pm\sqrt{16} = \pm 4$
$x = 3 \pm 4$
$\{-1, 7\}$

11. $4x^2+8x=45$ $x^2+2x=\frac{45}{4}$ $\left(\frac{b}{2}\right)^2=\left(\frac{2}{2}\right)^2=1$ $x^2+2x+1=\frac{45}{4}+\frac{4}{4}$ $(x+1)^2=\frac{49}{4}$ $x+1=\pm\sqrt{\frac{49}{4}}=\pm\frac{7}{2}$ $x=-1\pm\frac{7}{2}=-\frac{2}{2}\pm\frac{7}{2}$ $\left\{-\frac{9}{2},\frac{5}{2}\right\}$	12. $3x^2-12x+2=0$ $x^2-4x+\frac{2}{3}=0$ $x^2-4x=-\frac{2}{3}$ $\left(\frac{b}{2}\right)^2=\left(-\frac{4}{2}\right)^2=4$ $x^2-4x+4=-\frac{2}{3}+\frac{12}{3}$ $(x-2)^2=\frac{10}{3}$ $x-2=\pm\sqrt{\frac{10}{3}}=\pm\frac{\sqrt{30}}{3}$ $x=2\pm\frac{\sqrt{30}}{3}$ $\left\{2-\frac{\sqrt{30}}{3},2+\frac{\sqrt{30}}{3}\right\}$
13. $x=\frac{-9\pm\sqrt{9^2-4(10)(2)}}{2(10)}$ $x=\frac{-9\pm\sqrt{1}}{20}=\frac{-9\pm1}{20}$ $\left\{-\frac{1}{2},-\frac{2}{5}\right\}$	14. $x=\frac{-4\pm\sqrt{4^2-4(-3)(-1)}}{2(-3)}$ $x=\frac{-4\pm\sqrt{4}}{-6}=\frac{-4\pm2}{-6}$ $\left\{\frac{1}{3},1\right\}$
15. $x=\frac{-8\pm\sqrt{8^2-4(4)(-9)}}{2(4)}$ $x=\frac{-8\pm\sqrt{208}}{8}=\frac{-8\pm4\sqrt{13}}{8}=-1\pm\frac{\sqrt{13}}{2}$ $\left\{-1-\frac{\sqrt{13}}{2},-1+\frac{\sqrt{13}}{2}\right\}$	16. $x=\frac{12\pm\sqrt{(-12)^2-4(3)(2)}}{2(3)}$ $x=\frac{12\pm\sqrt{120}}{6}=\frac{12\pm2\sqrt{30}}{6}=2\pm\frac{\sqrt{30}}{3}$ $\left\{2-\frac{\sqrt{30}}{3},2+\frac{\sqrt{30}}{3}\right\}$

3.3 Graphs of Quadratic Functions

1. $x=\frac{-(-8)}{2(-2)}=-2$ $y=-2(-2)^2-8(-2)+3=11$ $x=-2$ and $(-2,11)$	2. $x=\frac{-(-2)}{2(-1)}=-1$ $y=-(-1)^2-2(-1)+1=2$ $x=-1$ and $(-1,2)$
3. For a parabola opening down, the maximum value is at the vertex. $x=\frac{-6}{2(-3)}=1$ $y=-3(1)^2+6(1)-2=1$ Maximum is 1	4. For a parabola opening up, the minimum value is at the vertex. $x=\frac{-(-20)}{2(5)}=2$ $y=5(2)^2-20(2)+14=-6$ Minimum is –6

5. 	6. 
7. 	8. 

3.4 Vertex Form and Transformations

1. (3)	
2. $a = 1$ $h = -\frac{b}{2a} = -3$ $k = (-3)^2 + 6(-3) + 10 = 1$ $y = (x+3)^2 + 1$ vertex: (–3,1)	3. $y = x^2 + 10x + 21$ $y - 21 = x^2 + 10x$ $\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = 25$ $y + 4 = x^2 + 10x + 25$ $y + 4 = (x+5)^2$ $y = (x+5)^2 - 4$ vertex: (–5,–4)
4. $y = (x+5)^2 + 3$ $y = (x+5)(x+5) + 3$ $y = x^2 + 10x + 25 + 3$ $y = x^2 + 10x + 28$	5. $y = -2(x-4)^2 - 5$ $y = -2(x-4)(x-4) - 5$ $y = -2(x^2 - 8x + 16) - 5$ $y = -2x^2 + 16x - 32 - 5$ $y = -2x^2 + 16x - 37$

3.5 Focus and Directrix [CC]

1. Vertex is $(2, -4)$
$p = \frac{1}{4a} = \frac{1}{4} \div \frac{1}{16} = 4$
Since $p > 0$, the directrix is below the vertex. Directrix is $y = -4 - 4 = -8$

2. $h = 0$ and $k = \frac{3+5}{2} = 4$
$p = 3 - 4 = -1$
$a = \frac{1}{4p} = -\frac{1}{4}$
$y = -\frac{1}{4}x^2 + 4$

3. $h = 4$ and $k = \frac{2-4}{2} = -1$
$p = 2 - (-1) = 3$
$a = \frac{1}{4p} = \frac{1}{12}$
$y = \frac{1}{12}(x-4)^2 - 1$

4. $x^2 + 6x = -4y - 5$
$x^2 + 6x + 9 = -4y - 5 + 9$
$(x+3)^2 = -4y + 4$
$(x+3)^2 - 4 = -4y$
$y = -\frac{1}{4}(x+3)^2 + 1$
Vertex is $(-3,1)$
$p = \frac{1}{4a} = \frac{1}{4} \div \left(-\frac{1}{4}\right) = -1$
Since $p < 0$, the focus is 1 unit below the vertex, at $(-3,0)$, and the directrix is 1 unit above the vertex, at $y = 2$.

5. Vertex is $(-1,2)$.
$p = 8 \div 4 = 2$
The equation of the directrix is
$y = k - p$, so $y = 2 - 2$, or $y = 0$

6. Vertex is $(-3,1)$
$p = -20 \div 4 = -5$
The equation of the directrix is
$y = k - p$, or $y = 6$.

7. $p = 1 - (-3) = 4$
$(x-2)^2 = 16(y-1)$

8. Vertex is $(2,1)$.
$4p = 2$, so $p = \frac{1}{2}$.
Focus is $\frac{1}{2}$ unit above the vertex, at $\left(2, \frac{3}{2}\right)$.
Directrix is $y = k - p$, or $y = \frac{1}{2}$.

Chapter 4. Imaginary Numbers

4.1 Set of Complex Numbers

1. $\sqrt{-25} = \sqrt{25}\sqrt{-1} = 5i$	2. $\sqrt{100}\sqrt{-1}\sqrt{3} = 10i\sqrt{3}$
3. $2 + \sqrt{-12} = 2 + \sqrt{4}\sqrt{-1}\sqrt{3} =$ $2 + 2i\sqrt{3}$	4. $-8 + \frac{3}{4}\sqrt{16}\sqrt{-1}\sqrt{5} = -8 + 3i\sqrt{5}$
5. $\sqrt{36}\sqrt{x^{16}}\sqrt{-1}\sqrt{5} = 6x^8 i\sqrt{5}$	6. $(3i)^3 = (3^3)(i^3) = 27(-i) = -27i$
7. $(2i)^4 = (2^4)(i^4) = 16(1) = 16$	8. $(-3)^3 i^{10} = (-27)(-1) = 27$

4.2 Operations with Complex Numbers

1. $8 - 2i$	2. $(3 + 2i)(2 - i) =$ $6 - 3i + 4i - 2(-1) =$ $8 + i$
3. $(3 - 7i)(3 - 7i) =$ $9 - 21i - 21i + 49(-1) =$ $-40 - 42i$	4. $\left(2\sqrt{2} + 5i\right)\left(5\sqrt{2} - 2i\right) =$ $20 - 4i\sqrt{2} + 25i\sqrt{2} - 10(-1) =$ $30 + 21i\sqrt{2}$
5. $(-1 + i)(-1 + i)(-1 + i) =$ $(1 - i - i - 1)(-1 + i) =$ $(-2i)(-1 + i) =$ $2i - 2(-1) =$ $2 + 2i$	6. $(3i)(2i)^2(2 + i) =$ $(3i)(-4)(2 + i) =$ $(-12i)(2 + i) =$ $-24i - 12(-1) =$ $12 - 24i$
7. $(x + i)(x + i) - (x - i)(x - i) =$ $(x^2 + 2xi - 1) - (x^2 - 2xi - 1) =$ $4xi$	8. $2xi(i - 4i^2) =$ $2xi(i + 4) =$ $-2x + 8xi$
9. $3x^2 + 48 = 3(x^2 + 16) =$ $3(x + 4i)(x - 4i)$	10. $-9x^2 - 81 = -9(x^2 + 9) =$ $-9(x + 3i)(x - 3i)$

4.3 Imaginary Roots

1. $x = \pm\sqrt{-25} = \pm\sqrt{25}\sqrt{-1} = \pm 5i$
   $\{-5i, 5i\}$

2. $x = \pm\sqrt{-16} = \pm\sqrt{16}\sqrt{-1} = \pm 4i$
   $\{-4i, 4i\}$

3. $x^2 = \frac{3}{2}(-18) = -27$
   $x = \pm\sqrt{-27} = \pm 3i\sqrt{3}$
   $\{-3i\sqrt{3}, 3i\sqrt{3}\}$

4. $x + 2 = \pm\sqrt{-9}$
   $x + 2 = \pm 3i$
   $x = -2 \pm 3i$

5. $x^2 - 12x = -40$
   $x^2 - 12x + 36 = -40 + 36$
   $(x - 6)^2 = -4$
   $x - 6 = \pm\sqrt{-4}$
   $x = 6 \pm 2i$

6. $x^2 + 8x = -25$
   $x^2 + 8x + 16 = -25 + 16$
   $(x + 4)^2 = -9$
   $x + 4 = \pm\sqrt{-9}$
   $x = -4 \pm 3i$

7. $x^2 - 4x = -9$
   $x^2 - 4x + 4 = -9 + 4$
   $(x - 2)^2 = -5$
   $x - 2 = \pm\sqrt{-5}$
   $x = 2 \pm i\sqrt{5}$

8. $n^2 + 6n + 12 = 0$
   $n^2 + 6n = -12$
   $n^2 + 6n + 9 = -12 + 9$
   $(n + 3)^2 = -3$
   $n + 3 = \pm\sqrt{-3} = \pm i\sqrt{3}$
   $n = -3 \pm i\sqrt{3}$
   $x = -1 \pm \frac{i\sqrt{3}}{3}$

9. $\frac{-(-3) \pm\sqrt{(-3)^2 - 4(1)(7)}}{2(1)} = \frac{3 \pm\sqrt{-19}}{2} = \frac{3}{2} \pm \frac{i\sqrt{19}}{2}$

10. $\frac{-(-12) \pm\sqrt{(-12)^2 - 4(4)(25)}}{2(4)} = \frac{12 \pm\sqrt{-256}}{8} = \frac{12 \pm 16i}{8} = \frac{3}{2} \pm 2i$

11. $3x^2 - 4x + 5 = 0$
    $\frac{-(-4) \pm\sqrt{(-4)^2 - 4(3)(5)}}{2(3)} = \frac{4 \pm\sqrt{-44}}{6} = \frac{4 \pm 2i\sqrt{11}}{6} = \frac{2}{3} \pm \frac{i\sqrt{11}}{3}$

12. $-3x^2 + 2x - 2 = 0$
    $\frac{-(2) \pm\sqrt{(2)^2 - 4(-3)(-2)}}{2(-3)} = \frac{-2 \pm\sqrt{-20}}{-6} = \frac{-2 \pm 2i\sqrt{5}}{-6} = \frac{1}{3} \pm \frac{i\sqrt{5}}{3}$

13. $-2 - 3i$ (its conjugate)

14. $(a + bi)(a - bi)$
    $= a^2 - abi + abi - (bi)^2$
    $= a^2 + b^2$

15. The roots are $1 - 2i\sqrt{2}$ and $1 + 2i\sqrt{2}$.
    $\left(x - \left(1 - 2i\sqrt{2}\right)\right)\left(x - \left(1 + 2i\sqrt{2}\right)\right) = 0$
    $\left((x - 1) + 2i\sqrt{2}\right)\left((x - 1) - 2i\sqrt{2}\right) = 0$
    $(x - 1)^2 - \left(2i\sqrt{2}\right)^2 = 0$
    $(x^2 - 2x + 1) - (-8) = 0$
    $x^2 - 2x + 9 = 0$
    $f(x) = x^2 - 2x + 9$

Chapter 5. Circles

5.1 Equations of Circles

1. Center is $(5, -2)$; radius is $\sqrt{36} = 6$.

2. Divide the equation by 3:
   $(x + 1)^2 + y^2 = 9$
   Center is $(-1, 0)$ and radius is $\sqrt{9} = 3$.

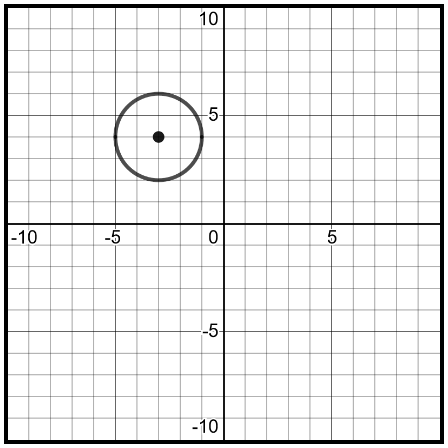
3. $(x + 8)^2 + (y - 6)^2 = 25$

4. Circle B has its center at $(0, -5)$, so the center of circle A is $(2, -2)$.

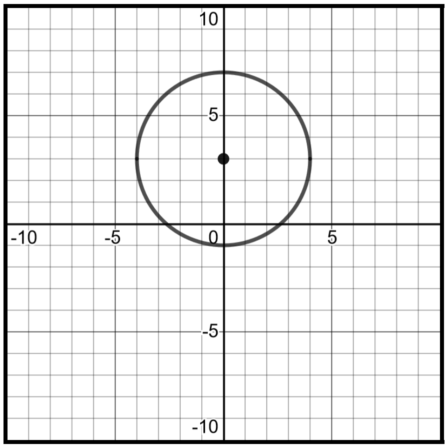
5. $x^2 - 6x + y^2 + 14y + 42 = 0$
   $x^2 - 6x + y^2 + 14y = -42$
   $(x^2 - 6x + 9) + (y^2 + 14y + 49) =$
   $-42 + 9 + 49$
   $(x - 3)^2 + (y + 7)^2 = 16$
   Center is $(3, -7)$, radius is $\sqrt{16} = 4$.

6. Divide by 2:
   $x^2 + 2x + y^2 - 10y - 23 = 0$
   $x^2 + 2x + y^2 - 10y = 23$
   $(x^2 + 2x + 1) + (y^2 - 10y + 25) =$
   $23 + 1 + 25$
   $(x + 1)^2 + (y - 5)^2 = 49$
   Center is $(-1,5)$, radius is $\sqrt{49} = 7$.

7.



8.



5.2 Circle-Linear Systems

1. $x^2 + (-x)^2 = 36$
   $x^2 + x^2 = 36$
   $2x^2 = 36$
   $x^2 = 18$
   $x = \pm\sqrt{18} = \pm 3\sqrt{2}$
   For $x = 3\sqrt{2}, y = -3\sqrt{2}$
   For $x = -3\sqrt{2}, y = 3\sqrt{2}$
   $\left(3\sqrt{2}, -3\sqrt{2}\right)$ and $\left(-3\sqrt{2}, 3\sqrt{2}\right)$

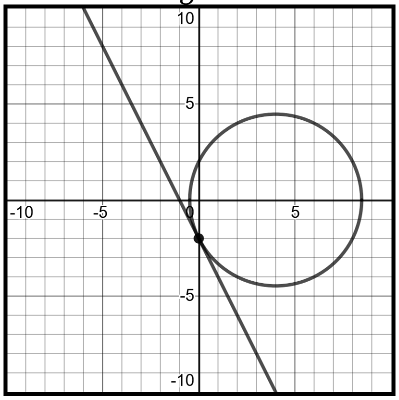
2. $(x - 1)^2 + (3x)^2 = 9$
   $x^2 - 2x + 1 + 9x^2 = 9$
   $10x^2 - 2x - 8 = 0$
   $5x^2 - x - 4 = 0$
   $$x = \frac{1 \pm \sqrt{(-1)^2 - 4(5)(-4)}}{2(5)} = \frac{1 \pm 9}{10}$$
   $x = \{1, -0.8\}$
   $y = 3(1) = 3$
   $y = 3(-0.8) = -2.4$
   $(1,3)$ and $(-0.8, -2.4)$

3. $x^2 + (-x + 2)^2 = 16$
$x^2 + x^2 - 4x + 4 = 16$
$2x^2 - 4x = 12$
$x^2 - 2x = 6$
$x^2 - 2x + 1 = 6 + 1$
$(x - 1)^2 = 7$
$x - 1 = \pm\sqrt{7}$
$x = 1 \pm \sqrt{7}$
$y = -\left(1 + \sqrt{7}\right) + 2 = 1 - \sqrt{7}$
$y = -\left(1 - \sqrt{7}\right) + 2 = 1 + \sqrt{7}$
$\left(1 + \sqrt{7},\ 1 - \sqrt{7}\right)$ and
$\left(1 - \sqrt{7},\ 1 + \sqrt{7}\right)$

4. $(x + 2)^2 + (2x - 5 - 1)^2 = 25$
$(x + 2)^2 + (2x - 6)^2 = 25$
$x^2 + 4x + 4 + 4x^2 - 24x + 36 = 25$
$5x^2 - 20x + 15 = 0$
$x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$
$x = \{3,1\}$
$y = 2(3) - 5 = 1$
$y = 2(1) - 5 = -3$
$(3, 1)$ and $(1, -3)$

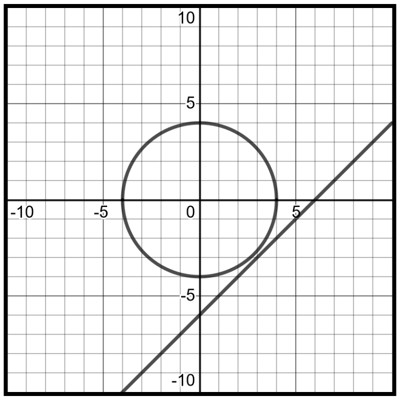
5. $(x - 2)^2 + \left(\frac{1}{2}x - 4\right)^2 = 9$
$x^2 - 4x + 4 + \frac{1}{4}x^2 - 4x + 16 = 9$
$\frac{5}{4}x^2 - 8x + 11 = 0$
$5x^2 - 32x + 44 = 0$ *multiply by 4*
$n^2 - 32n + 220 = 0$
$(n - 22)(n - 10) = 0$
$n = 22$ or $n = 10$
$x = \frac{22}{5} = 4.4$ or $x = \frac{10}{5} = 2$
$y = \frac{1}{2}(4.4) = 2.2$
$y = \frac{1}{2}(2) = 1$
$(4.4, 2.2)$ and $(2, 1)$

6. $-3y - 6 = 6x$
$y + 2 = -2x$ *divide by –3*
$y = -2x - 2$
$(x - 4)^2 + (-2x - 2)^2 = 20$
$x^2 - 8x + 16 + 4x^2 + 8x + 4 = 20$
$5x^2 = 0$
$x = 0$
$y = -2(0) - 2 = -2$
$(0, -2)$ *one point of intersection: the line is tangent to the circle*



7. $x^2 + (x - 6)^2 = 16$
$x^2 + x^2 - 12x + 36 = 16$
$2x^2 - 12x + 20 = 0$
$x^2 - 6x + 10 = 0$
$x^2 - 6x = -10$
$x^2 - 6x + 9 = -10 + 9$
$(x - 3)^2 = -1$
$x - 3 = \pm\sqrt{-1}$
$x = 3 \pm i$

No real solutions, so the line and circle do not intersect.



Chapter 6. Polynomials

6.1 Operations with Functions

1. a) $s(x) = 13x - 4$ b) $p(x) = 36x^2 - 11x - 5$	2. a) $d(x) = 3x^2 + 9x$ b) $q(x) = x + 4$
3. a) $2x + 6\sqrt{3}$ b) $10x\sqrt{3} + 15$ c) $\frac{2x+\sqrt{3}}{5\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2x\sqrt{3}+3}{15}$	4. a) $h(x) = 2(x^2 - 1) - 4(2x + 4) + 8$ $h(x) = 2x^2 - 2 - 8x - 16 + 8$ $h(x) = 2x^2 - 8x - 10$ b) $2x^2 - 8x - 10 = 0$ $x^2 - 4x - 5 = 0$ $(x + 1)(x - 5) = 0$ $\{-1, 5\}$
5. a) $P(x) = 6x - 170$ b) $6x - 170 > 0$ $6x > 170$ $x > 28.33$ Need to sell 29 headphones.	

6.2 Long Division

1. $5x-2$ $x+1\overline{)\;5x^2+3x-2}$ $\underline{-\left(5x^2+5x\right)}$ $-2x-2$ $\underline{-(-2x-2)}$ 0	2. $x-2$ $3x+1\overline{)\;3x^2-5x+1}$ $\underline{-\left(3x^2+x\right)}$ $-6x+1$ $\underline{-(-6x-2)}$ 3 Answer: $x - 2 + \frac{3}{3x+1}$

3.

$$\begin{array}{r} x+2 \\ x^2+x-6\overline{\big)\;x^3+3x^2-4x-12} \\ \underline{-\left(x^3+x^2-6x\right)} \\ 2x^2+2x-12 \\ \underline{-\left(2x^2+2x-12\right)} \\ 0 \end{array}$$

4.

$$\begin{array}{r} 2x+3 \\ x^2-4\overline{\big)\;2x^3+3x^2-8x-12} \\ \underline{-\left(2x^3\qquad -8x\right)} \\ 3x^2\qquad -12 \\ \underline{-\left(3x^2\qquad -12\right)} \\ 0 \end{array}$$

5.

$$\begin{array}{r} x^2+2x-2 \\ 2x+3\overline{\big)\;2x^3+7x^2+2x+9} \\ \underline{-\left(2x^3+3x^2\right)} \\ 4x^2+2x \\ \underline{-\left(4x^2+6x\right)} \\ -4x+9 \\ \underline{-(-4x-6)} \\ 15 \end{array}$$

$$x^2+2x-2+\frac{15}{2x+3}$$

6.

$$\begin{array}{r} x^2+x+3 \\ x-3\overline{\big)\;x^3-2x^2+0x-4} \\ \underline{-\left(x^3-3x^2\right)} \\ x^2+0x \\ \underline{-\left(x^2-3x\right)} \\ 3x-4 \\ \underline{-(3x-9)} \\ 5 \end{array}$$

$$x^2+x+3+\frac{5}{x-3}$$

7.

$$\begin{array}{r} x^3+2x^2+3 \\ x^3+2\overline{\big)\,x^6+2x^5+5x^3+4x^2+6} \\ \underline{-\left(x^6\qquad +2x^3\right)} \\ 2x^5+3x^3+4x^2 \\ \underline{-\left(2x^5\qquad +4x^2\right)} \\ 3x^3\qquad +6 \\ \underline{-\left(3x^3\qquad +6\right)} \\ 0 \end{array}$$

8.

$$\begin{array}{r} 2x+3 \\ x^2-4x+1\overline{\big)\;2x^3-5x^2+x-10} \\ \underline{-\left(2x^3-8x^2+2x\right)} \\ 3x^2-x-10 \\ \underline{-\left(3x^2-12x+3\right)} \\ 11x-13 \end{array}$$

$$2x+3+\frac{11x-13}{x^2-4x+1}$$

6.3 Synthetic Division

1. $\begin{array}{lrrrr} \boxed{2} & 3 & -4 & -7 & 6 \\ & & 6 & 4 & -6 \\ \hline & 3 & 2 & -3 & \mid\ 0 \end{array}$ Ans: $3x^2 + 2x - 3$	2. $\begin{array}{lrrrr} \boxed{4} & 2 & -5 & -11 & -4 \\ & & 8 & 12 & 4 \\ \hline & 2 & 3 & 1 & \mid\ 0 \end{array}$ Ans: $2x^2 + 3x + 1$
3. $\begin{array}{lrrrr} \boxed{1} & 3 & 1 & -6 & 2 \\ & & 3 & 4 & -2 \\ \hline & 3 & 4 & -2 & \mid\ 0 \end{array}$ Ans: $3x^2 + 4x - 2$	4. $\begin{array}{lrrrrr} \boxed{-2} & 3 & 7 & -1 & -5 & 5 \\ & & -6 & -2 & 6 & -2 \\ \hline & 3 & 1 & -3 & 1 & \mid\ 3 \end{array}$ Ans: $3x^3 + x^2 - 3x + 1 + \dfrac{3}{x+2}$
5. $\begin{array}{lrrrrr} \boxed{2} & 1 & 0 & -4 & -4 & 8 \\ & & 2 & 4 & 0 & -8 \\ \hline & 1 & 2 & 0 & -4 & \mid\ 0 \end{array}$ Ans: $x^3 + 2x^2 - 4$	6. $\begin{array}{lrrrrr} \boxed{-3} & 1 & 0 & -11 & -1 & 7 \\ & & -3 & 9 & 6 & -15 \\ \hline & 1 & -3 & -2 & 5 & \mid\ -8 \end{array}$ Ans: $x^3 - 3x^2 - 2x + 5 - \dfrac{8}{x+3}$

6.4 Remainder Theorem

1. $f(-2) = (-2)^3 + 2(-2)^2 + (-2) + 6$ $f(-2) = 4$, so the remainder is 4. This can be checked by synthetic division: $\begin{array}{lrrrr} \boxed{-2} & 1 & 2 & 1 & 6 \\ & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & \mid\ 4 \end{array}$	2. Substitute each a to see if $P(a) = 0$. $P(1) = 1^4 - 2(1^3) - 7(1^2) + 8(1) + 12$ $P(1) = 12$, so the correct answer is (3)
3. Enter the equation into the calculator as Y_1 and evaluate $Y_1(x)$ for each integer x between -3 and 3. $P(-3) = 0, P(-2) = 0, P(-1) = -16,$ $P(0) = 0, P(1) = 0, P(2) = -40$, and $P(3) = 0$ Therefore, the roots are $\{-3, -2, 0, 1, 3\}$	4. $\dfrac{a(x)}{x-2} = 3x + 13 + \dfrac{6}{x-2}$ Multiply both sides by $x - 2$. $a(x) = 3x(x-2) + 13(x-2) + 6$ $a(x) = 3x^2 - 6x + 13x - 26 + 6$ $a(x) = 3x^2 + 7x - 20$

