Answer Key

Algebra II

Regents Questions

through January, 2023

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Notation

A code next to each Regents Question number states from which Algebra II Common Core Regents exam the question came. For example, AUG '18 [25] means the question appeared on the August 2018 as question 25.

Chapter 1. Linear Functions

1.1 Linear Systems in Three Variables [CC]

1. AUG '16 [23] Ans: 2
2. JAN '18 [3] Ans: 4
3. JUN '19 [23] Ans: 2
4. JAN '20 [18] Ans: 1
5. JUN '22 [8] Ans: 2
6. SPR '15 [10]
   $7x + 7z = 35$ (1) + (2)
   $x + z = 5$
   $4x + 10z = 62$ (2) + (3)
   $-4(x + z = 5)$
   $\underline{4x + 10z = 62}$
   $6z = 42$
   $z = 7$
   $x + (7) = 5; x = -2$
   $(-2) + 3y + 5(7) = 45;$
   $3y = 12;\ y = 4$
7. JUN '17 [33]
   $4y - 4z = 12$ (1) + (3)
   $y - z = 3$
   $-2(x + y + z = 1)$
   $\underline{2x + 4y + 6z = 2}$
   $2y + 4z = 0$
   $y + 2z = 0$
   $y = -2z$
   $(-2z) - z = 3;\ z = -1$
   $y - (-1) = 3;\ y = 2$
   $x + (2) + (-1) = 1;\ x = 0$
8. AUG '18 [33]
   $2(2x + 3y - 4z = -1)$
   $\underline{-4x + y + z = 16}$
   $7y + 7z = 14$
   $y - z = 2$
   $y = z + 2$
   $4(x - 2y + 5z = 3)$
   $\underline{-4x + y + z = 16}$
   $-7y + 21z = 28$
   $y - 3z = -4$
   $y = 3z - 4$
   $z + 2 = 3z - 4$
   $2z = 6;\ z = 3$
   $y = (3) + 2;\ y = 5$
   $-4x + (5) + (3) = 16;$
   $-4x = 8;\ x = -2$
9. JAN '19 [33]
   Rewrite (3) as
   $-a + 6b + 2c = 14$
   $2b + 5c = 21$ (1) – (2)
   $8b + 3c = 16$ (2) + (3)
   $4(2b + 5c = 21)$
   $\underline{8b + 3c = 16}$
   $17c = 68$
   $c = 4$
   $2b + 5(4) = 21;\ 2b = 1;\ b = \frac{1}{2}$
   $a + 4\left(\frac{1}{2}\right) + 6(4) = 23;\ a + 26 = 23;$
   $a = -3$

Chapter 2. Irrational Expressions

2.1 Operations with Square Roots [CC]

There are no Regents exam questions on this topic.

2.2 Rationalize Monomial Denominators [CC]

There are no Regents exam questions on this topic.

2.3 Rationalize Binomial Denominators

There are no Regents exam questions on this topic.

Chapter 3. Quadratic Functions

3.1 Factor a Trinomial by Grouping [NG]

There are no Regents exam questions on this topic.

3.2 Solve Quadratics with a≠1 [NG]

1. AUG '22 [32]
$2x^2 - 7x + 4 = 11 - 2x$
$2x^2 - 5x - 7 = 0$
$2x^2 + 2x - 7x - 7 = 0$
$2x(x + 1) - 7(x + 1) = 0$
$(2x - 7)(x + 1) = 0$
$x = \left\{-1, \frac{7}{2}\right\}$
$y = 11 - 2(-1) = 13$
$y = 11 - 2\left(\frac{7}{2}\right) = 4$
$(-1,13)$ and $\left(\frac{7}{2}, 4\right)$

3.3 Graphs of Quadratic Functions

1. JAN '19 [22] Ans: 4

3.4 Vertex Form and Transformations

1. JUN '18 [8] Ans: 1

3.5 Focus and Directrix [CC]

1. SPR '15 [2] Ans: 4
2. AUG '16 [19] Ans: 4
3. JUN '17 [17] Ans: 4
4. AUG '17 [6] Ans: 2
5. JAN '18 [16] Ans: 1
6. JUN '18 [21] Ans: 4
7. AUG '18 [23] Ans: 4
8. JAN '19 [14] Ans: 3
9. JUN '22 [13] Ans: 1
10. AUG '22 [12] Ans: 3
11. JUN '16 [30]
The vertex is $(4, -3)$ and $p = 12 \div 4 = 3$. The *x*-coordinates of the focus and the vertex are the same. Since $p = 3$, the focus is 3 units up from the vertex, so the coordinates of the focus are (4,0).
12. JUN '19 [35]
The x-coordinate of the vertex is $\frac{-1-5}{2} = -3$, so the vertex is $(4, -3)$.
$p = y_{focus} - y_{vertex} = -1 + 3 = 2$.
$y = \frac{1}{8}(x - 4)^2 - 3$

13. JAN '20 [28]

The vertex is (3,6), which is 5 units up from the focus, so the directrix is 5 units above the vertex at $y = 11$.

Chapter 4. Imaginary Numbers

4.1 Set of Complex Numbers

There are no Regents exam questions on this topic.

4.2 Operations with Complex Numbers

1. JUN '16 [3] Ans: 2
2. JUN '17 [4] Ans: 2
3. AUG '17 [2] Ans: 3
4. JUN '18 [5] Ans: 3
5. AUG '18 [15] Ans: 3
6. JAN '19 [11] Ans: 1
7. JUN '19 [15] Ans: 1
8. JAN '20 [22] Ans: 1
9. JUN '22 [23] Ans: 4
10. AUG '22 [2] Ans: 4
11. SPR '15 [6]
$(4-3i)(5+2yi-5+2yi)$
$(4-3i)(4yi)$
$16yi-12yi^2$
$12y+16yi$
12. AUG '16 [27]
$xi(-6i)^2 = xi(36i^2) = -36xi$
13. JAN '17 [25]
$(1-i)(1-i)(1-i) =$
$(1-2i+i^2)(1-i) =$
$-2i(1-i) = -2i+2i^2 = -2-2i$
14. JAN '18 [25]
$i^2 = -1$, not 1.
$6+10i-4(-1) = 10+10i$
15. AUG '19 [27]
$-\frac{1}{2}i^3\left(\sqrt{-9}-4\right)-3i^2 =$
$\frac{1}{2}i(3i-4)+3 =$
$-\frac{3}{2}-2i+3 = \frac{3}{2}-2i$

4.3 Imaginary Roots

1. FALL '15 [4] Ans: 3
2. JUN '16 [12] Ans: 1
3. AUG '16 [1] Ans: 4
4. JAN '17 [11] Ans: 4
5. JUN '17 [7] Ans: 4
6. AUG '17 [3] Ans: 3
7. AUG '18 [9] Ans: 3
8. JAN '19 [5] Ans: 2
9. JAN '19 [9] Ans: 4
10. JUN '19 [12] Ans: 4
11. JAN '20 [20] Ans: 2
12. AUG '22 [8] Ans: 1
13. JUN '18 [27]
$x = \frac{-5 \pm \sqrt{5^2-4(2)(8)}}{2(2)} = -\frac{5}{4} \pm \frac{i\sqrt{39}}{4}$
14. JUN '22 [25]
Yes, because the discriminant
$b^2-4ac = (-4)^2-4(1)(13) = -36$
is negative.