5.

a) $(-4)^3 + 3(-4)^2 + (-4)k - 24 = 0$
$-64 + 48 - 4k - 24 = 0$
$-40 - 4k = 0$
$k = -10$

b) Find the other factor by division:

$$\begin{array}{r|rrrcr} \boxed{-4} & 1 & 3 & -10 & & -24 \\ & & -4 & 4 & & 24 \\ \hline & 1 & -1 & -6 & | & 0 \end{array}$$

$(x^2 - x - 6)(x + 4) = 0$
$(x - 3)(x + 2)(x + 4) = 0$
$\{-4, -2, 3\}$

6.5 Factor Polynomials

1. $x^2(x + 3) + 2(x + 3)$ $(x^2 + 2)(x + 3)$	2. $2x^2(2x + 5) - 5(2x + 5)$ $(2x^2 - 5)(2x + 5)$
3. $x^2(x + 3) - 4(x + 3)$ $(x^2 - 4)(x + 3)$ $(x + 2)(x - 2)(x + 3)$	4. $x^2(x - 2) - 9(x - 2)$ $(x^2 - 9)(x - 2)$ $(x + 3)(x - 3)(x - 2)$
5. $x^2(x + 2) - (x + 2)$ $(x^2 - 1)(x + 2)$ $(x + 1)(x - 1)(x + 2)$	6. $x^2(3x - 5) - 16(3x - 5)$ $(x^2 - 16)(3x - 5)$ $(x + 4)(x - 4)(3x - 5)$
7. $a(a + b) + c(a + b)$ $(a + c)(a + b)$	8. $a^3 + a^2 - ab - b$ $a^2(a + 1) - b(a + 1)$ $(a^2 - b)(a + 1)$
9. $x^4 - x^2 - 9x^2 + 9$ $x^2(x^2 - 1) - 9(x^2 - 1)$ $(x^2 - 9)(x^2 - 1)$ $(x + 3)(x - 3)(x + 1)(x - 1)$	10. $8y^4 - 10y^2 - 28y^2 + 35$ $2y^2(4y^2 - 5) - 7(4y^2 - 5)$ $(2y^2 - 7)(4y^2 - 5)$
11. $a^4 - a^2b^2 - 4a^2b^2 + 4b^4$ $a^2(a^2 - b^2) - 4b^2(a^2 - b^2)$ $(a^2 - 4b^2)(a^2 - b^2)$ $(a + 2b)(a - 2b)(a + b)(a - b)$	12. $2x^4 - 2x^2y^2 + x^2y^2 - y^4$ $2x^2(x^2 - y^2) + y^2(x^2 - y^2)$ $(2x^2 + y^2)(x^2 - y^2)$ $(2x^2 + y^2)(x + y)(x - y)$
13. $x^4(2x - 1) + 5x^2(2x - 1) + 4(2x - 1)$ $(x^4 + 5x^2 + 4)(2x - 1)$ $(x^4 + x^2 + 4x^2 + 4)(2x - 1)$ $[x^2(x^2 + 1) + 4(x^2 + 1)](2x - 1)$ $(x^2 + 4)(x^2 + 1)(2x - 1)$	
14. $(x + 5)(x^2 - 5x + 25)$	15. $(b + 4)(b^2 - 4b + 16)$
16. $(y - 6)(y^2 + 6y + 36)$	17. $(3x - 2)(9x^2 + 6x + 4)$
18. $2(8x^3 + 27) =$ $2(2x + 3)(4x^2 - 6x + 9)$	19. $(xy^2 - 4)(x^2y^4 + 4xy^2 + 16)$
20. $(8x - 7y)(64x^2 + 56xy + 49y^2)$	21. $2(x^3 + 64y^3) =$ $2(x + 4y)(x^2 - 4xy + 16y^2)$

22. Let $u = 2x^2$ $2u^2 - 3u - 2$ $(2u + 1)(u - 2)$ $(4x^2 + 1)(2x^2 - 2)$ $2(4x^2 + 1)(x^2 - 1)$ $2(4x^2 + 1)(x + 1)(x - 1)$	23. Let $u = x - 6$ $x^4 - u^2$ $(x^2 + u)(x^2 - u)$ $(x^2 + x - 6)(x^2 - x + 6)$ $(x + 3)(x - 2)(x^2 - x + 6)$
24. We can factor out the GCF of 2 from the middle terms: $(x^5 + 2x)^2 + 2(x^5 + 2x) + 1$ Now let $u = x^5 + 2x$ $u^2 + 2u + 1$ $(u + 1)(u + 1)$ $(x^5 + 2x + 1)(x^5 + 2x + 1)$	

6.6 Polynomial Identities [CC]

1.	$(x + a)(x + b)$	
	$= x^2 + ax + bx + ab$	multiply the binomials
	$= x^2 + (a + b)x + ab$	distributive property
2.	$(x^2 - 1)^2 + (2x)^2$	
	$= x^4 - 2x^2 + 1 + (2x)^2$	expand the square of the binomial
	$= x^4 - 2x^2 + 1 + 4x^2$	Power Rule
	$= x^4 + 2x^2 + 1$	combine like terms
	$= (x^2 + 1)^2$	rewrite as the square of a binomial
3.	$(a + b)^3$	
	$= (a + b)(a + b)(a + b)$	rewrite the cube as a product
	$= (a^2 + 2ab + b^2)(a + b)$	multiply the first two binomials
	$= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3$	multiply the trinomial and binomial
	$= a^3 + 3a^2b + 3ab^2 + b^3$	combine like terms
4.	$(a + b)^2 + (a - b)^2$	
	$= (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2)$	expand both squares of binomials
	$= 2a^2 + 2b^2$	combine like terms
	$= 2(a^2 + b^2)$	distributive property
5.	$(a + b)(a - b)[(a + b)^2 - 2ab]$	
	$= (a + b)(a - b)(a^2 + 2ab + b^2 - 2ab)$	expand the square of the binomial
	$= (a + b)(a - b)(a^2 + b^2)$	combine like terms
	$= (a^2 - b^2)(a^2 + b^2)$	express as a difference of two squares
	$= a^4 - b^4$	express as a difference of two squares
6.	$\left(x^2 + y^2 + \sqrt{2}xy\right)\left(x^2 + y^2 - \sqrt{2}xy\right)$	
	$= x^4 + x^2y^2 - \sqrt{2}x^2y +$ $x^2y^2 + y^4 - \sqrt{2}xy^2 +$ $\sqrt{2}x^2y + \sqrt{2}xy^2 - 2x^2y^2$	multiply by distributing each term
	$= x^4 + y^4$	combine like terms

Chapter 7. Polynomial Functions

7.1 Find Roots by Factoring

1. $x(x^2+x-2)=0$ $x(x+2)(x-1)=0$ $\{-2,0,1\}$	2. $x^2(2x-1)-4(2x-1)=0$ $(x^2-4)(2x-1)=0$ $(x+2)(x-2)(2x-1)=0$ $\left\{\frac{1}{2},\pm 2\right\}$
3. $4x^2(2x+1)-9(2x+1)=0$ $(4x^2-9)(2x+1)=0$ $(2x+3)(2x-3)(2x+1)=0$ $\left\{\pm\frac{3}{2},-\frac{1}{2}\right\}$	4. $x^3+5x^2-4x-20=0$ $x^2(x+5)-4(x+5)=0$ $(x^2-4)(x+5)=0$ $\{\pm 2,-5\}$
5. $(x^2+1)(x^2-4)=0$ $x^2+1=0$ or $x^2-4=0$ $\{\pm 2,\pm i\}$	6. $3x(x^4-16)=0$ $3x(x^2+4)(x^2-4)=0$ $3x(x^2+4)(x+2)(x-2)=0$ $\{0,\pm 2,\pm 2i\}$
7. $x(x^3+4x^2+4x+16)=0$ $x\big(x^2(x+4)+4(x+4)\big)=0$ $x(x^2+4)(x+4)=0$ $\{0,-4,\pm 2i\}$	8. $9x^2(x-10)+64(x-10)=0$ $(9x^2+64)(x-10)=0$ For the first factor, $x^2=-\frac{64}{9}$, so $x=\pm\frac{8}{3}i$ $\left\{10,\pm\frac{8}{3}i\right\}$
9. $(x+1)(x-2)(3x+1)(3x-2)=$ $(x^2-x-2)(3x+1)(3x-2)=$ $(3x^3-2x^2-7x-2)(3x-2)=$ $9x^4-12x^3-17x^2+8x+4$ $f(x)=9x^4-12x^3-17x^2+8x+4$	10. $x=-1+2i \quad\rightarrow\quad x+1-2i=0$ $x=-1-2i \quad\rightarrow\quad x+1+2i=0$ $x=-2 \quad\rightarrow\quad x+2=0$ $(x+1-2i)(x+1+2i)(x+2)=$ $[(x+1)^2-(2i)^2](x+2)=$ $(x^2+2x+5)(x+2)=$ x^3+x-10 To get a leading coefficient of 4, vertically dilate the function by multiplying by 4: $g(x)=4x^3+4x-40$

7.2 Root Theorems

1. Possible roots are
$\pm1, \pm2, \pm4, \pm5, \pm10, \pm20.$
$(-1)^3 - 8(-1)^2 + 11(-1) + 20 = 0$
$f(-1) = 0$, so -1 is a root and $(x+1)$ is a factor. By synthetic division,

$$\begin{array}{r|rrrrl} \boxed{-1} & 1 & -8 & 11 & 20 & \\ & & -1 & 9 & -20 & \\ \hline & 1 & -9 & 20 & \mid & 0 \end{array}$$

$(x+1)(x^2 - 9x + 20)$
$(x+1)(x-4)(x-5)$
Roots are -1, 4, and 5.

2. Possible roots are $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{3}, \pm\frac{2}{3}.$
$3(1)^3 - 10(1)^2 + 1 + 6 = 0$
$g(1) = 0$, so 1 is a root and $(x-1)$ is a factor. By synthetic division,

$$\begin{array}{r|rrrrl} \boxed{1} & 3 & -10 & 1 & 6 & \\ & & 3 & -7 & -6 & \\ \hline & 3 & -7 & -6 & \mid & 0 \end{array}$$

$(x-1)(3x^2 - 7x - 6)$
$(x-1)(3x^2 - 9x + 2x - 6)$
$(x-1)(x-3)(3x+2)$ [by grouping]
Roots are 1, 3, and $-\frac{2}{3}$.

3. Possible roots are
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24, \pm\frac{1}{2}, \pm\frac{3}{2}.$
$2(-2)^3 - (-2)^2 - 22(-2) - 24 = 0$
$h(-2) = 0$, so -2 is a root and $(x+2)$ is a factor. By synthetic division,

$$\begin{array}{r|rrrrl} \boxed{-2} & 2 & -1 & -22 & -24 & \\ & & -4 & 10 & 24 & \\ \hline & 2 & -5 & -12 & \mid & 0 \end{array}$$

$(x+2)(2x^2 - 5x - 12)$
$(x+2)(2x^2 - 8x + 3x - 12)$
$(x+2)(2x+3)(x-4)$ [by grouping]
Roots are -2, $-\frac{3}{2}$, and 4.

4. Possible roots are $\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}.$
$2(-1)^4 + (-1)^3 - 19(-1)^2 - 9(-1) + 9 = 0$
$f(-1) = 0$, so -1 is a root and $(x+1)$ is a factor. By synthetic division,

$$\begin{array}{r|rrrrrl} \boxed{-1} & 2 & 1 & -19 & -9 & 9 & \\ & & -2 & 1 & 18 & -9 & \\ \hline & 2 & -1 & -18 & 9 & \mid & 0 \end{array}$$

$(x+1)(2x^3 - x^2 - 18x + 9)$
$(x+1)(x^2 - 9)(2x - 1)$ [by grouping]
$(x+1)(x+3)(x-3)(2x-1)$
Roots are $-1, -3, 3$, and $\frac{1}{2}$.

5. $P(x)$ is cubic, so there are 3 roots.
$P(x)$ has two sign changes, so there are at most 2 positive real roots.
$P(-x) = -2x^3 + 3x^2 + 10x + 1$ has one sign change, so there is 1 negative real root.

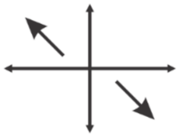
Positive Real Roots	Negative Real Roots	Imaginary Roots	Total Roots
2	1	0	3
0	1	2	3

6. $h(x)$ is a fifth-degree (quintic) polynomial, so there are 5 roots.
$h(x)$ has 3 sign changes, so there are at most 3 positive real roots.
$h(-x) = x^5 + x^4 + x^2 + x + 1$ has no sign changes, so 0 negative real roots.

Positive Real Roots	Negative Real Roots	Imaginary Roots	Total Roots
3	0	2	5
1	0	4	5

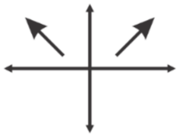
7.3 Properties of Polynomial Graphs

1. Degree $= 5$, leading coefficient $= -3$.



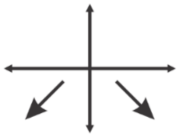
As $x \to -\infty$, $f(x) \to \infty$,
and as $x \to \infty$, $f(x) \to -\infty$

2. Degree $= 4$, leading coefficient $= 5$.



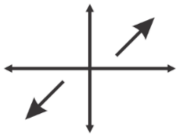
As $x \to -\infty$, $f(x) \to \infty$,
and as $x \to \infty$, $f(x) \to \infty$

3. Degree $= 4$, leading coefficient $= -16$.



As $x \to -\infty$, $f(x) \to -\infty$,
and as $x \to \infty$, $f(x) \to -\infty$

4. Degree $= 7$, leading coefficient $= 1$.



As $x \to -\infty$, $f(x) \to -\infty$,
and as $x \to \infty$, $f(x) \to \infty$

5. $h(0) = -4(-2)(-5)^2 = 200$,
so the y-intercept is $(0, 200)$.
$x^2 - 2 = 0 \quad \to \quad x = \pm\sqrt{2}$
$2x - 5 = 0 \quad \to \quad x = \frac{5}{2}$ *[double root]*
so the x-intercepts are $(-\sqrt{2}, 0)$, $(\sqrt{2}, 0)$, and $\left(\frac{5}{2}, 0\right)$.

6. $k(0) = (0)(1)(4)^2 = 0$,
so the y-intercept is $(0, 0)$.
$5x = 0 \quad \to \quad x = 0$
$x^2 + 1 = 0 \to$ imaginary roots $\pm i$
$x + 4 = 0 \quad \to \quad x = -4$ *[double root]*
so the x-intercepts are $(-4, 0)$ and $(0, 0)$.

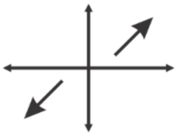
7. $(-2.31, 0)$, $(-0.76, 0)$, and $(0.57, 0)$

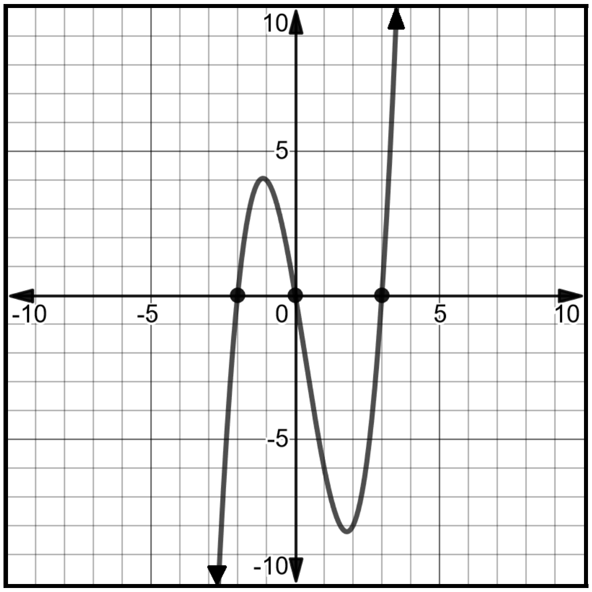
8. relative maximum at $(3, 5)$,
relative minimum at $(5, 1)$.
decreasing over $3 < x < 5$
increasing over $x < 3$ and $x > 5$

9. relative maximum at $(-2.5, 1.25)$,
relative minima at $(-3, 0)$ and $(-2, 0)$.
decreasing over $x < -3$ and $-2.5 < x < -2$
increasing over $3 < x < -2.5$ and $x > -2$

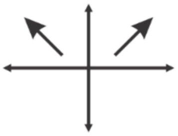
7.4 **Graph Polynomial Functions**

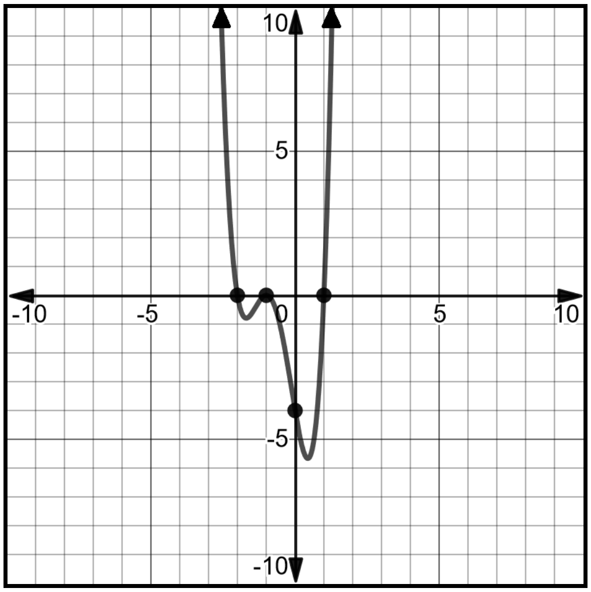
1. x-intercepts at -2, 0, and 3.
y-intercept at $f(0) = 0$.
Degree of 3 (odd), leading coefficient of 1 (positive), so end behavior of





2. x-intercepts at -2, -1, and 1, with -1 as a double root.
y-intercept at $f(0) = -4$.
Degree of 4 (even), leading coefficient of 2 (positive), so end behavior of

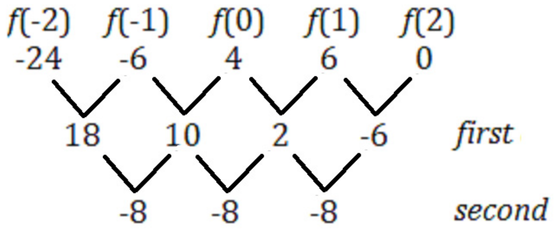




3. $c(x) = x^3 - 13x + 12$

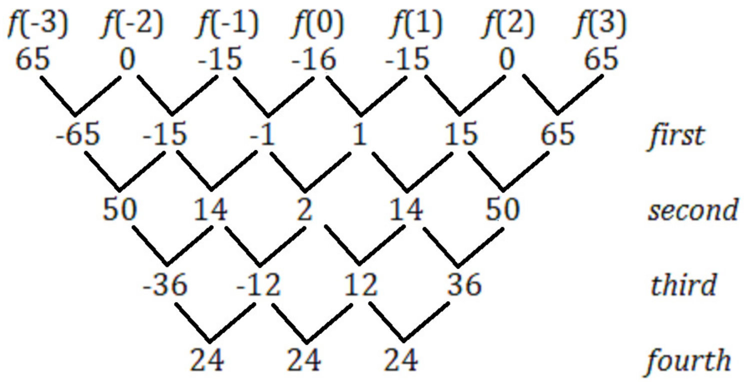
4. $q(x) = 2x^4 + 3x^3 - 3x^2 - 2x$

5.



Second degree (quadratic) function
$f(x) = -4x^2 + 6x + 4$

6.



Fourth degree (quartic) function
$g(x) = x^4 - 16$

7.5 Polynomial Transformations

1. (3)

2. (2)

3. (1)

4. The graph shifts 2 units to the right and 3 units up.

5. $g(x) = f(x-2) =$
$(x-2)(x+2)(x-5)$

6. Start with $y = 2x^4 - 3x^3 + x - 5$.
For a vertical dilation by a factor of 3, multiply the equation by 3, giving us
$y = 6x^4 - 9x^3 + 3x - 15$.
To reflect over the y-axis, replace each x with $-x$, giving us
$y = 6(-x)^4 - 9(-x)^3 + 3(-x) - 15$, or $y = 6x^4 + 9x^3 - 3x - 15$.
Finally, translate 6 unit up by adding 6:
$n(x) = 6x^4 + 9x^3 - 3x - 9$

7.6 Systems of Polynomial Functions

1. $x^2+2x-1=3x+5$
$x^2-x-6=0$
$(x+2)(x-3)=0$
$x=\{-2,3\}$
$y=3(-2)+5=-1$
$y=3(3)+5=14$
$(-2,-1)$ and $(3,14)$

2. $y+3x=1 \rightarrow \quad y=-3x+1$
$x^2+7x+22=-3x+1$
$x^2+10x+21=0$
$(x+7)(x+3)=0$
$x=\{-7,-3\}$
$y=-3(-7)+1=22$
$y=-3(-3)+1=10$
$(-7,22)$ and $(-3,10)$

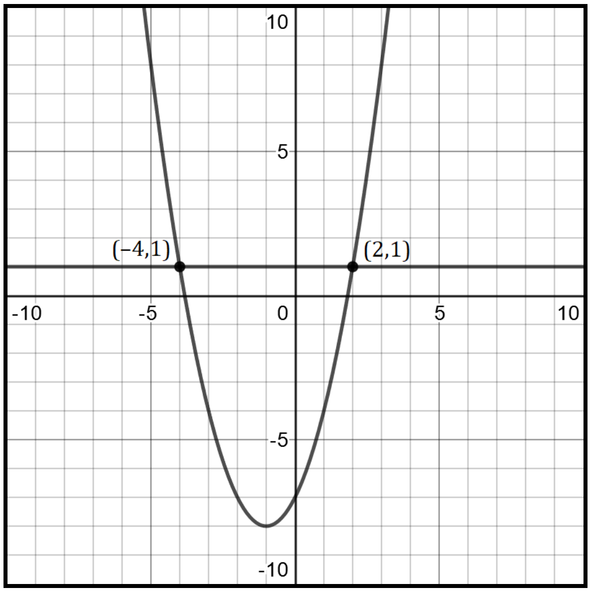
3. $x^2+2x=y+7 \rightarrow y=x^2+2x-7$
$x^2+2x-7=2x+1$
$x^2=8$
$x=\pm\sqrt{8}=\pm2\sqrt{2}$
$y=2\left(2\sqrt{2}\right)+1=1+4\sqrt{2}$
$y=2\left(-2\sqrt{2}\right)+1=1-4\sqrt{2}$
$\left(2\sqrt{2},1+4\sqrt{2}\right)$ and $\left(-2\sqrt{2},1-4\sqrt{2}\right)$

4. $x^3-6x+1=-2x+1$
$x^3-4x=0$
$x(x^2-4)=0$
$x(x+2)(x-2)=0$
$x=\{0,-2,2\}$
$y=-2(0)+1=1$
$y=-2(-2)+1=5$
$y=-2(2)+1=-3$
$(0,1),(-2,5),(2,-3)$

5. $y+2=3x \rightarrow y=3x-2$
$x^2-3x+9=3x-2$
$x^2-6x+11=0$
$x^2-6x=-11$
$x^2-6x+9=-11+9$
$(x-3)^2=-2$
$x-3=\pm\sqrt{-2}$
$x=3\pm i\sqrt{2}$
$y=3\left(3+i\sqrt{2}\right)-2$
$y=7+3i\sqrt{2}$
$y=3\left(3-i\sqrt{2}\right)-2$
$y=7-3i\sqrt{2}$
$\left(3+i\sqrt{2},7+3i\sqrt{2}\right)$ and
$\left(3-i\sqrt{2},7-3i\sqrt{2}\right)$

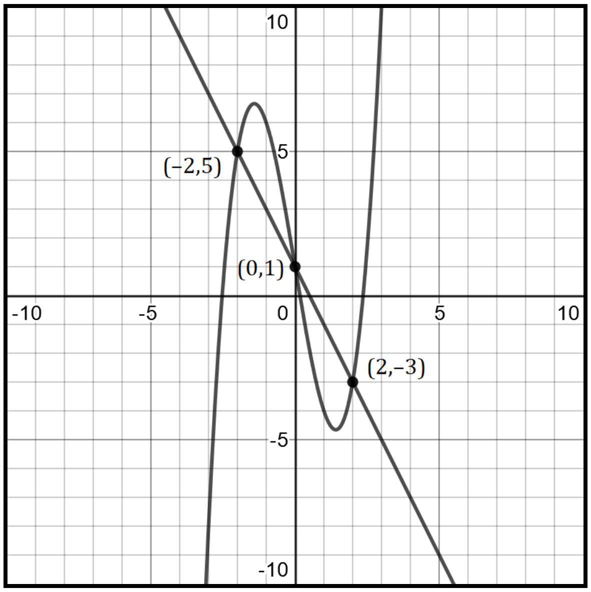
6. $x^3+5x^2+2=7x^2-5x+2$
$x^3-2x^2+5x=0$
$x(x^2-2x+5)=0$
$x=0 \qquad x^2-2x+5=0$
$x^2-2x=-5$
$x^2-2x+1=-5+1$
$(x-1)^2=-4$
$x-1=\pm\sqrt{-4}$
$x=1\pm2i$
$y=7(0)^2-5(0)+2=2$
$y=7(1+2i)^2-5(1+2i)+2=$
$7(1+4i-4)-5-10i+2=$
$-21+28i-3-10i=-24+18i$
$y=7(1-2i)^2-5(1-2i)+2=$
$7(1-4i-4)-5+10i+2=$
$-21-28i-3+10i=-24-18i$
$(0,2),(1+2i,-24+18i)$, and
$(1-2i,-24-18i)$

7.



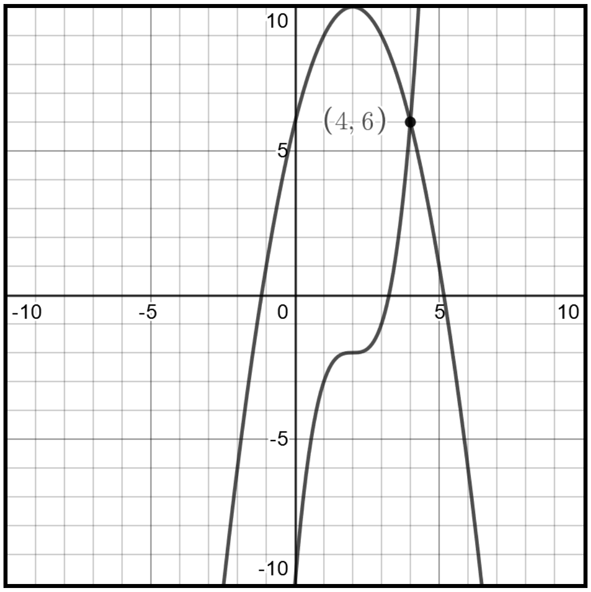
$(-4,1), (2,1)$

8.