Chapter 5. Circles

5.1 Equations of Circles

1. JUN '16 [19] Ans: 4

5.2 Circle-Linear Systems

1. AUG '17 [19] Ans: 1
2. AUG '19 [16] Ans: 3
3. JUN '16 [33]
$y = -x + 5$
$(x-3)^2 + (-x+5+2)^2 = 16$
$x^2 - 6x + 9 + x^2 - 14x + 49 = 16$
$2x^2 - 20x + 42 = 0$
$x^2 - 10x + 21 = 0$
$(x-3)(x-7) = 0$
$x = \{3, 7\}$
$y = -(3) + 5 = 2;$
$y = -(7) + 5 = -2$
$(3,2)$ and $(7, -2)$
4. AUG '18 [31]
$x^2 + (x-28)^2 = 400$
$x^2 + x^2 - 56x + 784 = 400$
$2x^2 - 56x + 384 = 0$
$x^2 - 28x + 192 = 0$
$(x-12)(x-16) = 0$
$x = \{12, 16\}$
$y = (12) - 28 = -16;$
$y = (16) - 28 = -12$
$(12, -16)$ and $(16, -12)$
5. JAN '20 [35]
$y = -x + 1$
$(x-2)^2 + (-x+1-3)^2 = 16$
$(x-2)^2 + (-x-2)^2 = 16$
$x^2 - 4x + 4 + x^2 + 4x + 4 = 16$
$2x^2 + 8 = 16$
$2x^2 = 8$
$x^2 = 4$
$x = \pm 2$
$y = -(-2) + 1 = 3;$
$y = -2 + 1 = -1$
$(-2,3)$ and $(2, -1)$
6. JUN '22 [36]
$y = 2x - 5$
$x^2 + (2x-5)^2 = 25$
$x^2 + 4x^2 - 20x + 25 = 25$
$5x^2 - 20x = 0$
$5x(x-4) = 0$
$x = \{0, 4\}$
$y = 2(0) - 5 = -5;$
$y = 2(4) - 5 = 3$
$(0, -5)$ and $(4,3)$

Chapter 6. Polynomials

6.1 Operations with Functions

1. JUN '16 [8] Ans: 4
2. JAN '17 [10] Ans: 3
3. JUN '17 [9] Ans: 2
4. JUN '18 [13] Ans: 1
5. AUG '18 [3] Ans: 4
6. JUN '22 [10] Ans: 3
7. JAN '18 [33]
$(2x^2 + x - 3)(x - 1) -$
$[(2x^2 + x - 3) + (x - 1)] =$
$(2x^3 - 2x^2 + x^2 - x - 3x + 3) -$
$(2x^2 + 2x - 4) =$
$2x^3 - 3x^2 - 6x + 7$

6.2 Long Division

1. FALL '15 [3] Ans: 1
2. JUN '16 [14] Ans: 2
3. AUG '17 [13] Ans: 1
4. JAN '18 [9] Ans: 4
5. AUG '19 [10] Ans: 1
6. JUN '22 [18] Ans: 2
7. JUN '18 [29]

$$
\begin{array}{r}
2a^2+5a+2 \\
3a-2\overline{)\ 6a^3+11a^2-4a-9} \\
\underline{-\left(6a^3-4a^2\right)} \\
15a^2-4a \\
\underline{-\left(15a^2-10a\right)} \\
6a-9 \\
\underline{-\left(6a-4\right)} \\
-5
\end{array}
$$

$2a^2 + 5a + 2 - \frac{5}{3a - 2}$

8. JUN '19 [30]
$p(x) = (x - 1)(x^2 + 7) + 5 =$
$x^3 - x^2 + 7x - 2$

6.3 Synthetic Division

1. AUG '16 [11] Ans: 2
2. AUG '18 [5] Ans: 3
3. JAN '20 [7] Ans: 3
4. JUN '22 [3] Ans: 1
5. AUG '22 [17] Ans: 2
6. JAN '17 [32]

$\boxed{2}$	3	7	-20
		6	26
	3	13	6

$3x + 13 + \frac{6}{x-2}$

7. JAN '19 [34]

$\boxed{-2}$	1	2	4	−10
		−2	0	−8
	1	0	4	| −18

$x^3 + 4 - \frac{18}{x+2}$

8. AUG '22 [35]

$\boxed{4}$	3	−4	2	−1
		12	32	136
	3	8	34	| 135

$3x^2 + 8x + 34 + \frac{135}{x-4}$

$x = 4$ is not a root because $r(x) \neq 0$.