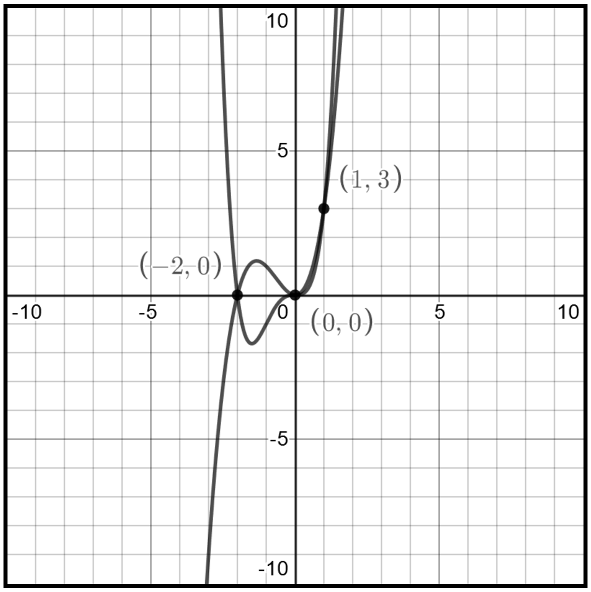
$(-2,5), (0,1), (2,-3)$

9.



$(4,6)$

10.



$(-2,0), (0,0), (1,3)$

Chapter 8. Radicals and Rational Exponents

8.1 *n*th Roots

1. 11	2. 6
3. 2.91	4. $\sqrt[4]{2\cdot\boxed{3\cdot3\cdot3\cdot3}} = 3\sqrt[4]{2}$
5. $x^2 = 25$ $\sqrt{x^2} = \pm\sqrt{25}$ $x = \pm5$	6. $\sqrt[5]{x^5} = \sqrt[5]{243}$ $x = 3$
7. $\sqrt[6]{x^6} = \pm\sqrt[6]{46{,}656}$ $x = \pm6$	8. $\sqrt[3]{x^3} = \sqrt[3]{515}$ $x \approx 8.02$
9. $4y^2$	10. $x^4y^5\sqrt[3]{y}$
11. $ab^2c\sqrt[5]{4a^3b^4}$	12. $\lvert x^3\rvert y^4$

8.2 Operations with Radicals

1. $9\sqrt[4]{2}$	2. $10\sqrt[3]{10}$
3. $\sqrt[3]{64} = 4$	4. $(\sqrt[4]{8})(3\sqrt[4]{6}) - (2\sqrt[4]{3}) = 3\sqrt[4]{48} - 2\sqrt[4]{3}$ $= 6\sqrt[4]{3} - 2\sqrt[4]{3} = 4\sqrt[4]{3}$
5. $\dfrac{3}{\sqrt[3]{9}}\cdot\dfrac{\left(\sqrt[3]{9}\right)^2}{\left(\sqrt[3]{9}\right)^2} = \dfrac{3\sqrt[3]{81}}{9} = \dfrac{\cancel{9}\sqrt[3]{3}}{\cancel{9}} = \sqrt[3]{3}$	6. $\dfrac{6\sqrt[4]{8}}{\sqrt[4]{16}} = \dfrac{6}{\sqrt[4]{2}}$ $\dfrac{6}{\sqrt[4]{2}}\cdot\dfrac{\left(\sqrt[4]{2}\right)^3}{\left(\sqrt[4]{2}\right)^3} = \dfrac{6\left(\sqrt[4]{2}\right)^3}{2} = 3\left(\sqrt[4]{2}\right)^3 = 3\sqrt[4]{8}$

8.3 Solve Equations with Radicals

1. $\left(\sqrt[3]{x}\right)^3 = 7^3$ $x = 343$	2. $3\sqrt[4]{x} = 18$ $\sqrt[4]{x} = 6$ $\left(\sqrt[4]{x}\right)^4 = 6^4$ $x = 1{,}296$
3. $x - 4 = 49$ $x = 53$	4. $\sqrt{2x-1} = 3$ $2x - 1 = 9$ $x = 5$

5. $\sqrt[3]{2x+3} = 3$ $2x + 3 = 27$ $2x = 24$ $x = 12$	6. $\left(\sqrt{x-a}\right)^2 = b^2$ $x - a = b^2$ $x = b^2 + a$
7. $x^2 - 3x + 3 = 1$ $x^2 - 3x + 2 = 0$ $(x-1)(x-2) = 0$ $\{1,2\}$	8. $x + 3 = (x+3)^2$ $x + 3 = x^2 + 6x + 9$ $x^2 + 5x + 6 = 0$ $(x+2)(x+3) = 0$ $\{-2,-3\}$
9. $2x - 4 = (x-2)^2$ $2x - 4 = x^2 - 4x + 4$ $x^2 - 6x + 8 = 0$ $(x-2)(x-4) = 0$ $\{2,4\}$	10. $x^2 = 4(2x-3)$ $x^2 - 8x + 12 = 0$ $(x-2)(x-6) = 0$ $\{2,6\}$
11. $9x + 10 = x^2$ $x^2 - 9x - 10 = 0$ $(x+1)(x-10) = 0$ $\{\cancel{-1}, 10\}$ -1 is extraneous	12. $5x + 29 = (x+3)^2$ $5x + 29 = x^2 + 6x + 9$ $x^2 + x - 20 = 0$ $(x+5)(x-4) = 0$ $\{\cancel{-5}, 4\}$ -5 is extraneous
13. $x^3 - 2x^2 - 5 = (x-1)^3$ $x^3 - 2x^2 - 5 = x^3 - 3x^2 + 3x - 1$ $x^2 - 3x - 4 = 0$ $(x+1)(x-4) = 0$ $\{-1, 4\}$	14. $15x^4 + 81 = (2x)^4$ $15x^4 + 81 = 16x^4$ $x^4 - 81 = 0$ $x^4 = 81$ $\{\cancel{-3}, 3\}$ -3 is extraneous
15. $\sqrt{x+4} = \sqrt{x-3} + 1$ $\left(\sqrt{x+4}\right)^2 = \left(\sqrt{x-3}+1\right)^2$ $x + 4 = x - 3 + 2\sqrt{x-3} + 1$ $2\sqrt{x-3} = 6$ $\sqrt{x-3} = 3$ $\left(\sqrt{x-3}\right)^2 = 3^2$ $x - 3 = 9$ $x = 12$	16. $\left(\sqrt{x+4}\right)^2 = \left(\sqrt{5x}-2\right)^2$ $x + 4 = 5x - 4\sqrt{5x} + 4$ $-4x = -4\sqrt{5x}$ $x = \sqrt{5x}$ $x^2 = 5x$ $x^2 - 5x = 0$ $x(x-5) - 0$ $\{\cancel{0}, 5\}$ 0 is extraneous
17. $\left(\sqrt{x+3}+1\right)^2 = \left(\sqrt{-2x}\right)^2$ $x + 3 + 2\sqrt{x+3} + 1 = -2x$ $2\sqrt{x+3} = -3x - 4$ $\left(2\sqrt{x+3}\right)^2 = (-3x-4)^2$ $4(x+3) = 9x^2 + 24x + 16$ $4x + 12 = 9x^2 + 24x + 16$ $9x^2 + 20x + 4 = 0$ $n^2 + 20n + 36 = 0$ $(n+18)(n+2) = 0$ $x = \frac{n}{9} = \left\{-2, \cancel{-\frac{2}{9}}\right\}$ $-\frac{2}{9}$ is extraneous	18. $\left(\sqrt{x+1} - \sqrt{x-4}\right)^2 = x - 7$ $x + 1 - 2\sqrt{x+1}\sqrt{x-4} + x - 4 = x - 7$ $2\sqrt{x+1}\sqrt{x-4} = x + 4$ $2\sqrt{x^2 - 3x - 4} = x + 4$ $4(x^2 - 3x - 4) = (x+4)^2$ $4x^2 - 12x - 16 = x^2 + 8x + 16$ $3x^2 - 20x - 32 = 0$ $n^2 - 20n - 96 = 0$ $(n-24)(n+4) = 0$ $x = \frac{n}{3} = \left\{8, \cancel{-\frac{4}{3}}\right\}$ $-\frac{4}{3}$ is extraneous

8.4 Graphs of Radical Functions

1. (1)

2. (2)

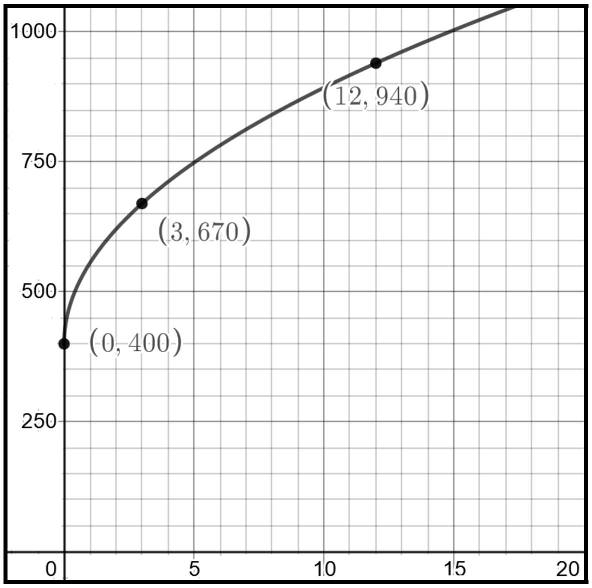
3. a) dilation by a factor of 2, translations 2 units left and 4 units down (in any order)
   b) $g(x) = 2\sqrt{x+2} - 4$

4.

(a)

x	y
0	400
3	670
6	781.8
9	867.7
12	940
15	1003.7

(b)



(c) 670
(d) 12

8.5 Negative Exponents

1. $\dfrac{1}{p^7}$

2. $\dfrac{1}{(5x)^2} = \dfrac{1}{25x^2}$

3. $1 + \dfrac{1}{3^2} = 1\frac{1}{9}$

4. $2^{-2} - 2^0 + 2^2 = \frac{1}{4} - 1 + 4 = 3\frac{1}{4}$

5. $-\dfrac{2m^3}{n^5}$

6. $5x^3$

7. $-\dfrac{3bc}{5}$

8. $\left(\dfrac{5y}{2x}\right)^3 = \dfrac{125y^3}{8x^3}$

9. $\left(\frac{3}{4}\right)^2 \cdot 4^2 = 9$

10. $\dfrac{a^6}{b^5}$

11. $\dfrac{3^{-2}}{(-2)^{-3}} = \dfrac{(-2)^3}{3^2} = -\dfrac{8}{9}$

12. $\dfrac{1}{5^{-2}a^3b^{-4}} = \dfrac{25b^4}{a^3}$

13. $\frac{y^6}{x^3}$	14. $\frac{x^4y^5}{3}$
15. $\frac{y^7}{2x^2}$	16. $\frac{3x^{-4}y^5}{(2x^3y^{-7})^{-2}} = \frac{3y^5(2x^3y^{-7})^2}{x^4} =$ $\frac{3y^5(4x^6y^{-14})}{x^4} = \frac{12x^6y^{-9}}{x^4} = \frac{12x^2}{y^9}$

8.6 Rational Exponents

1. $x^{\frac{4}{5}}$	2. $x^{\frac{7}{4}}$
3. $3x^{-\frac{1}{5}}$	4. $\sqrt{x^3}$
5. $\frac{1}{\sqrt[3]{x^2}}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\right) = \frac{\sqrt[3]{x}}{x}$	6. $\left(\frac{27}{64}\right)^{-\frac{2}{3}} = \left(\frac{64}{27}\right)^{\frac{2}{3}} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$
7. $\left(\frac{x^2}{x^{\frac{1}{9}}}\right)^3 = \left(\frac{x^{\frac{18}{9}}}{x^{\frac{1}{9}}}\right)^3 = \left(x^{\frac{17}{9}}\right)^3 = x^{\frac{17}{3}} =$ $\sqrt[3]{x^{17}}$	8. $\left(\frac{27x^4}{xy^{-\frac{2}{3}}}\right)^{\frac{1}{3}} = \left(27x^3y^{\frac{2}{3}}\right)^{\frac{1}{3}} = 3xy^{\frac{2}{9}} =$ $3x\sqrt[9]{y^2}$
9. $(9x^2y^6)^{-\frac{1}{2}} = \frac{1}{\sqrt{9x^2y^6}} = \frac{1}{3xy^3}$	10. $\left(x^{\frac{1}{2}}y^{-\frac{2}{3}}\right)^{-6} = x^{-3}y^4 = \frac{y^4}{x^3}$
11. $\left(\frac{x^{-5}}{x^{-9}}\right)^{\frac{1}{2}} = \left(\frac{x^9}{x^5}\right)^{\frac{1}{2}} = (x^4)^{\frac{1}{2}} = x^2$	12. $\frac{(m^6)^{-\frac{2}{3}}}{m^2} = \frac{1}{(m^6)^{\frac{2}{3}} \cdot m^2} = \frac{1}{m^4 \cdot m^2} = \frac{1}{m^6}$
13. $\frac{\sqrt[3]{x^2}}{\sqrt[6]{x}} = \frac{x^{\frac{2}{3}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{4}{6}}}{x^{\frac{1}{6}}} = x^{\frac{3}{6}} = x^{\frac{1}{2}} = \sqrt{x}$	14. $\frac{\sqrt{x^5}+x^3}{\sqrt[3]{x^7}} = \frac{x^{\frac{5}{2}}+x^3}{x^{\frac{7}{3}}} = \frac{x^{\frac{5}{2}}}{x^{\frac{7}{3}}} + \frac{x^3}{x^{\frac{7}{3}}} =$ $\frac{x^{\frac{15}{6}}}{x^{\frac{14}{6}}} + \frac{x^{\frac{9}{3}}}{x^{\frac{7}{3}}} = x^{\frac{1}{6}} + x^{\frac{2}{3}} = \sqrt[6]{x} + \sqrt[3]{x^2}$
15. $2x^{\frac{3}{4}} + 5 = 133$ $2x^{\frac{3}{4}} = 128$ $x^{\frac{3}{4}} = 64$ $\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = 64^{\frac{4}{3}}$ $x = 256$	16. $x^{\frac{7}{10}} \cdot \sqrt{x} = 729$ $x^{\frac{7}{10}} \cdot x^{\frac{1}{2}} = 729$ $x^{\left(\frac{7}{10}+\frac{5}{10}\right)} = 729$ $x^{\frac{6}{5}} = 729$ $\left(x^{\frac{6}{5}}\right)^{\frac{5}{6}} = \pm 729^{\frac{5}{6}}$ $x = \pm 243$

17. $y^{\frac{2}{3}} = x^{\frac{5}{6}}$ $\left(y^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(x^{\frac{5}{6}}\right)^{\frac{3}{2}}$ $y = x^{\frac{5}{4}}$	18. $V = a^2\sqrt{b^{\frac{1}{3}}}$ $\frac{V}{a^2} = \sqrt{b^{\frac{1}{3}}}$ $\frac{V^2}{a^4} = b^{\frac{1}{3}}$ $\frac{V^6}{a^{12}} = b$

Chapter 9. Rational Functions

9.1 Undefined Expressions

1. $x+2=0$, so $x=-2$	2. $3x+1=0$, so $x=-\frac{1}{3}$
3. $x^2-4=0$ $x^2=4$ $x=\pm\sqrt{4}=\pm 2$	4. $x^2-4x-12=0$ $(x+2)(x-6)=0$ $x=\{-2,6\}$
5. $9-x^2=0$ $-x^2=-9$ $x^2=9$ $x=\pm 3$	6. $x^2+5x-6=0$ $(x+6)(x-1)=0$ $x=\{-6,1\}$
7. $x+2=0$ or $x-1=0$ $x=\{-2,1\}$	8. $2x^2+1=0$ $2x^2=-1$ $x^2=-\frac{1}{2}$ $x=\pm\sqrt{-\frac{1}{2}}$ (not real numbers) The expression is defined for all possible real values of *x*.

9.2 Simplify Rational Expressions

1. $x+2$	2. $2x^2+3x+1$
3. $3x-9x^3$	4. x^4-9x^2+1
5. $3a^2b^2-6a$	6. $\frac{2x\cancel{(x-6)}}{\cancel{x-6}}=2x$
7. $\frac{25\cancel{(x-5)}}{(x+5)\cancel{(x-5)}}=\frac{25}{x+5}$	8. $\frac{{}^{2}\cancel{6}\cancel{(x+2)}}{\cancel{3}x\cancel{(x+2)}}=\frac{2}{x}$
9. $\frac{\cancel{(x+5)}(x+1)}{\cancel{(x+5)}(x-5)}=\frac{x+1}{x-5}$	10. $\frac{3x\cancel{(3x-5y)}}{(3x+5y)\cancel{(3x-5y)}}=\frac{3x}{3x+5y}$
11. $\frac{\cancel{(x+3)}(x-5)}{x\cancel{(x+3)}}=\frac{x-5}{x}$	12. $\frac{(x+2)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}}=\frac{x+2}{x-2}$
13. $\frac{2(x^2+5x-14)}{4(x+7)}=\frac{2\cancel{(x+7)}(x-2)}{4\cancel{(x+7)}}=$ $\frac{x-2}{2}$	14. $\frac{3-x}{2(x-3)}=\frac{-\cancel{(x-3)}}{2\cancel{(x-3)}}=-\frac{1}{2}$

15. $\frac{y-x}{(x+y)(x-y)} = \frac{-\cancel{(x-y)}}{(x+y)\cancel{(x-y)}} = -\frac{1}{x+y}$

16. $\frac{x(x-9)}{5x(9-x)} = \frac{x\cancel{(x-9)}}{-5x\cancel{(x-9)}} = -\frac{1}{5}$

17. $\frac{3y(y-4)}{y^2(4-y)} = \frac{3y\cancel{(y-4)}}{-y^2\cancel{(y-4)}} = -\frac{3}{y}$

18. $\frac{(xy+3)(xy-3)}{3-xy} = \frac{(xy+3)\cancel{(xy-3)}}{-\cancel{(xy-3)}} = -(xy+3) = -xy-3$

19. $\frac{x^2+8x+15}{x+5} = \frac{\cancel{(x+5)}(x+3)}{\cancel{x+5}} = x+3$

20. Base: $14x+21-2(4x+6) = 6x+9$
$\frac{6x+9}{14x+21} = \frac{3\cancel{(2x+3)}}{7\cancel{(2x+3)}} = \frac{3}{7}$

9.3 Multiply and Divide Rational Expressions

1. $\frac{7x^2}{3}\cdot\frac{9}{14x} = \frac{x}{1}\cdot\frac{3}{2} = \frac{3x}{2}$

2. $\frac{4x^2}{7y^2}\cdot\frac{21y^3}{20x^4} = \frac{1}{1}\cdot\frac{3y}{5x^2} = \frac{3y}{5x^2}$

3. $\frac{x^2-1}{x}\cdot\frac{4x^2}{x+1} = \frac{\cancel{(x+1)}(x-1)}{x}\cdot\frac{4x^2}{\cancel{x+1}} = \frac{x-1}{1}\cdot\frac{4x}{1} = 4x(x-1) = 4x^2-4x$

4. $\frac{4x}{x-1}\cdot\frac{x^2-1}{3x+3} = \frac{4x}{\cancel{x-1}}\cdot\frac{\cancel{(x+1)}\cancel{(x-1)}}{3\cancel{(x+1)}} = \frac{4x}{3}$

5. $\frac{x^2-1}{x+1}\cdot\frac{x+3}{3x-3} = \frac{\cancel{(x+1)}\cancel{(x-1)}}{\cancel{x+1}}\cdot\frac{x+3}{3\cancel{(x-1)}} = \frac{x+3}{3}$

6. $\frac{x+2}{2}\cdot\frac{4x+20}{x^2+6x+8} = \frac{\cancel{x+2}}{2}\cdot\frac{4(x+5)}{(x+4)\cancel{(x+2)}} = \frac{2(x+5)}{x+4} = \frac{2x+10}{x+4}$

7. $\frac{x+2}{3x+3}\cdot\frac{x^2+5x+4}{2x+4} = \frac{\cancel{x+2}}{3\cancel{(x+1)}}\cdot\frac{(x+4)\cancel{(x+1)}}{2\cancel{(x+2)}} = \frac{x+4}{6}$

8. $\frac{x^2-9}{x^2+9x+18}\cdot\frac{x}{x^2-3x} = \frac{\cancel{(x+3)}\cancel{(x-3)}}{(x+6)\cancel{(x+3)}}\cdot\frac{x}{x\cancel{(x-3)}} = \frac{1}{x+6}$

9. $\frac{x}{x+3}\div\frac{3x}{x^2-9} = \frac{x}{x+3}\cdot\frac{x^2-9}{3x} = \frac{x}{\cancel{x+3}}\cdot\frac{\cancel{(x+3)}(x-3)}{3x} = \frac{x-3}{3}$

10. $\frac{x}{x+4}\div\frac{2x}{x^2-16} = \frac{x}{x+4}\cdot\frac{x^2-16}{2x} = \frac{x}{\cancel{x+4}}\cdot\frac{\cancel{(x+4)}(x-4)}{2x} = \frac{x-4}{2}$

11. $\frac{9x^2}{x^2+12x+36} \div \frac{12x}{x^2+6x} =$ $\frac{9x^2}{x^2+12x+36} \cdot \frac{x^2+6x}{12x} =$ $\frac{9x^2}{\cancel{(x+6)}(x+6)} \cdot \frac{x\cancel{(x+6)}}{12x} =$ $\frac{3x^2}{4(x+6)} = \frac{3x^2}{4x+24}$	12. $\frac{3x+6}{4x+12} \div \frac{x^2-4}{x+3} =$ $\frac{3x+6}{4x+12} \cdot \frac{x+3}{x^2-4} =$ $\frac{3\cancel{(x+2)}}{4\cancel{(x+3)}} \cdot \frac{\cancel{x+3}}{\cancel{(x+2)}(x-2)} =$ $\frac{3}{4(x-2)} = \frac{3}{4x-8}$
13. $\frac{2x^2-8x-42}{6x^2} \cdot \frac{x^2-3x}{x^2-9} =$ $\frac{2\cancel{(x+3)}(x-7)}{6x^2} \cdot \frac{x\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x-3)}} = \frac{x-7}{3x}$	14. $\frac{3x^2+9x}{x^2+5x+6} \cdot \frac{x^2-x-6}{x^2-9} =$ $\frac{3x\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x+2)}} \cdot \frac{\cancel{(x+2)}\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} = \frac{3x}{x+3}$
15. $\frac{x^2+9x+14}{x^2-49} \cdot \frac{x^2+x-56}{3x+6} =$ $\frac{\cancel{(x+7)}\cancel{(x+2)}}{\cancel{(x+7)}\cancel{(x-7)}} \cdot \frac{(x+8)\cancel{(x-7)}}{3\cancel{(x+2)}} = \frac{x+8}{3}$	16. $\frac{2x-6}{2x+4} \cdot \frac{x^2+2x}{x^2+2x-15} =$ $\frac{2\cancel{(x-3)}}{2\cancel{(x+2)}} \cdot \frac{x\cancel{(x+2)}}{(x+5)\cancel{(x-3)}} = \frac{x}{x+5}$
17. $\frac{x^2+2x-15}{x^2-4x-45} \cdot \frac{x^2-5x-36}{x^2+x-12} =$ $\frac{\cancel{(x+5)}\cancel{(x-3)}}{\cancel{(x+5)}\cancel{(x-9)}} \cdot \frac{\cancel{(x+4)}\cancel{(x-9)}}{\cancel{(x+4)}\cancel{(x-3)}} = 1$	18. $\frac{x^2+4x+3}{2x^2-x-10} \cdot \frac{2x^2+4x^3}{x^2+3x} \cdot \frac{x^2+4x+4}{x^2+3x+2} =$ $\frac{\cancel{(x+3)}\cancel{(x+1)}}{(2x-5)\cancel{(x+2)}} \cdot \frac{2x^2(1+2x)}{x\cancel{(x+3)}} \cdot \frac{\cancel{(x+2)}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x+1)}} =$ $\frac{2x(2x+1)}{2x-5} = \frac{4x^2+2x}{2x-5}$

9.4 Add and Subtract Rational Expressions

1. $\frac{8}{x^2+1}$	2. $\frac{4x^2}{x-2}$
3. $\frac{x+12}{2x+4}$	4. $\frac{4}{5x}$
5. $\frac{2y+10}{y+5} = \frac{2\cancel{(y+5)}}{\cancel{y+5}} = 2$	6. $\frac{x^2}{x+1} + \frac{6x+5}{x+1} = \frac{x^2+6x+5}{x+1} =$ $\frac{(x+5)\cancel{(x+1)}}{\cancel{x+1}} = x+5$

7. $\underline{2^2} \cdot 3 \cdot \underline{7} \cdot \underline{a^3} \cdot b$ and $\underline{3^3} \cdot a^2 \cdot \underline{b^2}$
gives us an LCM of $2^2 \cdot 3^3 \cdot 7 \cdot a^3 \cdot b^2$, or
$756a^3b^2$

8. Factor as $(x+2)(x-2)$ and $(x+2)$.
So, LCM is $(x+2)(x-2)$.

9. Factor as $2x^2$, $x(x+1)$, and
$3x(x^2+1)$.
So, LCM is $6x^2(x+1)(x^2+1)$.

10. Factor as $(x+4)(x-1)$, $(x-1)^2$, and
$(x+4)(x+2)$.
So, the LCM is $(x-1)^2(x+4)(x+2)$.

11. LCM is 12

$$\frac{5x}{6}\left(\frac{2}{2}\right)+\frac{x}{4}\left(\frac{3}{3}\right)=\frac{10x}{12}+\frac{3x}{12}=\frac{13x}{12}$$

12. LCM is $5x$

$$\frac{3}{x}\left(\frac{5}{5}\right)-\frac{2}{5}\left(\frac{x}{x}\right)=\frac{15}{5x}-\frac{2x}{5x}=\frac{15-2x}{5x}$$

13. LCM is $21n$

$$\frac{3}{7n}\left(\frac{3}{3}\right)-\frac{7}{3n}\left(\frac{7}{7}\right)=\frac{9}{21n}-\frac{49}{21n}=-\frac{40}{21n}$$

14. LCM is $2x$

$$\frac{a}{x}\left(\frac{2}{2}\right)+\frac{b}{2x}=\frac{2a}{2x}+\frac{b}{2x}=\frac{2a+b}{2x}$$

15. LCM is $3b$

$$\frac{a}{b}\left(\frac{3}{3}\right)-\frac{1}{3}\left(\frac{b}{b}\right)=\frac{3a}{3b}-\frac{b}{3b}=\frac{3a-b}{3b}$$

16. LCM is $12x^2$

$$\frac{7}{12x}\left(\frac{x}{x}\right)-\frac{y}{6x^2}\left(\frac{2}{2}\right)=\frac{7x}{12x^2}-\frac{2y}{12x^2}=$$
$$\frac{7x-2y}{12x^2}$$

17.

$$\frac{6}{y-5}-\frac{y+5}{y^2-25}=$$
$$\frac{6}{y-5}-\frac{y+5}{(y+5)(y-5)}=$$
$$\frac{6}{y-5}-\frac{1}{y-5}=\frac{5}{y-5}$$

18.

$$\frac{5}{x(x+5)}+\frac{x}{x+5}$$

LCM is $x(x+5)$

$$\frac{5}{x(x+5)}+\frac{x}{x+5}\left(\frac{x}{x}\right)=$$
$$\frac{5}{x(x+5)}+\frac{x^2}{x(x+5)}=$$
$$\frac{x^2+5}{x(x+5)}=\frac{x^2+5}{x^2+5x}$$

19.

$$\frac{4x}{(x+1)(x-1)}-\frac{3x}{2(x+1)}$$

LCM is $2(x+1)(x-1)$

$$\frac{4x}{(x+1)(x-1)}\left(\frac{2}{2}\right)-\frac{3x}{2(x+1)}\left(\frac{x-1}{x-1}\right)=$$
$$\frac{8x}{2(x+1)(x-1)}-\frac{3x^2-3x}{2(x+1)(x-1)}=$$
$$\frac{-3x^2+11x}{2(x+1)(x-1)}=\frac{-3x^2+11x}{2(x^2-1)}=$$
$$\frac{-3x^2+11x}{2x^2-2}$$

20.

$$\frac{6x\cancel{(x+1)}}{\cancel{(x+1)}(x-2)}+\frac{(x+1)(x+1)}{3x(x-2)}=$$
$$\frac{6x}{(x-2)}+\frac{(x+1)(x+1)}{3x(x-2)}$$

LCM is $3x(x-2)$

$$\frac{6x}{(x-2)}\left(\frac{3x}{3x}\right)+\frac{(x+1)(x+1)}{3x(x-2)}=$$
$$\frac{18x^2}{3x(x-2)}+\frac{(x+1)(x+1)}{3x(x-2)}=$$
$$\frac{19x^2+2x+1}{3x(x-2)}=\frac{19x^2+2x+1}{3x^2-6x}$$

21. $\frac{y-5}{1}+\frac{3}{y+2}=\frac{y-5}{1}\left(\frac{y+2}{y+2}\right)+\frac{3}{y+2}=$

$$\frac{(y-5)(y+2)+3}{y+2}=\frac{y^2-3y-7}{y+2}$$

22. $\frac{3}{a-1}+\frac{3}{1-a}=\frac{3}{a-1}-\frac{3}{a-1}=$

$$\frac{0}{a-1}=0$$

23. $\frac{3x}{2x-6}+\frac{9}{6-2x}=\frac{3x}{2x-6}-\frac{9}{2x-6}=$

$$\frac{3x-9}{2x-6}=\frac{3\cancel{(x-3)}}{2\cancel{(x-3)}}=\frac{3}{2}$$

24. $\frac{x}{x-1}-\frac{1}{2-2x}=\frac{x}{x-1}-\frac{1}{2(1-x)}=$

$$\frac{x}{x-1}+\frac{1}{2(x-1)}=\frac{x}{x-1}\left(\frac{2}{2}\right)+\frac{1}{2(x-1)}=$$

$$\frac{2x}{2(x-1)}+\frac{1}{2(x-1)}=\frac{2x+1}{2(x-1)}=\frac{2x+1}{2x-2}$$

9.5 Simplify Complex Fractions

1. $\frac{x^2}{\frac{1}{x}}=x^2\div\frac{1}{x}=x^2\cdot x=x^3$

2. $\frac{\frac{2x}{y}}{\frac{4x}{y^2}}=\frac{2x}{y}\div\frac{4x}{y^2}=\frac{2x}{y}\cdot\frac{y^2}{4x}=\frac{y}{2}$

3. LCM is xy

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}\left(\frac{xy}{xy}\right)=\frac{y+x}{y-x}$$

4. LCM is x^2

$$\frac{1-\frac{1}{x^2}}{1+\frac{1}{x}}\left(\frac{x^2}{x^2}\right)=\frac{x^2-1}{x^2+x}=\frac{\cancel{(x+1)}(x-1)}{x\cancel{(x+1)}}=$$

$$\frac{x-1}{x}$$

5. LCM is $(x+1)(x-1)$

$$\frac{6-\frac{x}{x-1}}{4-\frac{x}{x+1}}\left(\frac{(x+1)(x-1)}{(x+1)(x-1)}\right)=$$

$$\frac{6(x+1)(x-1)-x(x+1)}{4(x+1)(x-1)-x(x-1)}=$$

$$\frac{6x^2-6-x^2-x}{4x^2-4-x^2+x}=\frac{5x^2-x-6}{3x^2+x-4}$$

[Note: both the numerator and denominator can be factored, but there are no common factors.]

6. LCM is $x-5$

$$1-\frac{1}{1-\frac{1}{x-5}}\left(\frac{x-5}{x-5}\right)=1-\frac{x-5}{x-5-1}$$

$$=1-\frac{x-5}{x-6}=\frac{x-6}{x-6}-\frac{x-5}{x-6}=\frac{-1}{x-6}$$

9.6 Solve Rational Equations

1. $16\left(\frac{x}{16}\right)+16\left(\frac{1}{4}\right)=16\left(\frac{1}{2}\right)$ $x+4=8$ $x=4$	2. $6\left(\frac{x}{2}\right)+6\left(\frac{x}{6}\right)=6(2)$ $3x+x=12$ $4x=12$ $x=3$
3. $5\left(\frac{3}{5}x\right)+5\left(\frac{2}{5}\right)=5(4)$ $3x+2=20$ $3x=18$ $x=6$	4. $4\left(\frac{3}{4}x\right)+4(2)=4\left(\frac{5}{4}x\right)-4(6)$ $3x+8=5x-24$ $32=2x$ $16=x$
5. $12\left(\frac{3}{4}x\right)=12\left(\frac{1}{3}x\right)+12(5)$ $9x=4x+60$ $5x=60$ $x=12$	6. $6n\left(\frac{5}{n}\right)-6n\left(\frac{1}{2}\right)=6n\left(\frac{3}{6n}\right)$ $30-3n=3$ $-3n=-27$ $n=9$
7. $15\left(\frac{2x}{5}\right)+15\left(\frac{1}{3}\right)=15\left(\frac{7x-2}{15}\right)$ $6x+5=7x-2$ $7=x$	8. $x\left(\frac{2}{x}\right)-x(3)=x\left(\frac{26}{x}\right)$ $2-3x=26$ $-3x=24$ $x=-8$
9. $6\left(\frac{2x}{3}\right)+6\left(\frac{x}{6}\right)=6(5)$ $4x+x=30$ $5x=30$ $x=6$	10. $21\left(\frac{1}{7}\right)+21\left(\frac{2x}{3}\right)=21\left(\frac{15x-3}{21}\right)$ $3+14x=15x-3$ $6=x$
11. $6\left(\frac{x}{3}\right)+6\left(\frac{x+1}{2}\right)=6(x)$ $2x+3(x+1)=6x$ $2x+3x+3=6x$ $5x+3=6x$ $3=x$	12. $12x\left(\frac{8}{3x}\right)-12x\left(\frac{x-1}{12}\right)=12x\left(\frac{1}{6x}\right)$ $32-x^2+x=2$ $x^2-x-30=0$ $(x+5)(x-6)=0$ $x=\{-5,6\}$
13. $4\cdot\frac{3}{4}(x+3)=4(9)$ $3(x+3)=36$ $3x+9=36$ $3x=27$ $x=9$	14. $5\cdot\frac{3}{5}(x+2)=5(x-4)$ $3(x+2)=5(x-4)$ $3x+6=5x-20$ $26=2x$ $13=x$
15. $10\left(\frac{m}{5}\right)+10\left(\frac{3(m-1)}{2}\right)=10\cdot 2(m-3)$ $2m+15(m-1)=20(m-3)$ $2m+15m-15=20m-60$ $17m-15=20m-60$ $45=3m$ $15=m$	16. $x^2(1)-x^2\left(\frac{6}{x^2}\right)=x^2\left(\frac{1}{x}\right)$ $x^2-6=x$ $x^2-x-6=0$ $(x+2)(x-3)=0$ $x=\{-2,3\}$

17. $x(x+2)\left[\frac{4x}{x+2}-\frac{12}{x}=1\right]$

$4x^2-12(x+2)=x(x+2)$

$4x^2-12x-24=x^2+2x$

$3x^2-14x-24=0$

$(3x+4)(x-6)=0$

$x=\left\{-\frac{4}{3},6\right\}$

18. $3x(x+1)\left[\frac{2}{3x}+\frac{4}{x}=\frac{7}{x+1}\right]$

$2(x+1)+12(x+1)=21x$

$2x+2+12x+12=21x$

$14x+14=21x$

$14=7x$

$2=x$

19. $3(x+3)(x-4)\left[\frac{3}{x+3}+\frac{2}{x-4}=\frac{4}{3}\right]$

$9(x-4)+6(x+3)=4(x+3)(x-4)$

$9x-36+6x+18=4(x^2-x-12)$

$15x-18=4x^2-4x-48$

$4x^2-19x-30=0$

$(4x+5)(x-6)=0$

$x=\left\{-\frac{5}{4},6\right\}$

20.

$(x+5)(x-5)\left[\frac{x}{x+5}+\frac{9}{x-5}=\frac{50}{(x+5)(x-5)}\right]$

$x(x-5)+9(x+5)=50$

$x^2-5x+9x+45=50$

$x^2+4x-5=0$

$(x+5)(x-1)=0$

$x=\{\cancel{-5},1\}$

-5 is an extraneous root, since

$x+5=(-5)+5=0$

21.

$(x+3)(x-4)\left[\frac{x}{x-4}-\frac{1}{x+3}=\frac{28}{(x+3)(x-4)}\right]$

$x(x+3)-(x-4)=28$

$x^2+3x-x+4=28$

$x^2+2x-24=0$

$(x+6)(x-4)=0$

$x=\{-6,\cancel{4}\}$

4 is an extraneous root.

22. $x(x+1)\left[\frac{4}{x}-\frac{3}{x+1}=7\right]$

$4(x+1)-3x=7x(x+1)$

$4x+4-3x=7x^2+7x$

$7x^2+6x-4=0$

$$x=\frac{-6\pm\sqrt{6^2-4(7)(-4)}}{2(7)}=\frac{-6\pm\sqrt{148}}{14}=$$

$$\frac{-6\pm2\sqrt{37}}{14}=\frac{-3\pm\sqrt{37}}{7}$$

23. $3x\left[\frac{x+3}{3}+\frac{x+3}{x}=2\right]$

$x^2+3x+3x+9=6x$

$x^2+9=0$

$x^2=-9$

$x=\pm3i$

24. $x\left[x+\frac{5}{x}=2\right]$

$x^2+5=2x$

$x^2-2x=-5$

$x^2-2x+1=-5+1$

$(x-1)^2=-4$

$x-1=\pm\sqrt{-4}$

$x-1=\pm2i$

$x=1\pm2i$

25. $x\left[x = 2 - \frac{8}{x}\right]$

$x^2 = 2x - 8$

$x^2 - 2x = -8$

$x^2 - 2x + 1 = -8 + 1$

$(x-1)^2 = -7$

$x - 1 = \pm\sqrt{-7}$

$x - 1 = \pm i\sqrt{7}$

$x = 1 \pm i\sqrt{7}$

26. $x\left[2x + \frac{3}{x} = -2\right]$

$2x^2 + 3 = -2x$

$2x^2 + 2x + 3 = 0$

$x^2 + x + \frac{3}{2} = 0$

$x^2 + x = -\frac{3}{2}$

$x^2 + x + \frac{1}{4} = -\frac{3}{2} + \frac{1}{4}$

$\left(x + \frac{1}{2}\right)^2 = -\frac{5}{4}$

$x + \frac{1}{2} = \pm\sqrt{-\frac{5}{4}}$

$x = -\frac{1}{2} \pm \frac{i\sqrt{5}}{2}$

27. $72x\left[\frac{x}{8} + \frac{8}{9x} = 0\right]$

$9x^2 + 64 = 0$

$x^2 = -\frac{64}{9}$

$x = \pm\sqrt{-\frac{64}{9}}$

$x = \pm\frac{8i}{3}$

28. $x^2\left[2 + \frac{5}{x^2} = \frac{6}{x}\right]$

$2x^2 + 5 = 6x$

$2x^2 - 6x = -5$

$x^2 - 3x = -\frac{5}{2}$

$x^2 - 3x + \frac{9}{4} = -\frac{5}{2} + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = -\frac{1}{4}$

$x - \frac{3}{2} = \pm\sqrt{-\frac{1}{4}}$

$x - \frac{3}{2} = \pm\frac{1}{2}i$

$x = \frac{3}{2} \pm \frac{1}{2}i$ or $x = \frac{3 \pm i}{2}$

9.7 Model Rational Expressions and Equations

1. $\frac{x - 3 + 7}{x + 7} = \frac{3}{4}$

$4(x + 4) = 3(x + 7)$

$4x + 16 = 3x + 21$

$x = 5$

Original fraction is $\frac{2}{5}$.

2. $x + \frac{64}{x} = 16$

$x\left[x + \frac{64}{x} = 16\right]$

$x^2 + 64 = 16x$

$x^2 - 16x + 64 = 0$

$(x - 8)^2 = 0$

$x = 8$

3. $\frac{5}{x} = \frac{7}{2x} + 3$ $2x\left[\frac{5}{x} = \frac{7}{2x} + 3\right]$ $10 = 7 + 6x$ $3 = 6x$ $x = \frac{1}{2}$	4. $\frac{6}{x} + \frac{7}{x+2} = 1$ $x(x+2)\left[\frac{6}{x} + \frac{7}{x+2} = 1\right]$ $6(x+2) + 7x = x(x+2)$ $13x + 12 = x^2 + 2x$ $x^2 - 11x - 12 = 0$ $(x+1)(x-12) = 0$ $x = \{\cancel{-1}, 12\}$ Reject -1. Numbers are 12 and 14.
5. $\frac{number\ of\ pets}{number\ of\ students} = 2$ $\frac{1\cdot 6 + 2\cdot 10 + 4k + 5\cdot 2}{22 + k} = 2$ $\frac{36 + 4k}{22 + k} = 2$ $36 + 4k = 2(22 + k)$ $36 + 4k = 44 + 2k$ $2k = 8$ $k = 4$	6. Written as a fraction, $2.25 = 2\frac{1}{4} = \frac{9}{4}$. Therefore, $\frac{1}{R_T} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$. $9x(x+3)\left[\frac{1}{x} + \frac{1}{x+3} = \frac{4}{9}\right]$ $9(x+3) + 9x = 4x(x+3)$ $9x + 27 + 9x = 4x^2 + 12x$ $4x^2 - 6x - 27 = 0$ $x = \frac{6\pm\sqrt{(-6)^2 - 4(4)(-27)}}{2(4)} = \frac{6\pm\sqrt{468}}{8}$ Positive solution $x \approx 3.5$.
7. $\frac{1}{6} + \frac{1}{4} = \frac{1}{x}$ $12x\left[\frac{1}{6} + \frac{1}{4} = \frac{1}{x}\right]$ $2x + 3x = 12$ $5x = 12$ $x = \frac{12}{5} = 2.4$ hrs	8. $\frac{1}{20} + \frac{1}{30} = \frac{1}{x}$ $60x\left[\frac{1}{20} + \frac{1}{30} = \frac{1}{x}\right]$ $3x + 2x = 60$ $5x = 60$ $x = 12$ mins $12 \times 50 = 600$ mins or 10 hrs
9. $\frac{1}{c} + \frac{1}{2c} = \frac{1}{5}$ $10c\left[\frac{1}{c} + \frac{1}{2c} = \frac{1}{5}\right]$ $10 + 5 = 2c$ $15 = 2c$ $c = \frac{15}{2} = 7.5$ hrs	10. $\frac{1}{x-5} + \frac{1}{x} = \frac{1}{6}$ $6x(x-5)\left[\frac{1}{x-5} + \frac{1}{x} = \frac{1}{6}\right]$ $6x + 6(x-5) = x(x-5)$ $6x + 6x - 30 = x^2 - 5x$ $x^2 - 17x + 30 = 0$ $x = \frac{17 \pm\sqrt{17^2 - 4(1)(30)}}{2(1)} = \frac{17 \pm\sqrt{169}}{2} = \frac{17 \pm 13}{2}$ $x = \{\cancel{2}, 15\}$ Reject $x = 2$ because $x - 5 > 0$. Faster machine takes $15 - 5 = 10$ hrs.

11.

	D	**R**	**T**
moped	40	$x + 20$	$\frac{40}{x+20}$
bicycle	15	x	$\frac{15}{x}$

$$\frac{40}{x+20} = \frac{15}{x}$$
$40x = 15(x + 20)$
$40x = 15x + 300$
$25x = 300$
$x = 12$
Bicycle is 12 mph, moped is 32 mph

12.

	D	**R**	**T**
upstream	4	$5 - c$	$\frac{4}{5-c}$
downstream	16	$5 + c$	$\frac{16}{5+c}$

$$\frac{4}{5-c} = \frac{16}{5+c}$$
$4(5 + c) = 16(5 - c)$
$20 + 4c = 80 - 16c$
$20c = 60$
$c = 3$ mph

13.

	D	**R**	**T**
with wind	1656	$x + 12$	$\frac{1656}{x+12}$
against wind	3168	$x - 12$	$\frac{3168}{x-12}$

$$\frac{1656}{x+12} = \left(\frac{1}{2}\right) \cdot \frac{3168}{x-12}$$
$$\frac{3312}{x+12} = \frac{3168}{x-12}$$
$3312(x - 12) = 3168(x + 12)$
$3312x - 39744 = 3168x + 38016$
$144x = 77760$
$x = 540$ mph

14.

	D	**R**	**T**
car	120	$x - 100$	$\frac{120}{x-100}$
train	120	x	$\frac{120}{x}$

75 mins = $\frac{5}{4}$ hours
$$\frac{120}{x} = \frac{120}{x-100} - \frac{5}{4}$$
$$4x(x - 100)\left[\frac{120}{x} = \frac{120}{x-100} - \frac{5}{4}\right]$$
$480(x - 100) = 480x - 5x(x - 100)$
$480x - 48{,}000 = 480x - 5x^2 + 500x$
$5x^2 - 500x - 48{,}000 = 0$
$x^2 - 100x - 9{,}600 = 0$
$(x - 160)(x + 60) = 0$
$x = \{160, \cancel{-60}\}$ reject negative value
Train travels 160 mph.

9.8 Graphs of Rational Functions

1. $x = -2$ and $x = -9$

2. This is in $y = \frac{a}{x-h} + k$ form.
Vertical asymptote at $x = -1$.
Horizontal asymptote at $y = 3$.

3. Degrees of numerator and denominator are equal (2), and leading coefficients are both 1. Asymptote is at $y = \frac{1}{1} = 1$.

4. The degree of the numerator (1) is smaller than the degree of the denominator (2). So, there is an asymptote at $y = 0$.

5. There is no horizontal asymptote because the degree of the numerator (2) is greater than the degree of the denominator (1).

6. $2x + 1 = 0$
Vertical asymptote at $x = -\frac{1}{2}$.
Horizontal asymptote at $y = \frac{4}{2} = 2$.

7. (2) Vertical asymptotes at –3 and 3, which are the roots of $x^2 - 9 = 0$, and a horizontal asymptote at 2.

8. $4x = 0$
x-intercept is 0
$f(0) = \frac{4(0)}{2(0) + 1} = 0$
y-intercept is 0

9. $x - 1 = 0$
x-intercept is 1
$g(0) = \frac{0 - 1}{0^2 - 4} = \frac{1}{4}$
y-intercept is $\frac{1}{4}$

10. $x^2 + 11x + 18 = 0$
$(x + 2)(x + 9) = 0$
x-intercepts at -2 and -9
$h(0) = \frac{0^2 + 11(0) + 18}{0^2 + 3(0)} = \frac{18}{0}$
undefined
There is no y-intercept

11. $\frac{x^2}{2x + 1} - 1 = 0$
$\frac{x^2}{2x + 1} = 1$
$x^2 = 2x + 1$
$x^2 - 2x = 1$
$x^2 - 2x + 1 = 2$
$(x - 1)^2 = 2$
$x - 1 = \pm\sqrt{2}$
$x = 1 \pm \sqrt{2}$
x-intercepts at $1 \pm \sqrt{2}$
$j(0) = \frac{0^2}{2(0) + 1} - 1 = -1$
y-intercept at –1

Chapter 10. Exponential Functions

10.1 Solve Simple Exponential Equations

1. $4^x = 64$ $4^x = 4^3$ $x = 3$	2. $3^x = 81$ $3^x = 3^4$ $x = 4$
3. $5^x = 25$ $5^x = 5^2$ $x = 2$	4. $\left(\frac{1}{2}\right)^x = \frac{1}{8}$ $\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^3$ $x = 3$
5. $2^{x+1} = 2^3$ $x + 1 = 3$ $x = 2$	6. $3^{2x-2} = 3^4$ $2x - 2 = 4$ $x = 3$
7. $3^{x-3} = 3^0$ $x - 3 = 0$ $x = 3$	8. $4^{3x+5} = 4^2$ $3x + 5 = 2$ $x = -1$
9. $3^{x+1} = 27$ $3^{x+1} = 3^3$ $x + 1 = 3$ $x = 2$	10. $(2^2)^4 = 2^{3x-1}$ $2^8 = 2^{3x-1}$ $8 = 3x - 1$ $x = 3$
11. $2x = x + 4$ $x = 4$	12. $2^x = 2^{2(x+1)}$ $x = 2(x + 1)$ $x = 2x + 2$ $x = -2$
13. $2^{2x} = 2^{3x+1}$ $2x = 3x + 1$ $x = -1$	14. $2^{6x} = 2^{x+5}$ $6x = x + 5$ $x = 1$
15. $3^{x-5} = 3^{2(x-3)}$ $x - 5 = 2x - 6$ $x = 1$	16. $2^{3(x-2)} = 2^x$ $3x - 6 = x$ $x = 3$
17. $2^1 = 2^{2x+1}$ $1 = 2x + 1$ $x = 0$	18. $2^{3(x-2)} = 2^{2x}$ $3x - 6 = 2x$ $x = 6$

10.2 Rewrite Exponential Expressions

1. $5^{2x} = (5^2)^x = 25^x$

2. $10(1.1)^{5x} = 10(1.1^5)^x = 10(1.61051)^x$

3. $2^{3x+2} = (2^3)^x \cdot 2^2 = 4(8)^x$

4. $4(3)^{x+1} = 4(3)^x(3^1) = 12(3)^x$

5. $\dfrac{3^{5x+1}}{9^x} = \dfrac{3^{5x+1}}{3^{2x}} = 3^{5x+1-2x} = 3^{3x+1}$
$= 3^{3x} \cdot 3^1 = 3(3^{3x}) = 3(27)^x$

6. $3^{2x-3} = \dfrac{(3^2)^x}{3^3} = \dfrac{9^x}{27}$

7. $5\left(4^{\frac{x}{2}+2}\right) \cdot 3^{3x}$
$= 5\left(4^{\frac{1}{2}}\right)^x \cdot (4^2) \cdot (3^3)^x$
$= 5\left(\sqrt{4}\right)^x \cdot (16) \cdot (27)^x$
$= (5 \cdot 16)(2^x)(27^x) = 80(54)^x$

8. $\dfrac{2^{4y+5}}{16^y} = \dfrac{2^{4y+5}}{2^{4y}} =$
$2^{4y+5-4y} = 2^5 = 32$

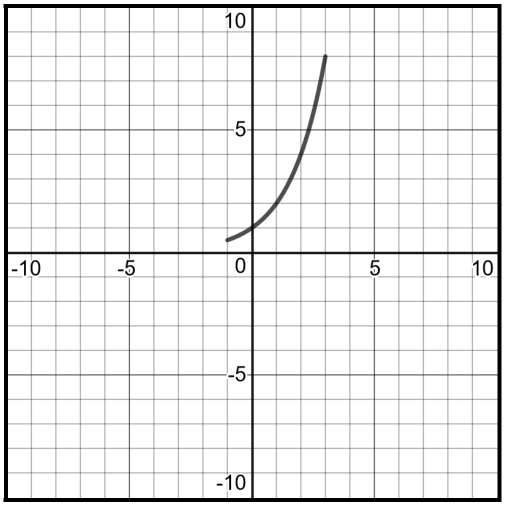
9. $4^x = \left(4^{\frac{1}{3}}\right)^{3x}$, so $k = 4^{\frac{1}{3}} = \sqrt[3]{4}$

10. $2^{x+3} - 2^x = 2^x(2^3) - 2^x = 2^x(2^3 - 1)$
$= 7 \cdot 2^x$, so $k = 7$

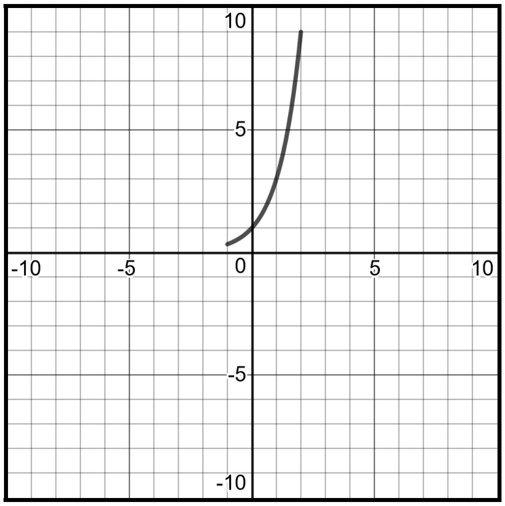
11. $\frac{1}{4}(2^x) = \frac{1}{4}\left(2^{4\cdot\frac{x}{4}}\right) = \frac{1}{4}\left(16^{\frac{x}{4}}\right) = \frac{1}{4}\left(16^{\frac{x}{4}-2+2}\right) = \frac{1}{4}\left(16^{\frac{x}{4}-2}\right)(16^2) = \frac{1}{4}\left(16^{\frac{x}{4}-2}\right)(256)$
$= 64\left(16^{\frac{x}{4}-2}\right)$, so $k = 64$ and $b = 16$.