6.4 Remainder Theorem

1. AUG '16 [21] Ans: 3
2. JAN '17 [20] Ans: 2
3. JUN '17 [11] Ans: 1
4. AUG '17 [20] Ans: 2
5. JAN '18 [19] Ans: 4
6. JUN '18 [12] Ans: 3
7. JUN '19 [7] Ans: 4
8. JUN '22 [6] Ans: 2
9. SPR '15 [7]
   By the Factor Theorem, $x - a$ is a factor of $f(x)$ only when $f(a) = 0$.
   $f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 0$
10. FALL '15 [15]
    $0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15$
    $0 = -750 + 25b + 260 + 15$
    $475 = 25b$
    $b = 19$
    $z(x) = 6x^3 + 19x^2 - 52x + 15$
    By the Factor Theorem, since $z(-5) = 0$, then $x + 5$ is a factor of $z(x)$.

$\boxed{-5}$	6	19	−52	15
		−30	55	−15
	6	−11	3	| 0

    $(x + 5)(6x^2 - 11x + 3) = 0$
    $(x + 5)(3x - 1)(2x - 3) = 0$
    $x = \left\{-5, \frac{1}{3}, \frac{3}{2}\right\}$
11. JUN '16 [27]
    By the Factor Theorem, $x - a$ is a factor of $f(x)$ only when $f(a) = 0$.
    $2(5)^3 - 4(5)^2 - 7(5) - 10 = 105$, so $x - 5$ is not a factor.
12. JUN '17 [25]
    $r(2) = (2)^3 - 4(2)^2 + 4(2) - 6 = -6$
    By the Factor Theorem, $x - a$ is a factor of $f(x)$ only when $f(a) = 0$.
    Since $r(2) = -6$, $x - 2$ is not a factor of $r(x)$.
13. AUG '18 [34]
    $j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48$
    $j(-1) = 0$
    By the Factor Theorem, since $j(-1) = 0$, then $x + 1$ is a factor of $j(x)$.

$\boxed{-1}$	2	−1	−35	16	48
		−2	3	32	−48
	2	−3	−32	48	| 0

    Factor by grouping:
    $2x^3 - 3x^2 - 32x + 48$
    $x^2(2x - 3) - 16(2x - 3)$
    $(x^2 - 16)(2x - 3)$
    $(x + 4)(x - 4)(2x - 3)$
    $(x + 1)(x + 4)(x - 4)(2x - 3) = 0$
    $x = \left\{-1, -4, 4, \frac{3}{2}\right\}$
14. AUG '19 [29]
    $P(-2) = 60$ and $Q(-2) = 0$; $(x + 2)$ is a factor of $Q(x)$ by the Factor Theorem since $Q(-2) = 0$.
15. JAN '10 [26]
    $m(3) = 3^3 - 3^2 - 5(3) - 3 = 0$; $x - 3$ is a factor of $m(x)$ by the Factor Theorem since $m(3) = 0$.