10.3 Graphs of Exponential Functions

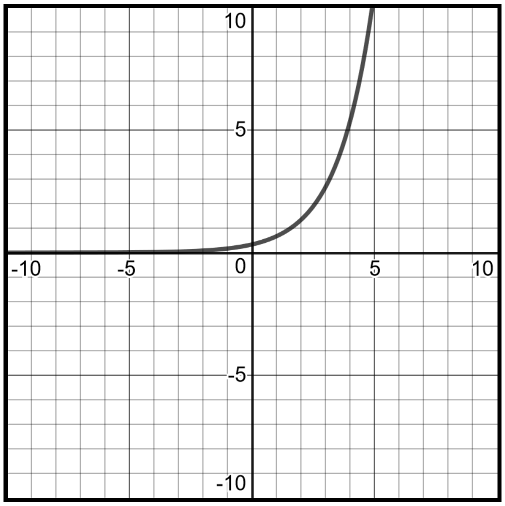
1.



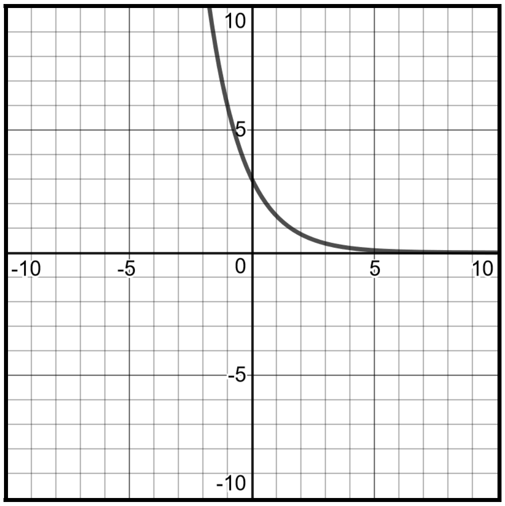
2.



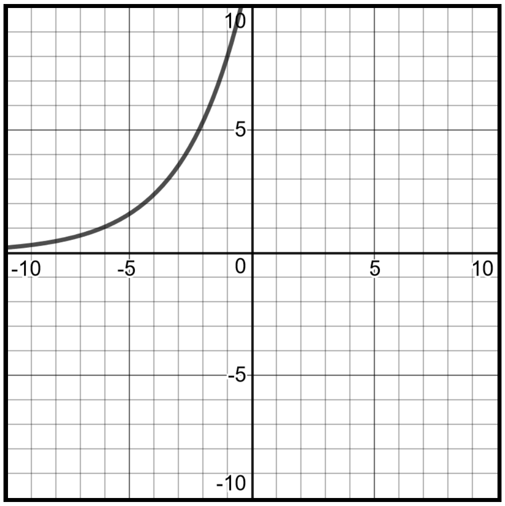
3.


10
5
-10
-5
0
5
10
-5
-10

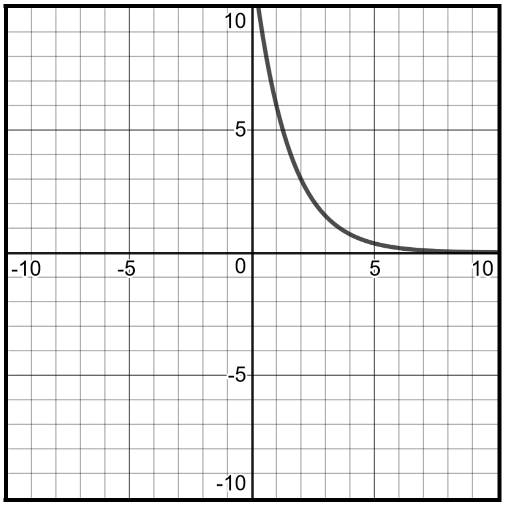
4.


10
5
-10
-5
0
5
10
-5
-10

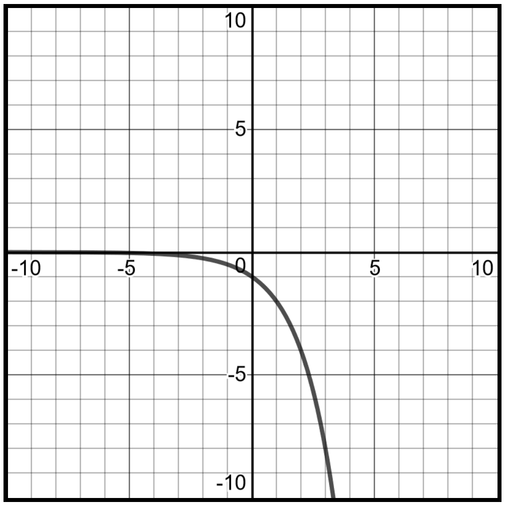
5.


10
5
-10
-5
0
5
10
-5
-10

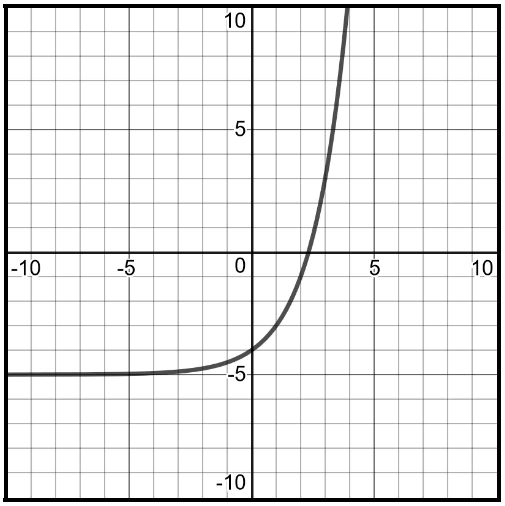
6.


10
5
-10
-5
0
5
10
-5
-10

7.


10
5
-10
-5
0
5
10
-5
-10

8.


10
5
-10
-5
0
5
10
-5
-10

10.4 Exponential Regression

1. $y = (0.25)^x$

2. $y = 0.1(4)^x$

3. Enter $(10, 4.072)$ and $(15, 5.197)$.
$y = 2.5(1.05)^x$

4. $y = 6.162(0.796)^x$

5. $f(x) = 1548.977(1.126)^x$
$f(10) \approx 5{,}079$

6. Shift the data down by 70°, so enter the data as

L1	0	5	10	15	20
L2	140	108	84	65	50

This gives us $y = 140(0.95)^x$. Shifting the function back up by adding 70° gives us $f(x) = 140(0.95)^x + 70$. After 30 minutes, the coffee is $f(30) \approx 100°$.

10.5 Exponential Growth or Decay

1. (3) because the base of the exponent t is less than 1.

2. $500(0.75)^5 \approx \$118.65$

3. $1(1.08)^{\frac{365}{10}} \approx 17$ feet

4. $250(0.85)^{\frac{24}{3}} \approx 68$ mg

5. "every 30 seconds" means twice a minute
$5000(1.10)^{2 \cdot 60} \approx 463{,}545{,}344$ cells

6. $x(1.005)^{\frac{52}{2}} = 28{,}461.49$
$x = \dfrac{28{,}461.49}{1.005^{26}} \approx \$25{,}000$

7. $f(t) = 10(0.5)^{\frac{t}{29}}$

8. $278(0.5)^{\frac{18}{1.8}} \approx 0.27$ MBq

9. a) 2%
b) $1.02^{\frac{1}{12}} = \sqrt[12]{1.02} \approx 1.0017$, or 0.17%

10. a) 15%
b) $\sqrt[52]{1.15} \approx 1.00269$, so 0.27%

11. $P\left(1.033^{\frac{1}{12}}\right)^{12t} \approx P(1.00271)^m$
Note: $m = 12t$

12. $\sqrt[365]{0.75} \approx 0.99921$
$1 - 0.99921 = 0.00079$, so 0.08%

10.6 Periodic Compound Interest

1. $A = 500\left(1 + \frac{0.04}{12}\right)^{(12)(3)} \approx \$563.64$

2. $A = 500\left(1 + \frac{0.04}{365}\right)^{(365)(3)} \approx \$563.74$

3. a) $\frac{0.03}{4} = 0.0075 = 0.75\%$
b) $A(t) = 2000\left(1 + \frac{0.03}{4}\right)^{4t}$
$A(5) \approx \$2322.37$

4. a) $\frac{0.05}{12} \approx 0.42\%$
b) $A(t) = 850\left(1 + \frac{0.05}{12}\right)^{12t}$
21 months = 1.75 years
$A(1.75) = \$927.56$

5. a) $f(t) = 2000(1.02)^{4t}$ b) $2000(1.02)^{4(1.5)} \approx 2252$ c) $(1.02)^4 - 1 \approx 0.0824 = 8.24\%$	6. a) $0.001 \times 365 = 0.365 = 36.5\%$ b) $100(1.001)^{365} \approx \$144$ c) $(1.001)^{365} - 1 \approx 1.44 = 44\%$
7. $1403.60 = P\left(1 + \frac{0.068}{12}\right)^{60}$ $P = \frac{1403.60}{\left(1 + \frac{0.068}{12}\right)^{60}} \approx \$1{,}000.00$	8. $1{,}000{,}000 = P\left(1 + \frac{0.0365}{365}\right)^{365\cdot 20}$ $1{,}000{,}000 = P(1.0001)^{7300}$ $P = \frac{1{,}000{,}000}{(1.0001)^{7300}} \approx \$481{,}926.58$
9. a) $1000(1.04)^5 = \$1{,}216.65$ b) $\frac{c - c\left(1 + \frac{r}{n}\right)^{nt}}{1 - \left(1 + \frac{r}{n}\right)} = \frac{200 - 200(1.04)^5}{1 - (1.04)} = \$1{,}083.26$ c) $1216.65 + 1083.26 = \$2{,}299.91$	

10.7 Continuous Growth or Decay

1. $A = 5000e^{(0.03)(4)} = \$5637.48$	2. $A = 550e^{(0.066)(10)} = \$1064.14$
3. a) $1000(1.025)^3 \approx \$1076.89$ b) $1000\left(1 + \frac{0.025}{12}\right)^{(12)(3)} \approx \$1077.80$ c) $1000e^{(0.025)(3)} \approx \$1077.88$	4. $30{,}000 = Pe^{(0.05)(6)}$ $P = \frac{30{,}000}{e^{0.3}} \approx \$22{,}225$

Chapter 11. Logarithms

11.1 Introduction to Logarithms

1. $4^x = 64$, so $x = 3$

2. $5^x = \frac{1}{125}$, so $x = -3$

3. $6^x = 1$, so $x = 0$

4. $k = \log_2 1$ means $2^k = 1$, so $k = 0$

5. $x = 3^4 = 81$

6. $x = 5^2 = 25 \qquad \sqrt{x} = 5$

7. $x + 1 = 2^3$
$x + 1 = 8$
$x = 7$

8. $5x - 7 = 2^3$
$5x - 7 = 8$
$5x = 15$
$x = 3$

9. $\log_{10}(x + 2) = 4$
$x + 2 = 10^4$
$x + 2 = 10{,}000$
$x = 9{,}998$

10.

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
y	−2	−1	0	1	2

11. $\log \frac{I_1}{I_2} = 8 - 5.5 = 2.5$
$\frac{I_1}{I_2} = 10^{2.5} \approx 316$ times as intense

12. $\log \frac{I_1}{I_2} = \frac{85-47}{10} = 3.8$
$\frac{I_1}{I_2} = 10^{3.8} \approx 6{,}310$ times as intense

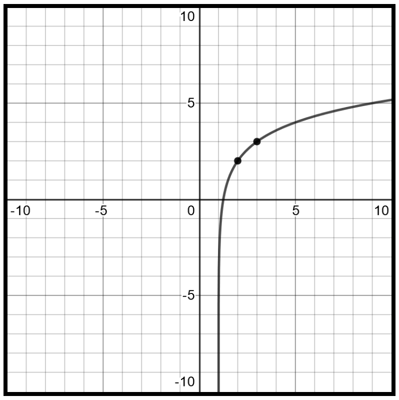
11.2 Graphs of Log Functions

1. a) (3) b) (2) c) (1) d) (4)

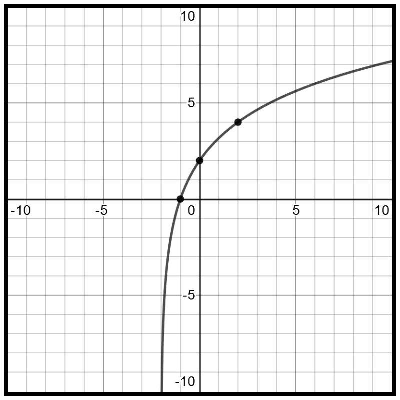
2. $g(x) = \log(x + 5) - 2$

3. $g(x) = \frac{1}{2}\log_2 x + 3$

4.



5.



6. y-intercept is $\log(0+1)+2=2$
To find the x-intercept:
$\log(x+1)+2=0$
$\log(x+1)=-2$
$x+1=10^{-2}$
$x+1=0.01$
$x=-0.99$

7. y-intercept is $2\log_2(0+5)\approx 4.64$
To find the x-intercept:
$2\log_2(x+5)=0$
$\log_2(x+5)=0$
$x+5=2^0=1$
$x=-4$

8. $\log 0$ is undefined, so there is no y-intercept.
$\log x-2=0$
$\log x=2$
$x=10^2=100$

9. $\log(-5)$ is undefined, so there is no y-intercept.
$\log_3(x-5)-3=0$
$\log_3(x-5)=3$
$x-5=3^3=27$
$x=32$

11.3 Properties of Logarithms

1. $2\log x+\log y$

2. $3\log a-\log b$

3. $\log 2+\log x+3\log y$

4. $\frac{1}{2}(\log x+\log y)$

5. $2x\log 4+\frac{1}{2}\log y$

6. $\log 5\,(2x)^2-\log(x+1)^3=$
$\log 5+2\log(2x)-3\log(x+1)=$
$\log 5+2\log 2+2\log x-3\log(x+1)$

7. $2\log_3 10-\log_3 20$
$=\log_3 10^2-\log_3 20$
$=\log_3\frac{10^2}{20}$
$=\log_3 5$

8. $3\log_b x+\log_b y-2\log_b z$
$=\log_b x^3+\log_b y-\log_b z^2$
$=\log_b\dfrac{x^3y}{z^2}$

9. $\frac{1}{2}\log x-2\log y$
$=\log\sqrt{x}-\log y^2$
$=\log\dfrac{\sqrt{x}}{y^2}$

10. $\dfrac{2\log x}{3}+\dfrac{3\log y}{4}$
$=\frac{2}{3}\log x+\frac{3}{4}\log y$
$=\log\sqrt[3]{x^2}+\log\sqrt[4]{y^3}$
$=\log\left(\sqrt[3]{x^2}\cdot\sqrt[4]{y^3}\right)$

11. $\log 3{,}000-\log 3=\log\frac{3{,}000}{3}=\log 1{,}000=3$

12. $\log_2 8+\log_2 2=\log_2(8\cdot 2)=\log_2 16=4$

13. (4) $\log\left(\frac{2I}{T}\right)=\log 2+\log I-\log T$

14. (2)
$\log[P(1+r)^t]=\log P+t\log(1+r)$

11.4 Use Logarithms to Solve Equations

1. $\log 2^x = \log 5$
$x \log 2 = \log 5$
$x = \dfrac{\log 5}{\log 2} \approx 2.32$

2. $\log 16^x = \log 88$
$x \log 16 = \log 88$
$x = \dfrac{\log 88}{\log 16} \approx 1.61$

3. $\log 13^x = \log 76$
$x \log 13 = \log 76$
$x = \dfrac{\log 76}{\log 13} \approx 1.69$

4. $\log 2^x = \log \frac{3}{16}$
$x \log 2 = \log \frac{3}{16}$
$x = \dfrac{\log \frac{3}{16}}{\log 2} \approx -2.42$

5. $20^x = 9$
$x \log 20 = \log 9$
$x = \dfrac{\log 9}{\log 20} \approx 0.73$

6. $3^x = 13$
$x \log 3 = \log 13$
$x = \dfrac{\log 13}{\log 3} \approx 2.33$

7. $\log 2^{3x} = \log 7$
$3x \log 2 = \log 7$
$3x = \dfrac{\log 7}{\log 2}$
$x = \dfrac{\log 7}{3 \log 2} \approx 0.94$

8. $\log 3^{2x-1} = \log 20$
$(2x - 1) \log 3 = \log 20$
$2x - 1 = \dfrac{\log 20}{\log 3} \approx 2.7268$
$2x \approx 3.7268$
$x \approx 1.863$

9. $3(5^{x+1}) = 125$
$5^{x+1} = \frac{125}{3}$
$\log 5^{x+1} = \log \frac{125}{3}$
$(x + 1) \log 5 = \log \frac{125}{3}$
$x + 1 = \dfrac{\log \frac{125}{3}}{\log 5} \approx 2.317$
$x \approx 1.317$

10. $1200(1.024)^t = 2400$
$(1.024)^t = 2$
$\log(1.024)^t = \log 2$
$t \log 1.024 = \log 2$
$t = \dfrac{\log 2}{\log 1.024} \approx 29.2$ weeks

11. $\left(1 + \frac{0.04}{4}\right)^{4t} = 2$
$(1.01)^{4t} = 2$
$\log(1.01)^{4t} = \log 2$
$4t \log 1.01 = \log 2$
$t = \dfrac{\log 2}{4 \log 1.01} \approx 17.4$ years

12. $95(0.90)^t = 5$
$(0.90)^t = \frac{5}{95} = \frac{1}{19}$
$t \log 0.90 = \log \frac{1}{19}$
$t = \dfrac{\log \frac{1}{19}}{\log 0.90} \approx 27.9$ hours

13. $8{,}000\left(1+\frac{0.06}{12}\right)^{12t} = 10{,}000$

$\left(1+\frac{0.06}{12}\right)^{12t} = 1.25$

$(1.005)^{12t} = 1.25$

$\log(1.005)^{12t} = \log 1.25$

$12t\log 1.005 = \log 1.25$

$t = \dfrac{\log 1.25}{12\log 1.005} \approx 3.7$ years

14. $8{,}000\left(1+\frac{0.07}{4}\right)^{4t} = 10{,}000$

$\left(1+\frac{0.07}{4}\right)^{4t} = 1.25$

$(1.0175)^{4t} = 1.25$

$\log(1.0175)^{4t} = \log 1.25$

$4t\log 1.0175 = \log 1.25$

$t = \dfrac{\log 1.25}{4\log 1.0175} \approx 3.2$ years

15. $97.656 = \frac{5^x}{2^x}$

$97.656 = \left(\frac{5}{2}\right)^x$

$\log 97.656 = \log 2.5^x$

$\log 97.656 = x\log 2.5$

$x = \dfrac{\log 97.656}{\log 2.5} \approx 5.0$

16. $\frac{551}{5} = \frac{9^x}{5^x}$

$110.2 = \left(\frac{9}{5}\right)^x$

$\log 110.2 = \log 1.8^x$

$\log 110.2 = x\log 1.8$

$x = \dfrac{\log 110.2}{\log 1.8} \approx 8.0$

17. $40{,}353{,}607 = (7^x)\left(7^{\frac{x}{2}}\right)$

$40{,}353{,}607 = 7^{\frac{3x}{2}}$

$\log 40{,}353{,}607 = \log 7^{\frac{3x}{2}}$

$\log 40{,}353{,}607 = \frac{3}{2}x\log 7$

$x = \dfrac{2\log 40{,}353{,}607}{3\log 7} = 6$

18. $\log[3.72(18)^x] = \log[5^{2x}]$

$\log 3.72 + \log 18^x = \log 5^{2x}$

$\log 3.72 + x\log 18 = 2x\log 5$

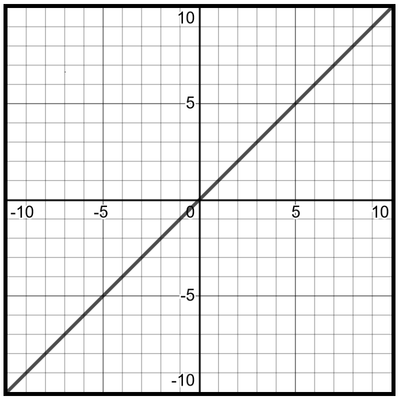
$\log 3.72 = 2x\log 5 - x\log 18$

$\log 3.72 = x(2\log 5 - \log 18)$

$x = \dfrac{\log 3.72}{2\log 5 - \log 18} \approx 4.0$

11.5 Natural Logarithms

1. Since $\ln e^x = x$, this is simply graphed as the line $y = x$.



2. $\ln e^x = \ln 15$

$x = \ln 15 \approx 2.71$

3. $\ln e^{2x} = \ln 13$

$2x = \ln 13$

$x = \dfrac{\ln 13}{2} \approx 1.28$

4. $4e^{2x} = 8$
$e^{2x} = 2$
$\ln e^{2x} = \ln 2$
$2x = \ln 2$
$x = \frac{\ln 2}{2} \approx 0.35$

5. $\frac{2050}{1500} = e^{4x}$
$\ln \frac{2050}{1500} = \ln e^{4x}$
$\ln \frac{2050}{1500} = 4x$
$x = \frac{\ln \frac{2050}{1500}}{4} \approx 0.078$

6. Use the power rule: $a \ln x = \ln x^a$.
$\ln x^2 = \ln 16$
$x^2 = 16$
$x = [\cancel{-4}, 4\}$
$f(x) = \ln x$ is restricted to $x > 0$,
so reject –4.
$x = 4$

7. Use the properties of logarithms:
$\ln(2x - 3) + \ln(x + 2) = 2 \ln x$
$\ln[(2x - 3)(x + 2)] = \ln x^2$
$(2x - 3)(x + 2) = x^2$
$2x^2 + x - 6 = x^2$
$x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$
$x = \{\cancel{-3}, 2\}$ *[reject $x = -3$]*
$x = 2$

8. $2 \ln x = \ln 9 + \ln e$ *[$\ln e = 1$]*
$\ln x^2 = \ln 9e$
$x^2 = 9e$
$x = 3\sqrt{e} \approx 4.95$ *[reject $-3\sqrt{e}$]*

9. $\ln e^{kt} = \ln 100^{2t}$
$kt = 2t \cdot \ln 100$
$k = 2 \ln 100 \approx 9.21$

10. $A = Pe^{rt}$
$\frac{A}{P} = e^{rt}$
$1.25 = e^{3r}$
$\ln 1.25 = \ln e^{3r}$
$\ln 1.25 = 3r$
$r = \frac{\ln 1.25}{3} \approx 7.4\%$

11. $A = Pe^{rt}$
$3000 = 2500e^{5r}$
$1.2 = e^{5r}$
$\ln 1.2 = \ln e^{5r}$
$\ln 1.2 = 5r$
$r = \frac{\ln 1.2}{5} \approx 3.6\%$

12. a) $f(10) = 40e^{-0.02877(10)} + 60 \approx 90°$
$f(20) = 40e^{-0.02877(20)} + 60 \approx 82.5°$
b) $70 = 40e^{-0.02877t} + 60$
$10 = 40e^{-0.02877t}$
$0.25 = e^{-0.02877t}$
$\ln 0.25 = \ln e^{-0.02877t}$
$\ln 0.25 = -0.02877t$
$-1.38629 \approx -0.02877t$
$t \approx 48$ minutes

13. $90{,}000e^{0.10t} = 80{,}000e^{0.11t}$
$\frac{9}{8}e^{0.10t} = e^{0.11t}$
$\frac{9}{8} = \frac{e^{0.11t}}{e^{0.10t}}$
$\frac{9}{8} = e^{0.01t}$
$\ln \frac{9}{8} = 0.01t$
$t = \frac{\ln \frac{9}{8}}{0.01} \approx 11.8$ years

11.6 Evaluate Loan Formulas

1. a) $M = 300{,}000\left(\frac{0.0025(1.0025)^{360}}{(1.0025)^{360}-1}\right)$ $M \approx \$1{,}265$

   b) $M = 275{,}000\left(\frac{0.0025(1.0025)^{360}}{(1.0025)^{360}-1}\right)$ $M \approx \$1{,}159$

   She can reduce her payments by \$106.

2. $M = 650{,}000 \cdot \frac{\left(\frac{0.025}{12}\right)\left(1+\frac{0.025}{12}\right)^{300}}{\left(1+\frac{0.025}{12}\right)^{300}-1} = 650{,}000 \cdot \frac{(0.00208)(1.00208)^{300}}{(1.00208)^{300}-1}$

   $M \approx \$2{,}915$

3. $1500 = (500{,}000 - x)\left(\frac{0.002(1.002)^{360}}{(1.002)^{360}-1}\right)$

   $1500\left(\frac{(1.002)^{360}-1}{0.002(1.002)^{360}}\right) = 500{,}000 - x$

   $x = -1500\left(\frac{(1.002)^{360}-1}{0.002(1.002)^{360}}\right) + 500{,}000$

   $x \approx 115{,}326.78$, so he should make a down payment of \$115,327.