6.5 Factor Polynomials

1. FALL '15 [5] Ans: 4
2. AUG '16 [5] Ans: 3
3. AUG '16 [15] Ans: 3
4. JAN '17 [3] Ans: 4
5. AUG '17 [15] Ans: 1
6. AUG '18 [14] Ans: 4
7. JAN '19 [3] Ans: 1
8. JUN '19 [11] Ans: 2
9. AUG '19 [1] Ans: 2
10. AUG '19 [4] Ans: 2
11. JAN '20 [6] Ans: 3
12. FALL '15 [12]
The expression is of the form $u^2 - 5u - 6$ or $(u - 6)(u + 1)$.
Let $u = 4x^2 + 5x$.
$(4x^2 + 5x - 6)(4x^2 + 5x + 1)$
$(4x - 3)(x + 2)(4x + 1)(x + 1)$
13. JUN '17 [27]
$x^2(4x - 1) + 4(4x - 1)$
$(x^2 + 4)(4x - 1)$
14. JAN '18 [28]
$3x^3 + x^2 + 3xy + y$
$x^2(3x + 1) + y(3x + 1)$
$(x^2 + y)(3x + 1)$
15. AUG '18 [25]
$(x^2 - 6)(x^2 + 2)$
16. JUN '22 [28]
$-x(2x^3 - x^2 - 18x + 9)$
$-x[x^2(2x - 1) - 9(2x - 1)]$
$-x(x^2 - 9)(2x - 1)$
$-x(x + 3)(x - 3)(2x - 1)$
17. AUG '22 [26]
$x^2(x - 2) - 9(x - 2)$
$(x^2 - 9)(x - 2)$
$(x + 3)(x - 3)(x - 2)$

6.6 Polynomial Identities [CC]

1. AUG '16 [20] Ans: 4
2. JAN '18 [6] Ans: 2
3. JUN '18 [22] Ans: 4
4. JUN '19 [2] Ans: 4
5. JAN '20 [3] Ans: 3
6. AUG '22 [19] Ans: 1
7. FALL '15 [11]
Let x and $x + 1$ represent the integers.
$(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 =$
$2x + 1$
$2x$ is an even integer, so $2x + 1$ is an odd integer.
8. JUN '16 [31]
$1 + \frac{1}{x^3 + 8} = \frac{x^3 + 8}{x^3 + 8} + \frac{1}{x^3 + 8} = \frac{x^3 + 9}{x^3 + 8}$
9. JAN '17 [33]
(1) $2x^3 - 10x^2 + 11x - 7 =$
$(x - 4)(2x^2 + hx + 3) + k$
(2) $2x^3 - 10x^2 + 11x - 7 =$
$2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k$
(3) $-2x^2 + 8x + 5 = hx^2 - 4hx + k$
(4) So, $h = -2$ and $k = 5$
10. AUG '17 [27]
$(x^2 - y^2)^2 + (2xy)^2$
$= x^4 - 2x^2y^2 + y^4 + (2xy)^2$
$= x^4 - 2x^2y^2 + y^4 + 4x^2y^2$
$= x^4 + 2x^2y^2 + y^4$
$= (x^2 + y^2)^2$
11. JAN '19 [27]
$(a + b)^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab(a + b)$
No. Erin's shortcut only works if $3ab(a + b) = 0$; that is, only if $a = 0$, $b = 0$, or $a = -b$.

Chapter 7. Polynomial Functions

7.1 Find Roots by Factoring

1. JUN '16 [6] Ans: 1
2. AUG '17 [8] Ans: 4
3. AUG '18 [21] Ans: 4
4. AUG '19 [21] Ans: 4
5. JAN '20 [19] Ans: 4
6. AUG '22 [6] Ans: 2
7. JUN '19 [33]
$(4x^2 + 9)(2x + 3)(2x - 3)$;
No, because $4x^2 + 9 = 0$ leads to roots of $\pm\frac{3}{2}i$

7.2 Root Theorems

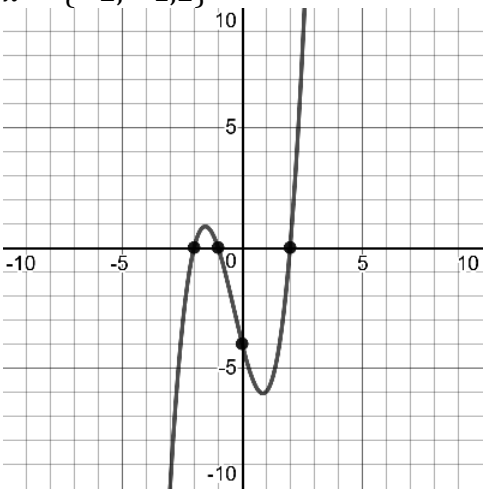
There are no Regents exam questions on this topic.