4. $1386.50 = 250{,}000 \cdot \frac{\left(\frac{0.03}{12}\right)\left(1+\frac{0.03}{12}\right)^{n}}{\left(1+\frac{0.03}{12}\right)^{n}-1}$

   $1386.50 = 250{,}000 \cdot \frac{(0.0025)(1.0025)^{n}}{(1.0025)^{n}-1}$

   $1386.50 = 625 \cdot \frac{(1.0025)^{n}}{(1.0025)^{n}-1}$

   $2.2184 = \frac{(1.0025)^{n}}{(1.0025)^{n}-1}$

   $2.2184(1.0025)^{n} - 2.2184 = (1.0025)^{n}$

   $1.2184(1.0025)^{n} = 2.2184$

   $(1.0025)^{n} = \frac{2.2184}{1.2184}$

   $n \log 1.0025 = \log \frac{2.2184}{1.2184}$

   $n = \frac{\log \frac{2.2184}{1.2184}}{\log 1.0025} \approx 240$ mos, or 20 yrs

Chapter 12. Trigonometric Functions

12.1 Trigonometric Ratios

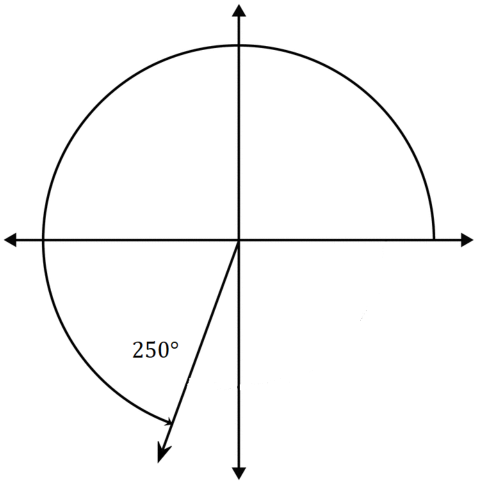
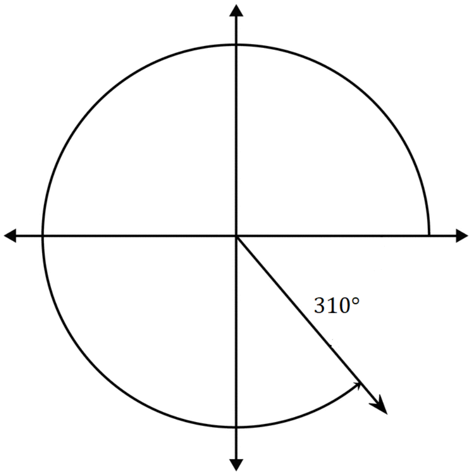
1. $\cot A = \frac{adj}{opp} = \frac{2}{4} = \frac{1}{2}$	2. $\csc A = \frac{hyp}{opp} = \frac{25}{7}$
3. $\csc 45^\circ = \frac{1}{\sin 45^\circ} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$	4. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{4}}{\frac{\sqrt{15}}{4}} = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$
5. $\cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	6. $\csc 60^\circ = \frac{1}{\sin 60^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
7. $\sec 35^\circ = \frac{1}{\cos 35^\circ} \approx 1.221$	8. $\csc 35^\circ = \frac{1}{\sin 35^\circ} \approx 1.743$
9. sin and cos are cofunctions, so $\sin x = \cos(90^\circ - x)$. Therefore, $\cos 20^\circ \approx 0.9397$.	10. $\sec x = \csc(90^\circ - x)$, so $a = 90 - 28 = 62^\circ$
11. a) $\theta = \frac{360}{12} = 30^\circ$ b) $\cot\theta = \frac{h}{\frac{x}{2}} = \frac{2h}{x}$, so $h = \frac{1}{2}x\cot\theta$ c) $\cot 30^\circ = \sqrt{3}$, so $h = \frac{\sqrt{3}}{2}x$ d) $A_\triangle = \frac{1}{2}bh = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$ e) $A_\hexagon = 6\left(\frac{\sqrt{3}}{4}x^2\right) = \frac{3\sqrt{3}}{2}x^2$	

12.2 Radians

1. a) $45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$ rad b) $270 \cdot \frac{\pi}{180} = \frac{3\pi}{2}$ rad c) $150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$ rad d) $-210 \cdot \frac{\pi}{180} = -\frac{7\pi}{6}$ rad	2. a) $\frac{\pi}{6} \cdot \frac{180}{\pi} = 30^\circ$ b) $\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225^\circ$ c) $-\frac{3\pi}{5} \cdot \frac{180}{\pi} = -108^\circ$ d) $\frac{5\pi}{9} \cdot \frac{180}{\pi} = 100^\circ$ e) $\frac{8\pi}{5} \cdot \frac{180}{\pi} = 288^\circ$
3. $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \approx 0.866$	4. $\csc\left(-\frac{5\pi}{6}\right) = \frac{1}{\sin\left(-\frac{5\pi}{6}\right)} = -2$

5. cofunctions of complementary angles are equal, so $\frac{\pi}{6}+x=\frac{\pi}{2}$ $\left[\frac{\pi}{2}\text{ rad } = 90^\circ\right]$ $x=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$ rad	6. $\theta+\left(\theta+\frac{\pi}{3}\right)=\frac{\pi}{2}$ *[cofunctions]* $2\theta+\frac{\pi}{3}=\frac{\pi}{2}$ $2\theta=\frac{\pi}{6}$ $\theta=\frac{\pi}{12}$
7. $L=\frac{\pi}{4}\cdot 12=3\pi\approx 9.4$ inches	8. $L=2\cdot 4=8$ inches
9. $8\pi=\theta\cdot 10$ $\theta=\frac{4\pi}{5}$ rad	10. $65=5r$ $r=13$ feet

12.3 Unit Circle

1. (4)	2. (2)
3. (4)	4. (2) $(\cos 30^\circ, \sin 30^\circ)=\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
5.  $-\cos 70^\circ$	6.  $-\sin 50^\circ$
7. $\tan 50^\circ$ *[230° − 180° = 50°]*	8. $\sin 135^\circ=\sin 45^\circ=\frac{\sqrt{2}}{2}$
9. $\sin\frac{4\pi}{3}=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$	10. $\sin\frac{3\pi}{2}+\cos\frac{2\pi}{3}=-1+\left(-\frac{1}{2}\right)=-\frac{3}{2}$
11. $\cos 120^\circ=-\cos 60^\circ=-\frac{1}{2}$	12. $\sin 145^\circ=\sin 35^\circ\approx 0.574$
13. $(-\cos 30^\circ,-\sin 30^\circ)=\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	14. $400^\circ-360^\circ=40^\circ$ $(\cos 40^\circ,\sin 40^\circ)\approx(0.766, 0.643)$

12.4 Solve Simple Trigonometric Equations

1. $\cos\theta = \frac{1}{2}$ $\cos^{-1}\left(\frac{1}{2}\right) = 60°$	2. $\cos\theta = \frac{\sqrt{2}}{2}$ $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45°$
3. $\cos\theta = \frac{\sqrt{3}}{2}$ $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$ $m\angle\theta = 180° + 30° = 210°$	4. $\cos\theta = 0.6$ $\cos^{-1}(0.6) \approx 53°$ $m\angle\theta \approx 360° - 53° \approx 307°$
5. Points are in Quadrants I and IV $R = \cos^{-1}(0.25) \approx 75.5°$ $360° - 75.5° = 284.5°$ Solutions: 75.5° and 284.5°	6. Points are in Quadrants II and III $R = \cos^{-1}(0.75) \approx 41.4°$ $180° - 41.4° = 138.6°$ and $180° + 41.4° = 221.4°$ Solutions: 138.6° and 221.4°
7. Points are in Quadrants I and II $R = \sin^{-1}(0.99) \approx 81.9°$ $180° - 81.9° = 98.1°$ Solutions: 81.9° and 98.1°	8. Points are in Quadrants III and IV $R = \sin^{-1}\left(\frac{1}{3}\right) \approx 19.5°$ $180° + 19.5° = 199.5°$ and $360° - 19.5° = 340.5°$ Solutions: 199.5° and 340.5°

12.5 Circles of Any Radius

1. $\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$	2. $\tan\theta = \frac{y}{x} = \frac{-\sqrt{2}}{-\sqrt{2}} = 1$
3. $r = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5$ $\sin\theta = \frac{y}{r} = \frac{4}{5}$	4. $r = \sqrt{(-4)^2 + 0^2} = \sqrt{16} = 4$ $\sec\theta = \frac{r}{x} = \frac{4}{-4} = -1$
5. $r = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$ $\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$	6. $r = \sqrt{(-8)^2 + 5^2} = \sqrt{89}$ $\sec\theta = \frac{r}{x} = -\frac{\sqrt{89}}{8}$
7. $r = \sqrt{(-7)^2 + 5^2} = \sqrt{74}$ $\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{74}} = \frac{5\sqrt{74}}{74}$ $\cos\theta = \frac{x}{r} = -\frac{7}{\sqrt{74}} = -\frac{7\sqrt{74}}{74}$ $\tan\theta = \frac{y}{x} = -\frac{5}{7}$	8. $r = \sqrt{3^2 + (-7)^2} = \sqrt{58}$ $\csc\theta = \frac{r}{y} = -\frac{\sqrt{58}}{7}$ $\sec\theta = \frac{r}{x} = \frac{\sqrt{58}}{3}$ $\cot\theta = \frac{x}{y} = -\frac{3}{7}$

9. In Quadrant I, $x > 0$ and $y > 0$.
$\sin\theta = \frac{y}{r}$, so $y = 4$ and $r = 5$
$x^2 + 4^2 = 5^2$, so $x = 3$
$\cot\theta = \frac{x}{y} = \frac{3}{4}$

10. In Quadrant IV, $x > 0$ and $y < 0$.
$\cos\theta = \frac{x}{r}$, so $x = \sqrt{3}$ and $r = 2$
$\left(\sqrt{3}\right)^2 + y^2 = 2^2$, so $y = -1$
$\sin\theta = \frac{y}{r} = -\frac{1}{2}$

11. In Quadrant III, $x < 0$ and $y < 0$.
$\sec\theta = \frac{r}{x}$, so $x = -2$ and $r = 5$
$(-2)^2 + y^2 = 5^2$, so $y = -\sqrt{21}$
$\sin\theta = \frac{y}{r} = -\frac{\sqrt{21}}{5}$

12. $\cos\theta = \frac{x}{r}$, so $x = \sqrt{2}$ and $r = 3$
$\left(\sqrt{2}\right)^2 + y^2 = 3^2$, so $y = -\sqrt{7}$
[reject $y = \sqrt{7}$ because $y = \sin\theta < 0$]
$\tan\theta = \frac{y}{x} = -\frac{\sqrt{7}}{\sqrt{2}} = -\frac{\sqrt{14}}{2}$

13. In Quadrant III, $x < 0$ and $y < 0$.
$\tan\theta = \frac{y}{x}$, so $x = -5$ and $y = -3$
$5^2 + 3^2 = r^2$, so $r = \sqrt{34}$
[reject $r = -\sqrt{34}$ because $r > 0$]
$\sec\theta = \frac{r}{x} = -\frac{\sqrt{34}}{5}$

14. In Quadrant II, $x < 0$ and $y > 0$.
$\cot\theta = \frac{x}{y}$, so $x = -3\sqrt{2}$ and $y = 2$
$\left(-3\sqrt{2}\right)^2 + 2^2 = r^2$, so $r = \sqrt{22}$.
[reject $r = -\sqrt{22}$ because $r > 0$]
$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{22}} = \frac{2\sqrt{22}}{22} = \frac{\sqrt{22}}{11}$

15. $(x, y) = (r\cos\theta, r\sin\theta)$
$2\cos\frac{\pi}{6} = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$ and $2\sin\frac{\pi}{6} = 2\cdot\frac{1}{2} = 1$
$P\left(\sqrt{3}, 1\right)$

12.6 Pythagorean Identity

1. $\sin^2\theta = 1 - \cos^2\theta$
$\sin^2\theta = 1 - \left(\frac{1}{2}\right)^2$
$\sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$
$\sin\theta = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$
(sin is negative in Quadrant IV)

2. $\cos^2\theta = 1 - \sin^2\theta$
$\cos^2\theta = 1 - \left(-\frac{\sqrt{2}}{2}\right)^2$
$\cos^2\theta = 1 - \frac{1}{2} = \frac{1}{2}$
$\cos\theta = -\sqrt{\frac{1}{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$
(cos is negative in Quadrant III)

3. $\sin^2\theta = 1 - \cos^2\theta$
$\sin^2\theta = 1 - \left(\frac{1}{3}\right)^2$
$\sin^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$
$\sin\theta = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{3}$
(sin is positive in Quadrant I)

4. $\cos^2\theta = 1 - \sin^2\theta$
$\cos^2\theta = 1 - \left(\frac{\sqrt{3}}{4}\right)^2$
$\cos^2\theta = 1 - \frac{3}{16} = \frac{13}{16}$
$\cos\theta = -\sqrt{\frac{13}{16}} = -\frac{\sqrt{13}}{4}$
(cos is negative in Quadrant II)

5. If $\sec\theta = 2$, then $\cos\theta = \frac{1}{2}$ $\sin^2\theta = 1 - \left(\frac{1}{2}\right)^2$ $\sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$ $\sin\theta = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$ (sin is negative in Quadrant IV) $\csc\theta = \frac{1}{\sin\theta} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$	6. If $\csc\theta = -\sqrt{5}$, then $\sin\theta = -\frac{1}{\sqrt{5}}$ $\cos^2\theta = 1 - \left(-\frac{1}{\sqrt{5}}\right)^2$ $\cos^2\theta = 1 - \frac{1}{5} = \frac{4}{5}$ $\cos\theta = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}}$ (cos is negative in Quadrant III) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{1}{\sqrt{5}}}{-\frac{2}{\sqrt{5}}} = \frac{1}{2}$
7. $\cos^2\theta = 1 - \left(\frac{9}{10}\right)^2$ $\cos^2\theta = 1 - \frac{81}{100} = \frac{19}{100}$ $\cos\theta = \frac{\sqrt{19}}{10}$ (cos is positive)	8. $\sin^2\theta = 1 - \left(\frac{4}{9}\right)^2$ $\sin^2\theta = 1 - \frac{16}{81} = \frac{65}{81}$ $\sin\theta = -\frac{\sqrt{65}}{9}$ (sin is negative)

12.7 Simplify Trigonometric Expressions

1. $\sec x \cot x = \frac{1}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x} = \frac{1}{\sin x} = \csc x$	2. $\tan x \csc x = \frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cancel{\sin x}} = \frac{1}{\cos x} = \sec x$
3. $\sin x \cos x \tan x =$ $\sin x \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} = \sin^2 x$	4. $\frac{\sin\theta}{\csc\theta} = \frac{\sin\theta}{\frac{1}{\sin\theta}} = \sin^2\theta$
5. $\sin^2\theta \csc\theta = \sin^2\theta \cdot \frac{1}{\sin\theta} = \sin\theta$	6. $\frac{\tan\theta}{\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\sin\theta} = \frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}} = \frac{1}{\cos\theta} = \sec\theta$
7. $\frac{\cot x}{\csc x} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x} = \cos x$	8. $\frac{\cot x \sin x}{\sec x} = \frac{\frac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x}}{\frac{1}{\cos x}} = \cos^2 x$
9. $\frac{1 - \sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$	10. $\frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 1$
11. $\frac{\sin^2 x + \cos^2 x}{1 - \sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$	12. $\cos x\,(\cos x + 1) + \sin^2 x =$ $\cos^2 x + \cos x + \sin^2 x =$ $\cos x + 1$
13. $\cos x\,(\sec x - \cos x) =$ $\cos x\left(\frac{1}{\cos x} - \cos x\right) =$ $1 - \cos^2 x = \sin^2 x$	14. $\sin^2 x\,(1 + \cot^2 x) =$ $\sin^2 x\,(1 + \frac{\cos^2 x}{\sin^2 x}) =$ $\sin^2 x + \cos^2 x = 1$

15. $\frac{2-2\sin^2 x}{\cos x}=\frac{2(1-\sin^2 x)}{\cos x}=\frac{2\cos^2 x}{\cos x}=$ $2\cos x$	16. $\sec x-\tan x\sin x=$ $\frac{1}{\cos x}-\frac{\sin x}{\cos x}\cdot\sin x=\frac{1}{\cos x}-\frac{\sin^2 x}{\cos x}=$ $\frac{1-\sin^2 x}{\cos x}=\frac{\cos^2 x}{\cos x}=\cos x$
17. $\csc^2 x\,(1+\sin x)(1-\sin x)=$ $\frac{1}{\sin^2 x}(1-\sin^2 x)=\frac{\cos^2 x}{\sin^2 x}=\cot^2 x$	18. $\frac{\tan^2 x-\sec^2 x}{\cot^2 x-\csc^2 x}=\frac{\frac{\sin^2 x}{\cos^2 x}-\frac{1}{\cos^2 x}}{\frac{\cos^2 x}{\sin^2 x}-\frac{1}{\sin^2 x}}=\frac{\frac{\sin^2 x-1}{\cos^2 x}}{\frac{\cos^2 x-1}{\sin^2 x}}$ $=\frac{\frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}}}{\frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}}}=\frac{1}{1}=1$

12.8 Graphs of Parent Trig Functions

1. (4)	2. (3)
3. (3)	4. (4)
5. (2)	6. (4)

12.9 Trigonometric Transformations

1. amplitude is 2; period is 2π	
2. amplitude is 3; frequency is 2; each period has a length of π	
3. (3)	4. (2)
5. $p=\frac{2\pi}{\lvert 4\rvert}=\frac{\pi}{2}$	6. $f=\frac{2\pi}{\pi}=2$
7. $p=\frac{2\pi}{\lvert 2(-4)\rvert}=\frac{\pi}{4}$	8. $f=\frac{2\pi}{\frac{\pi}{6}}=2\pi\cdot\frac{6}{\pi}=12$
9. amplitude is $\frac{2}{3}$ frequency is 4 each period is $\frac{2\pi}{4}=\frac{\pi}{2}$ (or 90°) long	10. midline is $y=4$ frequency is $\lvert 2\cdot 3\rvert=6$ each period is $\frac{2\pi}{6}=\frac{\pi}{3}$ (or 60°) long
11. 3; There is no vertical shift and the amplitude is 3.	12. $-\frac{1}{3}$; There is no vertical shift and the amplitude is $\frac{1}{3}$.

13. 6; The amplitude is 2 plus there is a vertical shift of 4.

14. -5; The amplitude is 2 and there is a vertical shift of 3 units down.
[The negation of a does not affect the amplitude; it merely reflects the graph over the midline.]

15. midline is $y = 0$
amplitude is 4
period is π
phase shift is $\frac{\pi}{2}$ to the right

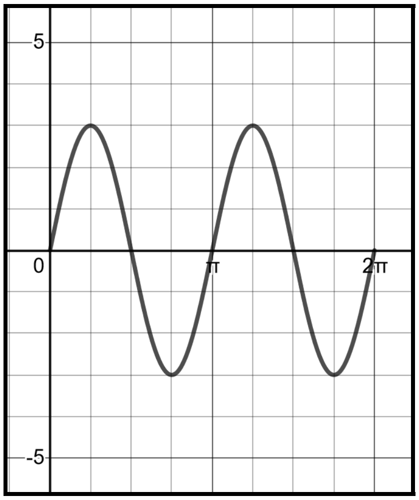
16. midline is $y = -5$
amplitude is $\frac{1}{2}$
period is $\frac{2\pi}{3}$
phase shift is $\frac{\pi}{6}$ to the left

17. (2)

18. If the sine graph is shifted left by $\frac{\pi}{2}$, then it will coincide with the cosine graph, so $h = \frac{\pi}{2}$.

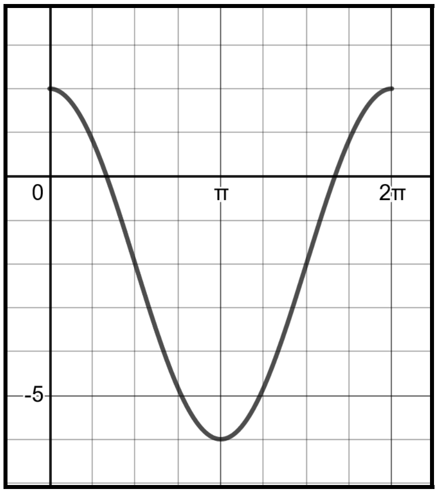
12.10 Graph Trigonometric Functions

1.



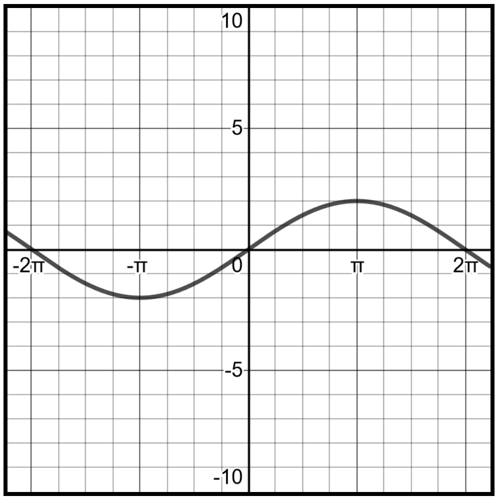
$y = 3\sin 2x$

2.

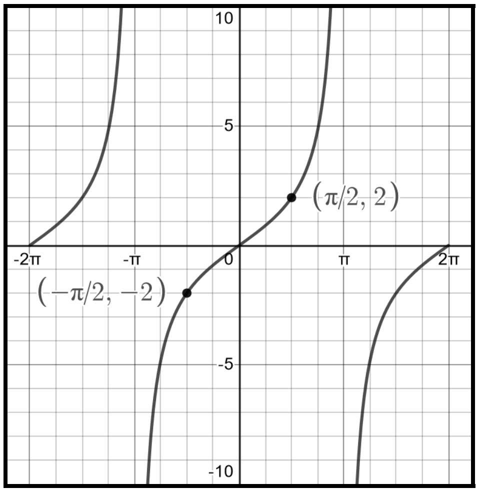


$y = 4\cos x - 2$

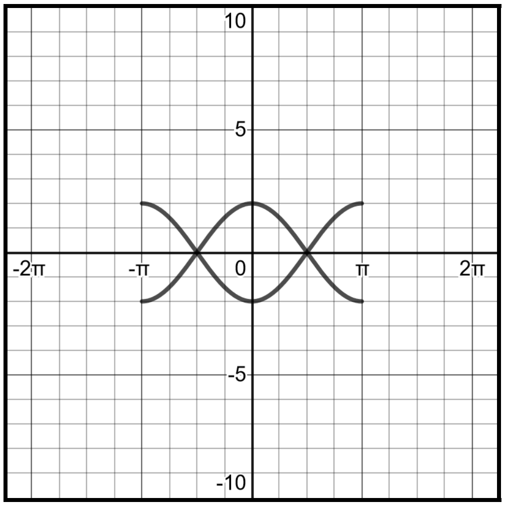
3.



4.



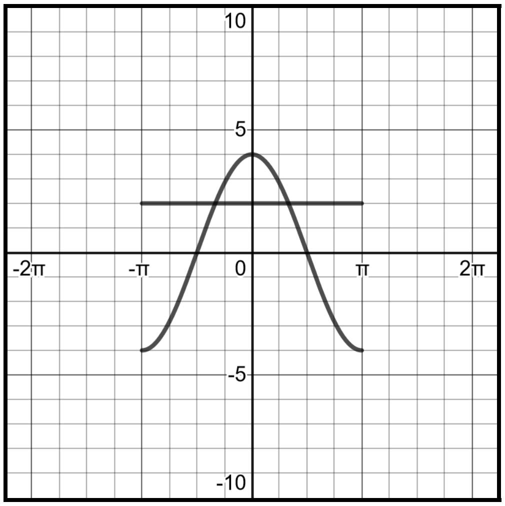
5.



2 points of intersection.

$g\left(\frac{\pi}{6}\right) = -2\cos\frac{\pi}{6} = -2 \cdot \frac{\sqrt{3}}{2} = -\sqrt{3}$

6.



Solve for x:

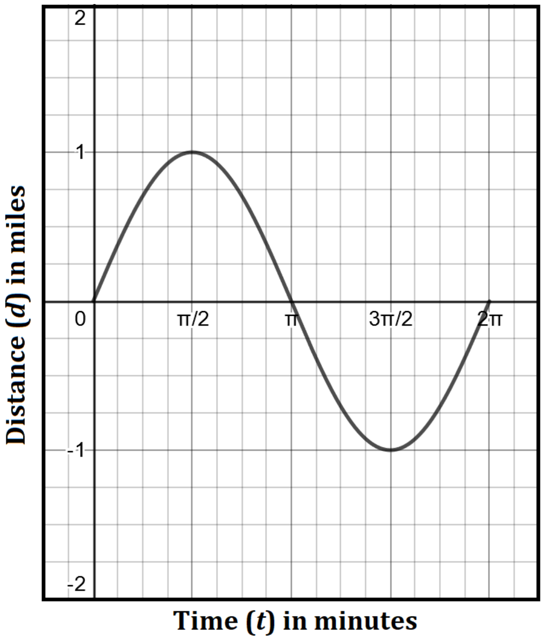
$4\cos x \geq 2$

$\cos x \geq \frac{1}{2}$

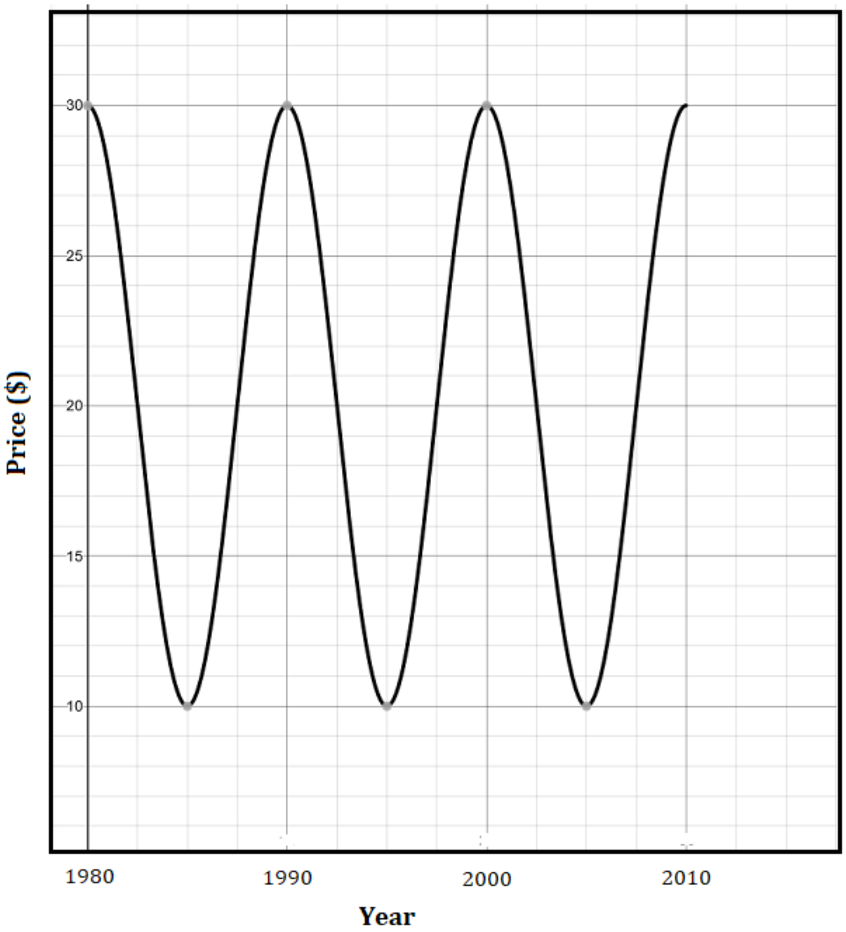
$x \geq \cos^{-1}\frac{1}{2}$

$-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

12.11 Model Trigonometric Functions

1. (4)	2. $30 + 27 = 57$ feet.
3. $\frac{2\pi}{\frac{\pi}{3}} = 6$ seconds	4. a) 1 cycle is $\frac{2\pi}{\frac{\pi}{5}} = 10$ secs, so a 40-sec ride would complete 4 revolutions. b) Radius is 20 (the amplitude). c) The base is $24 - 20 = 4$ feet off the ground.
5.  $d(t) = \sin t$	

6.



The minimum value of the cosine function is –1.
$10(-1) + 20 = 10$, so the minimum price is $10.

The price is $10 when

$10\cos\left(\frac{\pi}{5}t\right) + 20 = 10$

$10\cos\left(\frac{\pi}{5}t\right) = -10$

$\cos\left(\frac{\pi}{5}t\right) = -1$

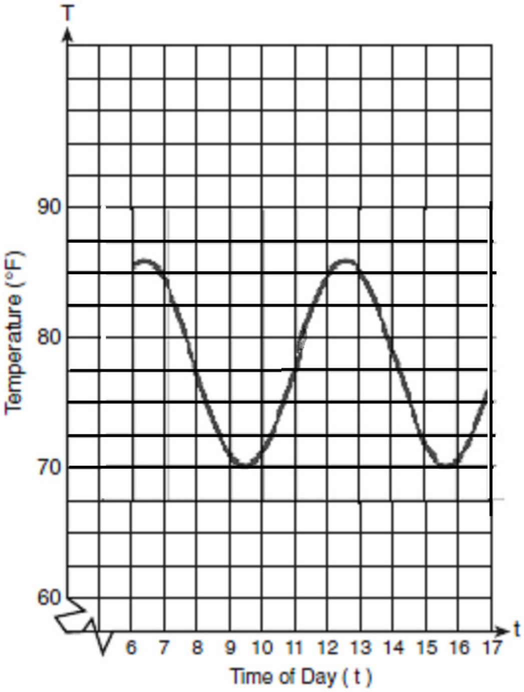
$\cos^{-1}(-1) = \pi$,

so $\frac{\pi}{5}t = \pi$ or $t = 5$ (year 1985).

Each period is $\frac{2\pi}{\frac{\pi}{5}} = 10$, so the price cycles every 10 years.

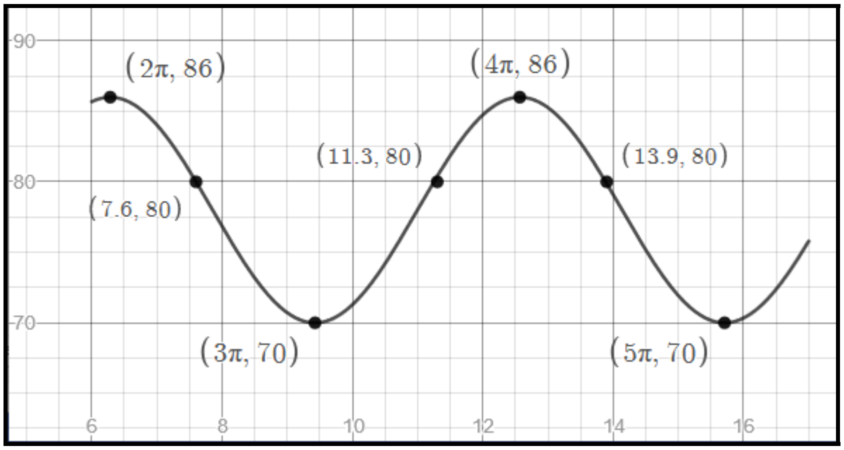
The minimum price of $10 occurs in 1985, 1995, and 2005.

7.



$8 \cos t + 78 = 80$
$8 \cos t = 2$
$\cos t = \frac{1}{4}$
$\cos^{-1} \frac{1}{4} \approx 1.3$ (left of the interval)

Since the period is 2π, the graph intersects
$y = 80$ at:
$x = 2\pi + 1.3 \approx 7.6$
$x = 4\pi - 1.3 \approx 11.3$
$x = 4\pi + 1.3 \approx 13.9$
(See below)



Chapter 13. Properties of Functions

13.1 Even and Odd Function Graphs

1. even (all even powers of x including the constant)	2. neither (odd powers of x except the constant)
3. odd	4. neither
5. even	6. odd
7. (2)	8. (1)

13.2 Algebraically Determine Even or Odd [CC]

1. even $f(-x) = (-x)^4 - 3(-x)^2 + 7 =$ $x^4 - 3x^2 + 7 = f(x)$	2. neither $f(-x) = (-x)^5 - 3(-x)^3 + 7 =$ $-x^5 + 3x^3 + 7$
3. odd $f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x)$	4. even $f(-x) = \left\lvert\frac{1}{-x}\right\rvert = \frac{1}{x} = f(x)$
5. neither $f(-x) = 2^{-x} - 1 = \frac{1}{2^x} - 1$	6. odd $f(-x) = \frac{(-x)^2 + 4}{(-x)^3 - (-x)} = \frac{x^2 + 4}{-x^3 + x} =$ $\frac{x^2 + 4}{-(x^3 - x)} = -\left(\frac{x^2 + 4}{x^3 - x}\right) = -f(x)$

13.3 Inverse Functions

1. (2)	
2. $x = 5y + 2$ $x - 2 = 5y$ $\frac{x - 2}{5} = y$ $f^{-1}(x) = \frac{x - 2}{5}$	3. $x = \frac{2y + 5}{3}$ $3x = 2y + 5$ $3x - 5 = 2y$ $\frac{3x - 5}{2} = y$ $f^{-1}(x) = \frac{3x - 5}{2}$

4. $x = -\frac{2}{3}y$ $-\frac{3}{2}x = y$ $f^{-1}(x) = -\frac{3}{2}x$	5. $x = \frac{1}{3}y + 2$ $x - 2 = \frac{1}{3}y$ $3(x - 2) = y$ $3x - 6 = y$ $f^{-1}(x) = 3x - 6$
6. $x = y^2 - 5$ $x + 5 = y^2$ $\pm\sqrt{x + 5} = y$ $f^{-1}(x) = \pm\sqrt{x + 5}$	7. $x = (y - 2)^3$ $\sqrt[3]{x} = y - 2$ $\sqrt[3]{x} + 2 = y$ $f^{-1}(x) = \sqrt[3]{x} + 2$
8. $x = 3^y + 1$ $x - 1 = 3^y$ $\log_3(x - 1) = \log_3 3^y$ $\log_3(x - 1) = y$ $f^{-1}(x) = \log_3(x - 1)$	9. $x = \log(y - 2) + 3$ $x - 3 = \log\,(y - 2)$ $10^{(x-3)} = y - 2$ $10^{(x-3)} + 2 = y$ $f^{-1}(x) = 10^{(x-3)} + 2$
10. $x = \dfrac{y}{y + 2}$ $x(y + 2) = y$ $xy + 2x = y$ $2x = y - xy$ $2x = y(1 - x)$ $\dfrac{2x}{1 - x} = y$ $f^{-1}(x) = \dfrac{2x}{1 - x}$	11. $x = \sin y - 1$ $x + 1 = \sin y$ $\arcsin(x + 1) = y$ $p^{-1}(x) = \arcsin(x + 1)$ Note: parentheses are important here.

12. a) $x = \sqrt{2}$

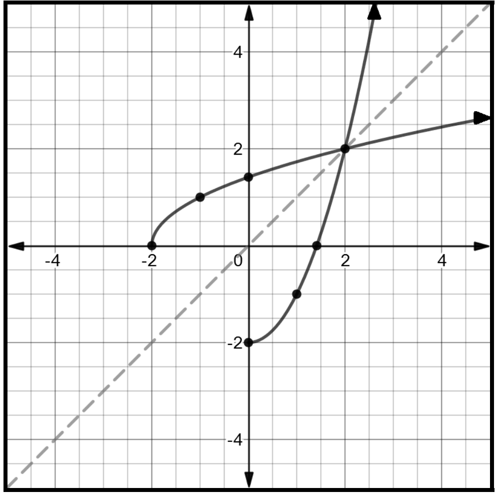
b) $x = y^2 - 2$

$x + 2 = y^2$

$\sqrt{x+2} = y$

$f^{-1}(x) = \sqrt{x+2}$

c)



13. a) $2\sqrt{x+1} - 1 = 0$

$\sqrt{x+1} = \frac{1}{2}$

$x + 1 = \frac{1}{4}$

$x = -\frac{3}{4}$

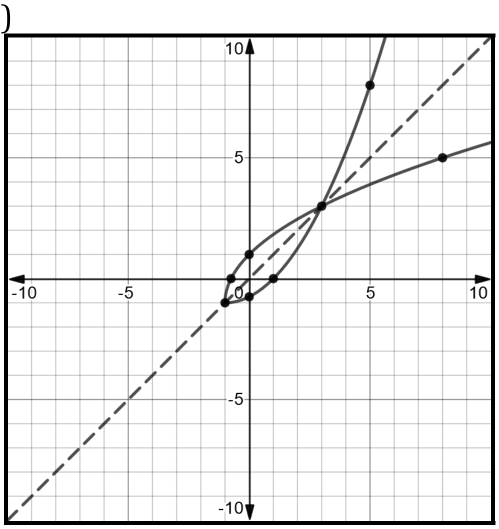
b) $x = 2\sqrt{y+1} - 1$

$\frac{x+1}{2} = \sqrt{y+1}$

$y = \left(\frac{x+1}{2}\right)^2 - 1$

$f^{-1}(x) = \left(\frac{x+1}{2}\right)^2 - 1$

c)



13.4 Average Rate of Change

1. a) $\frac{139-79}{80-60} = \frac{60}{20} = 3$ b) $\frac{49-139}{90-80} = \frac{-90}{10} = -9$ c) $\frac{49-79}{90-60} = \frac{-30}{30} = -1$

2. a) $\frac{5.06-3.91}{1999-1987} = \frac{1.15}{12} \approx 0.096$ b) $\frac{7.50-5.06}{2009-1999} = \frac{2.44}{10} \approx 0.244$

3. $f(5) = 5^2 + 2 = 27$

$f(15) = 15^2 + 2 = 227$

$R = \frac{227-27}{15-5} = \frac{200}{10} = 20$

4. $f(-3) = (-3)^2 + 10(-3) + 16 = -5$

$f(3) = 3^2 + 10(3) + 16 = 55$

$R = \frac{55-(-5)}{3-(-3)} = \frac{60}{6} = 10$

5. $f(-1) = (-1)^4 + 2(-1)^3 = -1$

$f(1) = 1^4 + 2(1^3) = 3$

$R = \frac{3-(-1)}{1-(-1)} = \frac{4}{2} = 2$

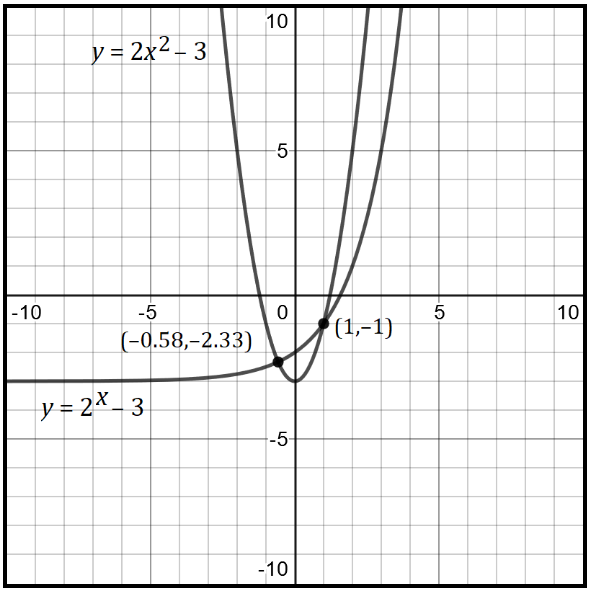
6. $f\left(\frac{\pi}{2}\right) = 2 \cdot 1 = 2$

$f\left(\frac{3\pi}{2}\right) = 2 \cdot (-1) = -2$

$R = \frac{-2-2}{\frac{3\pi}{2} - \frac{\pi}{2}} = -\frac{4}{\pi}$

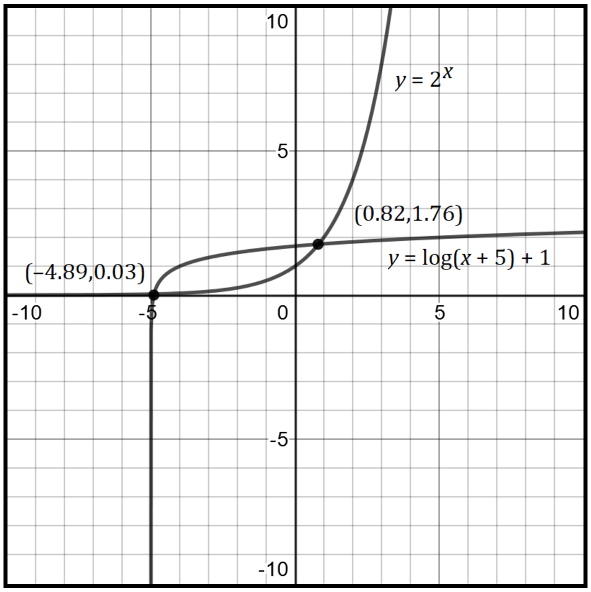
13.5 Solutions to Equation of Two Functions

1.



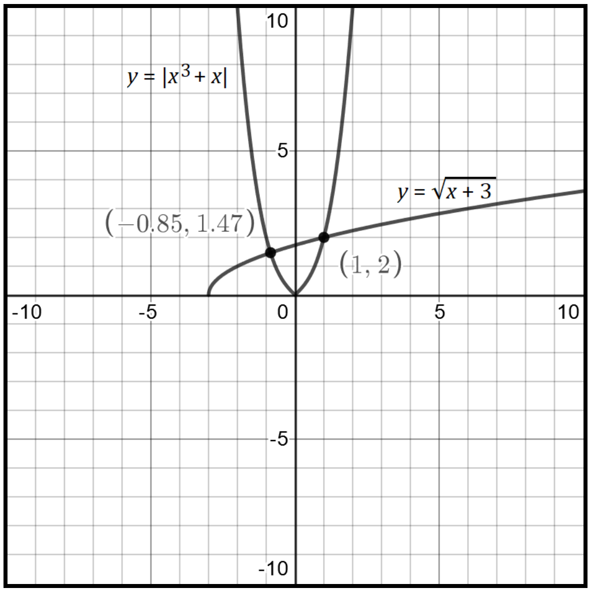
Solutions: $\{-0.58, 1\}$

2.



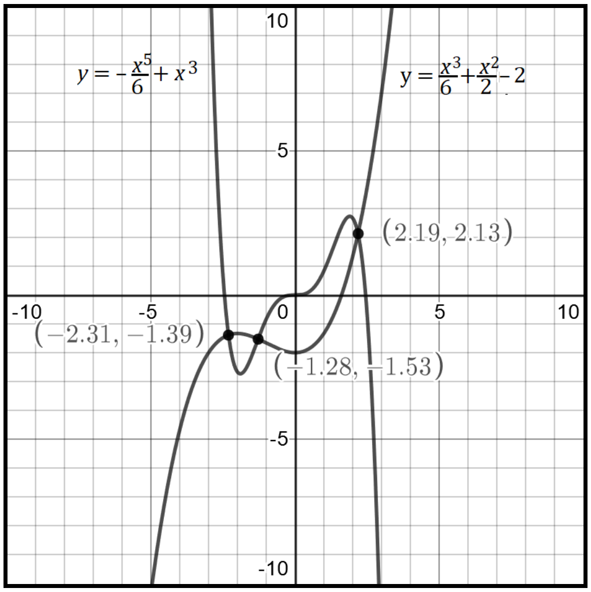
Solutions: $\{-4.89, 0.82\}$

3.



Solutions: $\{-0.85, 1\}$

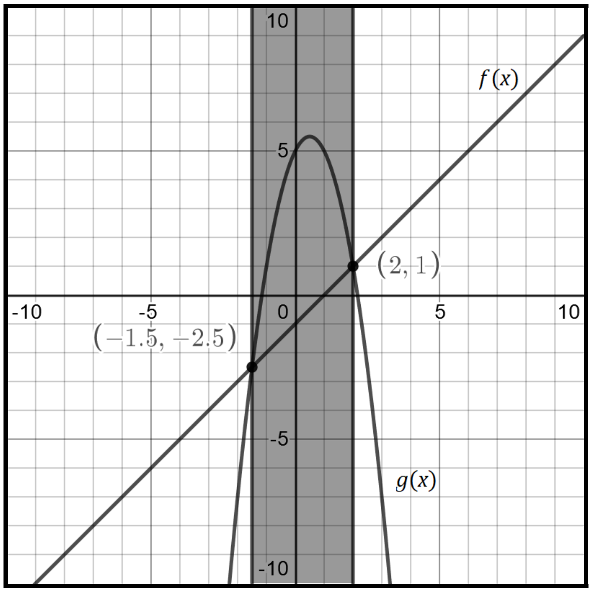
4.



Solutions: $\{-2.31, -1.28, 2.19\}$

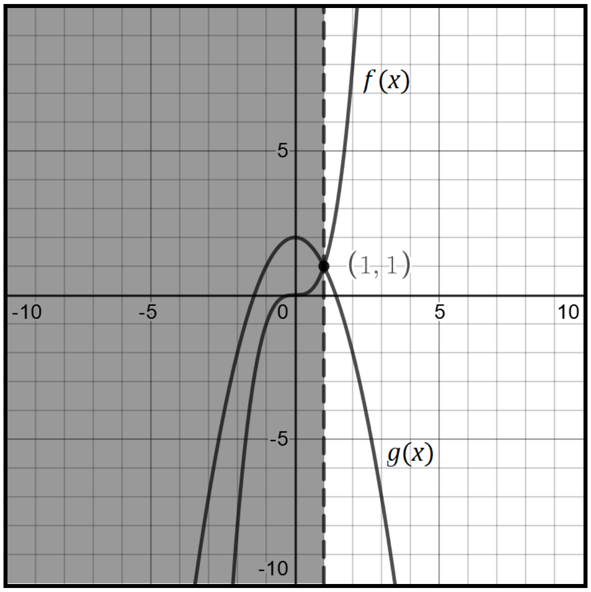
13.6 Solutions to Inequality of Two Functions [NG]

1.



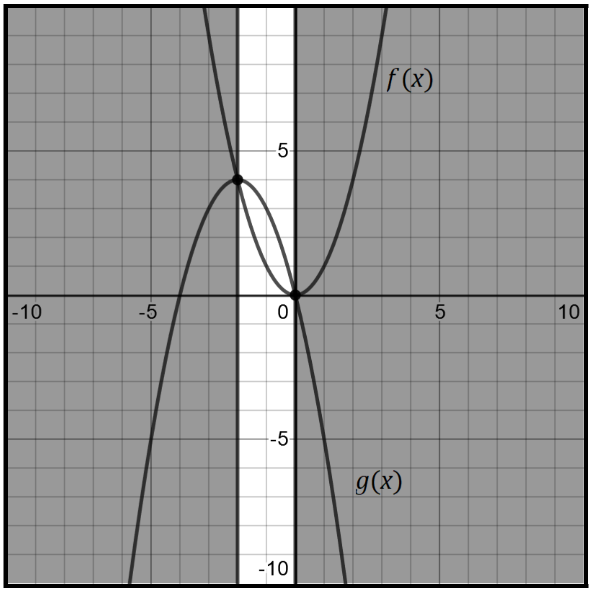
Solution: $-1.5 \leq x \leq 2$

2.



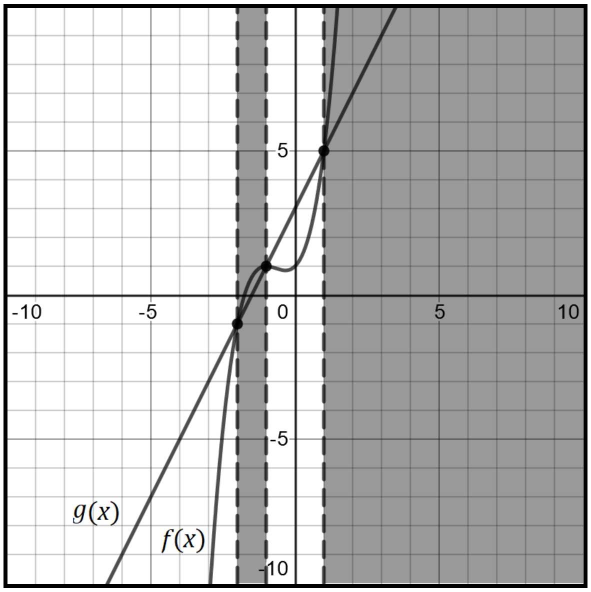
Solution: $x < 1$

3.



Solution: $x \leq -2$ or $x \geq 0$

4.



Solution: $-2 < x < -1$ or $x > 1$

Chapter 14. Sequences and Series

14.1 Arithmetic Sequences

1. $a_1 = 32$ and $d = 5$
$a_n = a_1 + (n-1)d$
$a_n = 32 + (n-1) \cdot 5$
$a_n = 32 + 5n - 5$
$a_n = 5n + 27$

2. $a_1 = 24$ and $d = -7$
$a_n = a_1 + (n-1)d$
$a_n = 24 + (n-1)(-7)$
$a_n = 24 - 7n + 7$
$a_n = -7n + 31$

3. $a_1 = -1$ and $d = \frac{1}{2}$
$a_n = a_1 + (n-1)d$
$a_n = -1 + (n-1) \cdot \frac{1}{2}$
$a_n = -1 + \frac{1}{2}n - \frac{1}{2}$
$a_n = \frac{1}{2}n - \frac{3}{2}$

4. $a_n = a_1 + (n-1)d$
$a_8 = 21 + (8-1) \cdot 9 = 84$

5. $a_n = a_1 + (n-1)d$
$a_{12} = 16 + (12-1) \cdot 11 = 137$

6. $a_n = a_1 + (n-1)d$
$a_9 = 35 + (9-1) \cdot (-5) = -5$

7. $a_n = a_1 + (n-1)d$
$a_{27} = 5 + (27-1) \cdot 3 = 83$

8. $a_n = a_1 + (n-1)d$
$a_{20} = -8 + (20-1) \cdot 6 = 106$

9. $(6, 10)$ and $(21, 55)$
$d = \frac{55-10}{21-6} = \frac{45}{15} = 3$
$a_n = a_1 + (n-1)d$
$10 = a_1 + (6-1) \cdot 3$
$10 = a_1 + 15$
$a_1 = -5$
$a_n = a_1 + (n-1)d$
$a_n = -5 + (n-1) \cdot 3$
$a_n = -5 + 3n - 3$
$a_n = 3n - 8$

10. $(4, -23)$ and $(22, 49)$
$d = \frac{49-(-23)}{22-4} = \frac{72}{18} = 4$
$a_n = a_1 + (n-1)d$
$-23 = a_1 + (4-1) \cdot 4$
$-23 = a_1 + 12$
$a_1 = -35$
$a_n = a_1 + (n-1)d$
$a_n = -35 + (n-1) \cdot 4$
$a_n = -35 + 4n - 4$
$a_n = 4n - 39$

11. $(3, 29)$ and $(15, 53)$
a) $d = \frac{53-29}{15-3} = \frac{24}{12} = 2$
$a_n = a_1 + (n-1)d$
$29 = a_1 + (3-1) \cdot 2$
$29 = a_1 + 4$
$a_1 = 25$
$a_n = 25 + (n-1) \cdot 2$
$a_n = 2n + 23$

b) $a_{10} = 2(10) + 23 = 43$
There are 43 seats in the tenth row.