7.3 Properties of Polynomial Graphs

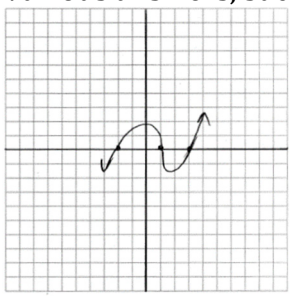
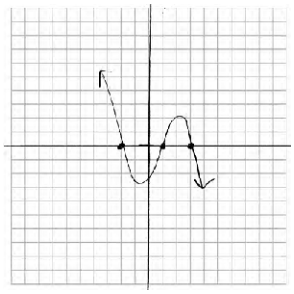
1. SPR '15 [1] Ans: 1
2. JUN '16 [4] Ans: 3
3. JUN '16 [20] Ans: 2
4. JUN '17 [1] Ans: 1
5. AUG '17 [12] Ans: 4
6. JAN '18 [17] Ans: 3
7. JUN '18 [16] Ans: 2
8. AUG '18 [4] Ans: 1
9. JAN '19 [8] Ans: 1
10. JAN '19 [19] Ans: 1
11. JUN '19 [21] Ans: 4
12. AUG '19 [8] Ans: 2
13. JAN '20 [5] Ans: 3

7.4 Graph Polynomial Functions

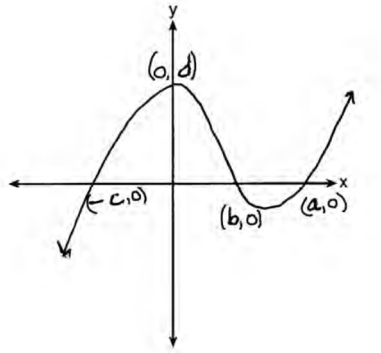
1. AUG '16 [33]
$0 = x^2(x + 1) - 4(x + 1)$
$0 = (x^2 - 4)(x + 1)$
$0 = (x + 2)(x - 2)(x + 1)$
$x = \{-2, -1, 2\}$



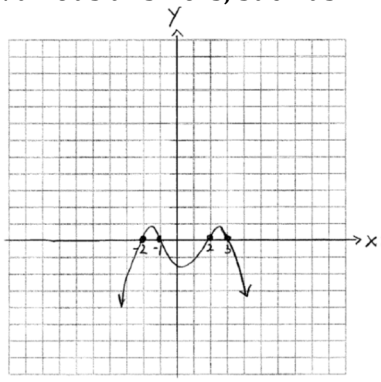
2. JAN '17 [29]
Various answers, such as


or


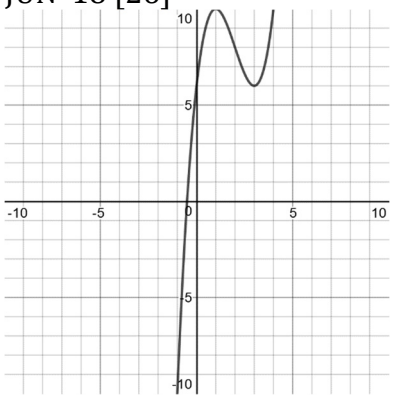
3. AUG '17 [32]
Various answers, such as