14.2 Geometric Sequences

1. $a_n = a_1r^{n-1}$ $a_n = 6(3)^{n-1}$ $a_n = \frac{6(3)^n}{3}$ $a_n = 2(3)^n$	2. $a_n = a_1r^{n-1}$ $a_n = 12(0.5)^{n-1}$ $a_n = \frac{12(0.5)^n}{0.5}$ $a_n = 24(0.5)^n$
3. $a_n = a_1r^{n-1}$ $a_n = -2(-4)^{n-1}$ $a_n = \frac{-2(-4)^n}{-4}$ $a_n = 0.5(-4)^n$	4. $a_n = a_1r^{n-1}$ $a_n = -1(-2)^{n-1}$ $a_n = \frac{-1(-2)^n}{-2}$ $a_n = 0.5(-2)^n$
5. $a_n = 4(2.5)^{n-1}$ $a_n = \frac{4(2.5)^n}{2.5}$ $a_n = 1.6(2.5)^n$	6. $a_n = a_1r^{n-1}$ $a_9 = 16(2)^{9-1} = 16(2)^8 = 4{,}096$
7. $a_n = a_1r^{n-1}$ $a_5 = 12(-3)^4 = 972$	8. $a_n = a_1r^{n-1}$ $a_7 = 8(1.5)^6 = 91.125$
9. $a_n = a_1r^{n-1}$ $a_{15} = 5(-2)^{14} = 81{,}920$	10. $a_n = a_1r^{n-1}$ $a_7 = 6\left(-\frac{1}{2}\right)^6 = 6\left(\frac{1}{2^6}\right) = \frac{6}{64} = \frac{3}{32}$ (Using the calculator, $a_7 = 0.09375$.)
11. a) $r^{6-3} = \frac{3645}{135}$ $r^3 = 27$ $r = \sqrt[3]{27} = 3$ $a_6r^4 = a_{10}$ $a_{10} = 3645(3)^4 = 295{,}245$ b) $a_n = a_1r^{n-1}$ $135 = a_1(3)^{3-1}$ $135 = a_1(9)$ $a_1 = 15$ $a_n = 15(3)^{n-1} = \frac{15(3)^n}{3} = 5(3)^n$	12. a) $r^{10-5} = \frac{112{,}640}{3{,}520}$ $r^5 = 32$ $r = \sqrt[5]{32} = 2$ $a_5r^3 = a_8$ $a_8 = 3520(2)^3 = 28{,}160$ b) $a_n = a_1r^{n-1}$ $3520 = a_1(2)^{5-1}$ $3520 = a_1(16)$ $a_1 = 220$ $a_n = 220(2)^{n-1} = \frac{220(2)^n}{2} = 110(2)^n$

14.3 Recursively Defines Sequences

1. $a_1 = 6$ $a_n = a_{n-1} + 4$	2. $a_1 = 8$ $a_n = 3a_{n-1}$
3. $a_1 = 16$ $a_n = -0.5a_{n-1}$	4. $a_1 = 3$ $a_2 = 2(3) + 1 = 7$ $a_3 = 2(7) + 1 = 15$ $a_4 = 2(15) + 1 = 31$ $3, 7, 15, 31$
5. $a_1 = 3$ $a_2 = 2(3) - 4 = 2$ $a_3 = 2(2) - 4 = 0$ $a_4 = 2(0) - 4 = -4$ $3, 2, 0, -4$	6. $a_2 = 3 + 2 = 5$ $a_3 = 5 + 3 = 8$ $a_4 = 8 + 4 = 12$ $3, 5, 8, 12$ Neither; there is no common difference nor common ratio.
7. $a_2 = 3(2) + 2 = 8$ $a_3 = 3(8) + 3 = 27$ $a_4 = 3(27) + 4 = 85$ $2, 8, 27, 85$	8. $a_2 = 2(-3) - 2 = -8$ $a_3 = 2(-8) - 3 = -19$ $a_4 = 2(-19) - 4 = -42$ $-3, -8, -19, -42$
9. $a_1 = 2$ $a_2 = 3$ $a_n = a_{n-2} \cdot a_{n-1}$ for $n > 2$	10. a) $a_1 = 40$ $a_2 = 8$ $a_n = \frac{1}{2}(a_{n-2} + a_{n-1})$, for $n > 2$ b) $40, 8, 24, 16, 20, 18, 19, \ldots$ First odd term is a_7.

14.4 Sigma Notation

1. $3(2) + 3(3) + 3(4) + 3(5) =$ $6 + 9 + 12 + 15 = 42$	2. $(2^2 - 1) + (3^2 - 1) + (4^2 - 1) + (5^2 - 1)$ $= 3 + 8 + 15 + 24 = 50$
3. $-2(1) + 100 - 2(2) + 100 - 2(3) +$ $100 - 2(4) + 100 - 2(5) + 100 =$ $-30 + 500 = 470$	4. $(2^3 + 2) + (2^4 + 2) + (2^5 + 2) =$ $10 + 18 + 34 = 62$
5. $(1^2 + 1) + (2^2 + 2) + (3^2 + 3) +$ $(4^2 + 4) + (5^2 + 5) =$ $2 + 6 + 12 + 20 + 30 = 70$	6. $(2 - 1)^2 + (2 - 2)^2 + (2 - 3)^2 =$ $1 + 0 + 1 = 2$
7. $3(2)^0 + 3(2)^1 + 3(2)^2 =$ $3 + 6 + 12 = 21$	8. $(-3^2 + 3) + (-4^2 + 4) + (-5^2 + 5) =$ $-6 - 12 - 20 = -38$
9. $(-1^4 - 1) + (-2^4 - 2) + (-3^4 - 3) =$ $-2 - 18 - 84 = -104$	10. $(2 \cdot 1 + 1)^0 + (2 \cdot 2 + 1)^1 +$ $(2 \cdot 3 + 1)^2 = 1 + 5 + 49 = 55$
11. (3)	12. (4)

13. (2)	14. (4)
15. $13567(0) + 294 = 294$ $13567(1) + 294 = 13861$ $13567(2) + 294 = 27428$ Sum is \$41,583	

14.5 Arithmetic Series

1. $\sum_{k=1}^{5}(2k+1)$ $S_5 = \frac{5(3 + 11)}{2} = 35$	2. $\sum_{k=1}^{7}(4k+3)$ $S_7 = \frac{7(7 + 31)}{2} = 133$
3. $\sum_{k=1}^{6}(4k+21)$ $S_6 = \frac{6(25 + 45)}{2} = 210$	4. $\sum_{k=1}^{n}(2k+3)$ $2n + 3 = 43$ $n = 20$ $\sum_{k=1}^{20}(2k+3)$
5. $\sum_{k=1}^{n}(2k-1)$ $2n - 1 = 39$ $n = 20$ $\sum_{k=1}^{20}(2k-1)$	6. $\sum_{k=1}^{n}(12k-1)$ $12n - 1 = 119$ $n = 10$ $\sum_{k=1}^{10}(12k-1) = 650$
7. $\sum_{k=1}^{n}7k$ $7n = 105$ $n = 15$ $\sum_{k=1}^{15}7k = 840$	8. $\sum_{k=1}^{n}(8k+10)$ $8n + 10 = 122$ $n = 14$ $\sum_{k=1}^{14}(8k+10) = 980$
9. $S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$ $S_{19} = \frac{19(3 + 3 + (19 - 1) \cdot 7)}{2} = 1{,}254$	10. $S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$ $S_{20} = \frac{20(5 + 5 + (20 - 1) \cdot 9)}{2} = 1{,}810$
11. $S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$ $S_{30} = \frac{30(15 + 15 + (30 - 1) \cdot 2)}{2} = 1{,}320$	12. $S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$ $S_{21} = \frac{21(18 + 18 + (21 - 1) \cdot 2)}{2} = 798$

14.6 Geometric Series

1. $\sum_{k=1}^{5} 7(3)^{k-1}$	2. $\sum_{n=1}^{6} (-2)^{n-1}$
3. $\sum_{k=1}^{4} 200\left(\frac{1}{2}\right)^{k-1}$	4. $S_7 = \frac{9 - 9(4)^7}{1 - 4} = 49{,}149$ $\sum_{k=1}^{7} 9(4)^{k-1} = 49{,}149$
5. $S_8 = \frac{3 - 3(-4)^8}{1 - (-4)} = -39{,}321$	6. $\sum_{k=1}^{n} 10(2)^{k-1}$ $10(2)^{n-1} = 5120$ $2^{n-1} = 512$ $\log 2^{n-1} = \log 512$ $(n-1)\log 2 = \log 512$ $n - 1 = \frac{\log 512}{\log 2} = 9$ $n = 10$ $\sum_{k=1}^{10} 10(2)^{k-1}$
7. $\sum_{k=1}^{n} \frac{1}{2}(2)^{k-1}$ $\frac{1}{2}(2)^{n-1} = 1024$ $2^{n-1} = 2048$ $\log 2^{n-1} = \log 2048$ $n - 1 = \frac{\log 2048}{\log 2} = 11$ $n = 12$ $\sum_{k=1}^{12} \frac{1}{2}(2)^{k-1}$	8. $\sum_{k=1}^{n} 6(5)^{k-1}$ $6(5)^{n-1} = 93{,}750$ $5^{n-1} = 15{,}625$ $\log 5^{n-1} = \log 15{,}625$ $n - 1 = \frac{\log 15{,}625}{\log 5} = 6$ $n = 7$ $\sum_{k=1}^{7} 6(5)^{k-1} = 117{,}186$
9. $a_1 = 9$ and $r = -3$ a) $\sum_{n=1}^{7} 9(-3)^{n-1} = 4{,}923$ b) $S_7 = \frac{9 - 9(-3)^7}{1 - (-3)} = 4{,}923$	
10. (1)	

Chapter 15. Probability

15.1 Theoretical and Empirical Probability

1. $\frac{1}{4}$	2. $\frac{6}{10} = \frac{3}{5}$
3. $\frac{6}{22} = \frac{3}{11}$	4. $\frac{3}{6} = \frac{1}{2}$
5. $\frac{1}{6}$	6. $\frac{23}{29}$
7. $\frac{6}{20} = \frac{3}{10}$	8. $\frac{13}{52} = \frac{1}{4}$
9. $\frac{5}{8}$	10. $P(red) = \frac{30}{90}$ $P(white) = \frac{31}{90}$ $P(blue) = \frac{29}{90}$ White is the most likely to be picked.
11. $\frac{2{,}000}{80{,}000} = \frac{1}{40}$	12. $\frac{8}{20} = \frac{2}{5}$
13. The trials in this case are 100 products per month for 10 months, or 1,000. The empirical probability of a faulty bulb is $\frac{20}{1000} = \frac{1}{50}$.	14. $20\% = \frac{2}{10}$. There are 10 numbers from 0 to 9, so any two numbers (such as 0 and 1) can represent the event occurring.

15.2 Probability Involving And or Or

1. $\frac{6}{11}$	2. $\frac{4}{5}$
3. $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$	4. $P(pen\ or\ red) =$ $P(pen) + P(red) - P(red\ pen) =$ $\frac{6}{14} + \frac{9}{14} - \frac{4}{14} = \frac{11}{14}$

5. a) $P(A\ and\ B) = P(A \cap B) =$

$$P(\{5, 8\}) = \frac{2}{10} = \frac{1}{5}$$

b) $P(A\ or\ B) = P(A \cup B) =$

$$P(\{2, 3, 4, 5, 7, 8, 9\}) = \frac{7}{10}$$

6. $P(A) = 0.05, P(B) = 0.08$, and $P(A\ and\ B) = 0.004$

a) not mutually exclusive because $P(A\ and\ B) \neq 0$

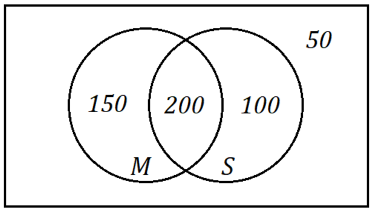
b) $P(A\ or\ B) = 0.05 + 0.08 - 0.004 = 0.126$

7. $P(G\ or\ A)$

$$= P(G) + P(A) - P(G\ and\ A)$$

$$= \frac{11}{20} + \frac{9}{20} - \frac{5}{20} = \frac{15}{20} = \frac{3}{4}$$

8.



$$\frac{50}{500} = \frac{1}{10}$$

9. $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$P(A\ or\ B) + P(A\ and\ B) = P(A) + P(B)$ *[add $P(A\ and\ B)$ to both sides]*

$P(A\ and\ B) = P(A) + P(B) - P(A\ or\ B)$ *[subtract $P(A\ or\ B)$ from both sides]*

15.3 Two-Way Frequency Tables

1. a) $\frac{15}{113} \approx 13.3\%$ of the students are undecided.

b) $\frac{31}{60} \approx 51.7\%$ of the 9th graders are watching.

2. Given data in bold below.

	Coca-Cola	Sprite	Total
Table	16	**14**	30
Garbage	34	8	**42**
Total	**50**	22	**72**

3. a) $P(F) = \frac{72}{240} = \frac{3}{10}$ *[from the Total row]*

b) $P(C) = \frac{80}{240} = \frac{1}{3}$ *[from the Total column]*

c) $P(F|C) = \frac{24}{80} = \frac{3}{10}$ *[from the first row]*

d) $P(C|F) = \frac{24}{72} = \frac{1}{3}$ *[from the first column]*

e) $P(C\ and\ F) = \frac{24}{240} = \frac{1}{10}$ *[from the one cell and the grand total]*

f) $P(F|C) = P(F) = \frac{3}{10}$ and $P(C|F) = P(C) = \frac{1}{3}$, so they appear to be independent.

4. It is helpful to calculate the totals first:

	Dogs	Cats	Rabbits	Total
Girls	53	72	25	150
Boys	62	28	40	130
Total	115	100	65	280

a) $P(G|R) = \frac{25}{65} = \frac{5}{13}$.

b) $P(R|G) = \frac{25}{150} = \frac{1}{6}$.

c) $P(B|D \text{ or } C) = \frac{62+28}{115+100} = \frac{90}{215} = \frac{18}{43}$ *[from the first two columns]*

15.4 Series of Events [CC]

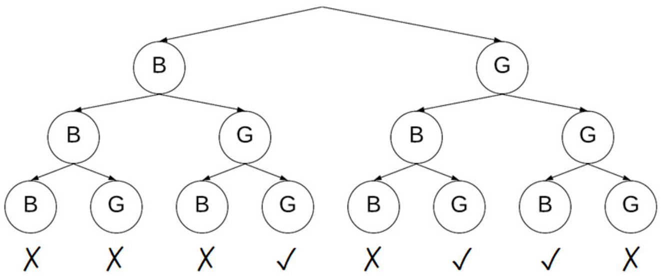
1. $\frac{1}{3} \times \frac{3}{7} = \frac{1}{7}$

2. $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

3. $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

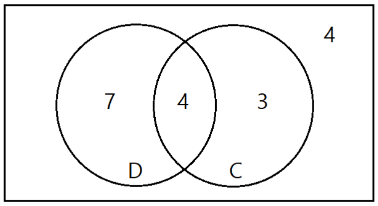
4. $0.95 \times 0.93 \times 0.98 \approx 87\%$

5.



a) $\frac{3}{8}$ (see check marks above) b) $\frac{7}{8}$ (all except the first leaf)

6.



a) 3 b) 4

7. $P(\text{at least one blue}) =$

$1 - P(\text{red or white on all 5 picks}) =$

$1 - \left(\frac{2}{3}\right)^5 = 1 - \frac{32}{243} = \frac{211}{243} \approx 87\%$

8. $\frac{1}{20} \times \frac{1}{19} = \frac{1}{380}$

9. $\frac{5}{7} \times \frac{2}{6} = \frac{5}{21}$

10. $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

11. $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

12. $P(M|S) = \frac{P(S\ and\ M)}{P(S)} = \frac{\frac{3}{10}}{\frac{3}{5}} = \frac{15}{30} = \frac{1}{2}$

13. $P(H_1\ and\ H_2) = P(H_1) \cdot P(H_2|H_1) =$
    $\frac{13}{52} \times \frac{12}{51} = \frac{156}{2652} = \frac{1}{17}$

14. $P(same\ suit) =$
    $P(2Hs\ or\ 2Ds\ or\ 2Cs\ or\ 2Ss) =$
    $\frac{1}{17} + \frac{1}{17} + \frac{1}{17} + \frac{1}{17} = \frac{4}{17}$

15. a) $\frac{10}{25} \times \frac{10}{25} \times \frac{10}{25} \times \frac{10}{25} \times \frac{10}{25} = \left(\frac{10}{25}\right)^5 = \frac{32}{3125}$
    b) $\frac{10}{25} \times \frac{9}{24} \times \frac{8}{23} \times \frac{7}{22} \times \frac{6}{21} = \frac{6}{1265}$

16. Let A = the patient has arthritis and H = the patient has hay fever.
    We want to find $P(A|H)$.
    $P(A) = 0.10$, $P(H) = 0.05$, and $P(H|A) = 0.07$
    $P(A|H) = \frac{P(A\ and\ H)}{P(H)} = \frac{P(A) \times P(H|A)}{P(H)} = \frac{(0.10)(0.07)}{(0.05)} = 0.14$

Chapter 16. Statistics

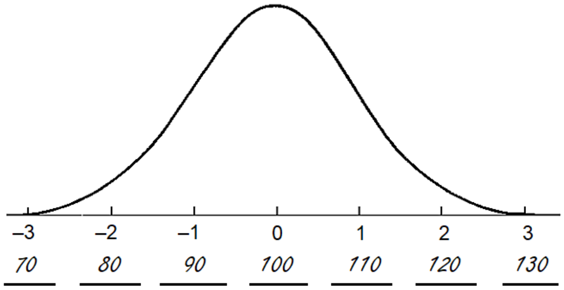
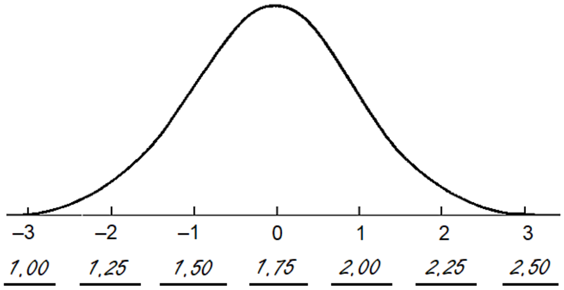
16.1 Data Collection

1. (4)	2. (2)
3. (2)	
4. a) The control group of plants would receive the normal level of CO2 (300 ppm). There should be two experimental groups, one which is exposed to 400 ppm and one which is exposed to 500 ppm. b) The independent variable is level of CO_2 exposure. The dependent variable is the rate of photosynthesis.	

16.2 Bias

1. (3) Seniors or physics students may be biased by aspects of class scheduling specific to their groups. Selecting only students from the cafeteria would omit students who have already chosen not to eat there.	
2. (4) Allowing subjects to self-select their participation can lead to bias. Honors calculus students may tend to spend more (or less) time on homework due to the nature of their courses. Surveying only teenagers at a movie theater would omit other age groups as well as people who don't like to go to movie theaters.	
3. (4) People who attend a football game are more likely to prefer an increase in the sports budget since they are sports fans.	
4. (2)	5. (1)

16.3 Normal Distribution

1. 	2. 
3. $74 + 6 = 80$	4. $85 - 2(4) = 77$

5. The interval from 115 to 125 is 1 standard deviation from the mean, which is about 68% of the data.	6. 95% of the data is within 2 standard deviations from the mean, so this is the interval between $66 - 2(4) = 58$ and $66 + 2(4) = 74$ inches.
7. $\frac{1}{2}(80 - 50) = 15$	8. $\frac{1}{4}(92 - 78) = 3.5$
9. $\frac{1}{4}(69 - 63) = 1.5$	10. $SD = \frac{1}{3}(81 - 57) = 8$ $57 + 8 = 65$ Mean = 65 (check: $81 - 2(8) = 65$ ✓)
11. From $56 - 2(5)$ to $56 + 2(5)$, or between 46 and 66.	12. Interval is within 1 SD of the mean, representing about 68% of the homes. 75 × 68% = 51 homes.
13. a) 50% b) 68% c) 2.5%	14. a) $\frac{1}{2}(68\%) + \frac{1}{2}(95\%) = 81.5\%$ b) $\frac{1}{2}(100\% - 99.7\%) = 0.15\%$ c) $0.15\% \times 424 = 0.636 \approx 1$ student d) $\frac{1}{2}(68\%) + \frac{1}{2}(99.7\%) = 83.85\%$ $83.85\% \times 424 = 355.5 \approx 356$ students

16.4 Areas Under Normal Curves

1. normalcdf(60, 73, 65, 5) ≈ 0.787	2. normalcdf(620, 1E99, 500, 100) ≈ 0.115
3. normalcdf(54.3, 63.5, 54.3, 4.6) ≈ 48%	4. normalcdf(74, 82, 80, 4) ≈ 0.62
5. normalcdf(80, 100, 72, 9) ≈ 19%	6. normalcdf(12.5, 1E99, 11, 1.5) ≈ 0.16
7. normalcdf(3, 1E99, 2.75, 0.42) ≈ 0.28	8. a) normalcdf(90, 1E99, 75, 8) ≈ 3.04% b) normalcdf(80, 90, 75, 8) ≈ 23.56% c) normalcdf(−1E99, 60, 75, 8) ≈ 3.04%
9. normalcdf (42, 1E99, 35, 2.8) ≈ 0.62% 0.62% × 3000 ≈ 19	10. normalcdf(550, 1E99, 510, 110) ≈ .358 0.358 × 1000 = 358

16.5 Plausible Outcomes

1. $5.6 - 2(0.2)$ and $5.6 + 2(0.2)$, or between 5.2 and 6.0.
   Yes, 5.9 is within the margin of error.
2. $CI = 11.3095 \pm 2(0.7625)$, so the interval is approximately 9.78 to 12.83.
   The claim is plausible because 10 is within this interval.

16.6 Difference in Means [CC]

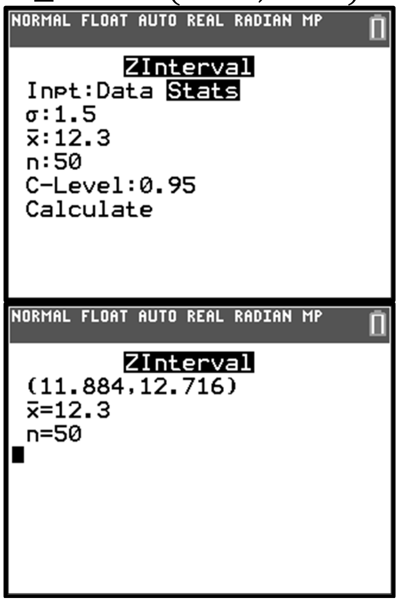
1. a) $3.2 - 2.5 = 0.7$
   b) According to the graph, at least 18 of the 100 rerandomized groups (18%) showed a mean difference of 1.0 or higher. Therefore, a mean difference of 0.7 is certainly within the 95% interval and is not statistically significant. There is no strong evidence of the success of the drug despite an observed difference between the effects on the groups in the sample.

16.7 Estimate Population Parameters

1. $SE = \frac{s}{\sqrt{n}} = \frac{12}{\sqrt{200}} \approx 0.848$
   $ME = (1.96)(0.848) \approx 1.66$

2. $SE = \frac{s}{\sqrt{n}} = \frac{0.125}{\sqrt{50}} \approx 0.018$
   $ME = (1.96)(0.018) \approx 0.035$

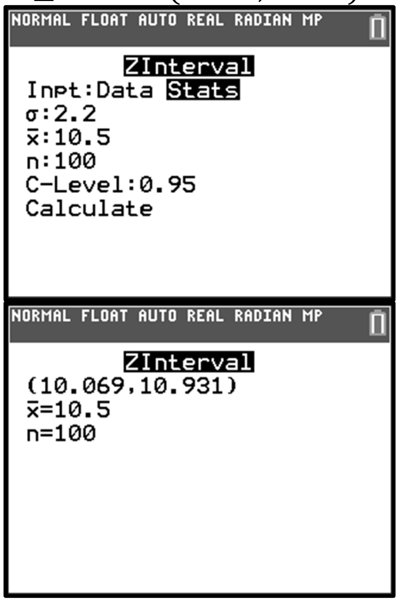
3. $SE = \frac{s}{\sqrt{n}} = \frac{1.5}{\sqrt{50}} \approx 0.212$
   $ME = (1.96)(0.212) \approx 0.42$
   $12.3 \pm 0.42 \rightarrow (11.88, 12.72)$



```
NORMAL FLOAT AUTO REAL RADIAN MP
          ZInterval
 Inpt:Data Stats
 σ:1.5
 x̄:12.3
 n:50
 C-Level:0.95
 Calculate
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
          ZInterval
 (11.884,12.716)
 x̄=12.3
 n=50
```

4. $SE = \frac{s}{\sqrt{n}} = \frac{2.2}{\sqrt{100}} \approx 0.22$
   $ME = (1.96)(0.22) \approx 0.43$
   $10.5 \pm 0.43 \rightarrow (10.07, 10.93)$



```
NORMAL FLOAT AUTO REAL RADIAN MP
          ZInterval
 Inpt:Data Stats
 σ:2.2
 x̄:10.5
 n:100
 C-Level:0.95
 Calculate
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
          ZInterval
 (10.069,10.931)
 x̄=10.5
 n=100
```

5. $SE = \frac{s}{\sqrt{n}} = \frac{10.2}{\sqrt{100}} = 1.02$
   $ME = (1.96)(1.02) \approx 2.00$
   $44.25 - 2.00 < \mu < 44.25 + 2.00$, or $(42.25, 46.25)$

6. $SE = \frac{s}{\sqrt{n}} = \frac{2.5}{\sqrt{49}} \approx 0.357$
   $ME = (1.96)(0.357) \approx 0.7$
   $12 - 0.7 < \mu < 12 + 0.7$, or $(11.3, 12.7)$

7. a) $\hat{p} = \frac{245}{350} = 0.7$
   b) $\hat{q} = 1 - \hat{p} = 0.3$
   c) $SE = \sqrt{\frac{(0.7)(0.3)}{350}} \approx 0.024$

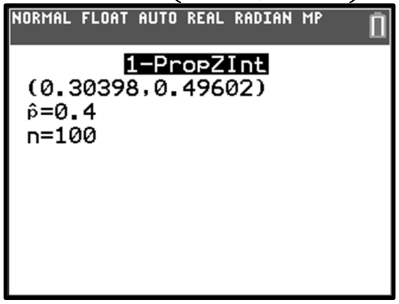
8. a) $\hat{p} = \frac{290}{500} = 0.58$
   b) $\hat{q} = 1 - \hat{p} = 0.42$
   c) $SE = \sqrt{\frac{(0.58)(0.42)}{500}} \approx 0.022$

9. $SE = \sqrt{\frac{(0.4)(0.6)}{100}} \approx 0.049$

$ME = (1.96)(0.049) \approx 0.096$

$0.4 - 0.096 < p < 0.4 + 0.096$

Population proportion should fall within the interval $(0.304, 0.496)$.



```
NORMAL FLOAT AUTO REAL RADIAN MP
         1-PropZInt
 (0.30398,0.49602)
 p̂=0.4
 n=100
```

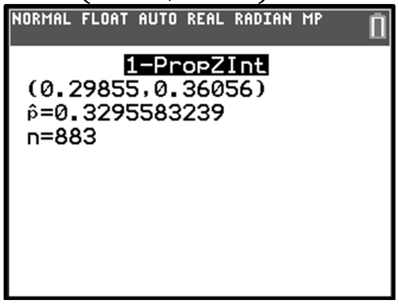
10. $\hat{p} = \frac{291}{883} \approx 0.330$

$SE = \sqrt{\frac{(0.33)(0.67)}{883}} \approx 0.0158$

$ME = (1.96)(0.0158) \approx 0.031$

$0.330 - 0.031 < p < 0.330 + 0.031$

$CI = (0.299, 0.361)$



```
NORMAL FLOAT AUTO REAL RADIAN MP
         1-PropZInt
 (0.29855,0.36056)
 p̂=0.3295583239
 n=883
```

11. $\hat{p} = \frac{180}{300} = 0.6 = 60\%$

$ME = 1.96\sqrt{\frac{(0.60)(0.40)}{300}} \approx 0.055$

$0.60 - 0.055 \leq p \leq 0.60 + 0.055$

Population proportion should fall within the interval $(0.545, 0.655)$.

54.5% of 1,800 is 981 and 65.7% of 1,800 is 1179, so we can estimate that between 981 and 1,179 students own a pet.