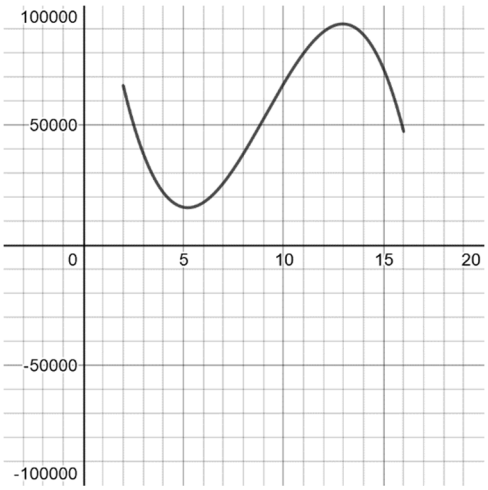
4. JAN '18 [31]
Various answers, such as



5. JUN '18 [26]

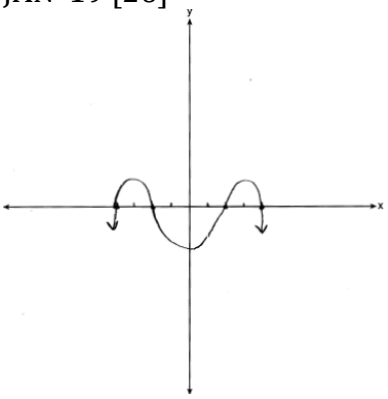


6. AUG '18 [37]
$P(x) = R(x) - C(x) = -330x^3 + 9{,}000x^2 - 67{,}000x + 167{,}000$

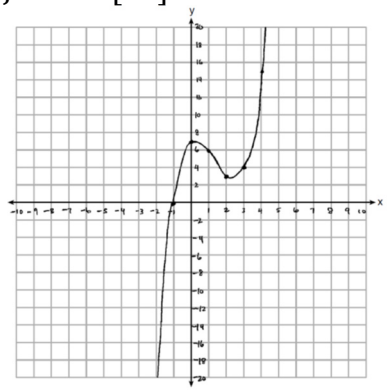


Least profitable at year 5 because there is a minimum at $P(5)$. Most profitable at year 13 because there is a maximum at $P(13)$.

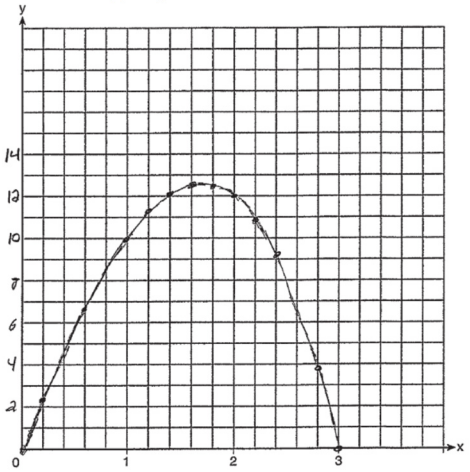
7. JAN '19 [26]



8. JAN '20 [32]



9. AUG '22 [34]



maximum is 12.6 cu. in.

7.5 Polynomial Transformations

1. AUG '18 [17] Ans: 4
2. JAN '18 [36]
   $f(x) = x^2(x+4)(x-3)$
   $g(x) = f(x+2) = (x+2)^2(x+6)(x-1)$

7.6 Systems of Polynomial Functions

1. JUN '16 [22] Ans: 4
2. AUG '16 [6] Ans: 3
3. JAN '17 [5] Ans: 4
4. JUN '18 [1] Ans: 2
5. JUN '19 [3] Ans: 3
6. FALL '15 [7]
   $-2x + 1 = -2x^2 + 3x + 1$
   $2x^2 - 5x = 0$
   $x(2x-5) = 0$
   $x = \left\{0, \frac{5}{2}\right\}$
7. AUG '19 [36]
   $x^2 - 6x = -17$
   $x^2 - 6x + 9 = -17 + 9$
   $(x-3)^2 = -8$
   $x - 3 = \pm 2i\sqrt{2}$
   $x = 3 \pm 2i\sqrt{2}$
   A real solution would appear as a point of intersection on the graph; since the parabola and line do not intersect, the solutions are imaginary.

Chapter 8. Radicals and Rational Exponents

8.1 *n*th Roots

There are no Regents exam questions on this topic.

8.2 Operations with Radicals

1. AUG '22 [13] Ans: 1

8.3 Solve Equations with Radicals

1. JUN '16 [5] Ans: 3
2. AUG '17 [4] Ans: 2
3. JAN '18 [2] Ans: 3
4. AUG '18 [7] Ans: 3
5. JUN '19 [19] Ans: 2
6. JAN '20 [10] Ans: 2
7. SPR '15 [8]
   $\sqrt{x-5} = -x+7$
   $x-5 = x^2-14x+49$
   $x^2-15x+54=0$
   $(x-6)(x-9)=0$
   $x=\{6,9\}$
   Reject $x=9$ because
   $\sqrt{9-5}+9 \neq 7$.
   $x=6$
8. AUG '16 [35]
   $\left(\sqrt{2x-7}\right)^2 = (5-x)^2$
   $2x-7 = 25-10x+x^2$
   $x^2-12x+32=0$
   $(x-4)(x-8)=0$
   $x=\{4,8\}$
   Reject $x=8$ because
   $\sqrt{2(8)-7}+8 \neq 5$.
   $x=4$
9. JAN '17 [37]
   $0 = \sqrt{t}-2t+6$
   $\sqrt{t} = 2t-6$
   $\left(\sqrt{t}\right)^2 = (2t-6)^2$
   $t = 4t^2-24t+36$
   $4t^2-25t+36=0$
   $(4t-9)(t-4)=0$
   $t=\left\{\frac{9}{4},4\right\}$
   Reject $t=\frac{9}{4}$ because $2\left(\frac{9}{4}\right)-6<0$.
   $\sqrt{1}-2(1)+6=5$
   $\sqrt{3}-2(3)+6=\sqrt{3}$
   $5-\sqrt{3}\approx 3.268$
   327 mph
10. JUN '17 [30]
    $\sqrt{x-4} = -x+6$
    $x-4 = x^2-12x+36$
    $x^2-13x+40=0$
    $(x-5)(x-8)=0$
    $x=\{5,8\}$
    8 is extraneous because
    $\sqrt{8-4} \neq -8+6$
11. JUN '18 [33]
    $\sqrt{6-2x}+x = 2x+30-9$
    $\sqrt{6-2x} = x+21$
    $6-2x = x^2+42x+441$
    $x^2+44x+435=0$
    $(x+29)(x+15)=0$
    $x=\{-29,-15\}$
    –29 is extraneous because
    $\sqrt{6-2(-29)}-29 \neq 2(-14)-9$.

12. JAN '19 [36]
$3\sqrt{x} - 2x = -5$
$3\sqrt{x} = 2x - 5$
$9x = 4x^2 - 20x + 25$
$4x^2 - 29x + 25 = 0$
$(4x - 25)(x - 1) = 0$
$x = \left\{\frac{25}{4}, \cancel{1}\right\}$
1 is extraneous because
$3\sqrt{1} - 2(1) \neq -5$

13. AUG '19 [37]
$B = 1.69\sqrt{30 + 4.45} - 3.49 \approx 6$,
which is a steady breeze;
$15 = 1.69\sqrt{s + 4.45} - 3.49$
$18.49 = 1.69\sqrt{s + 4.45}$
$\frac{18.49}{1.69} = \sqrt{s + 4.45}$
$\left(\frac{18.49}{1.69}\right)^2 = s + 4.45$
$s = \left(\frac{18.49}{1.69}\right)^2 - 4.45 \approx 115$
B values of 9.5 to 10.49 would round to a 10
$9.5 = 1.69\sqrt{s + 4.45} - 3.49$ solves to $s \approx 55$ and
$10.49 = 1.69\sqrt{s + 4.45} - 3.49$ solves to $s \approx 64$, so the range is 55 to 64 mph

14. JUN '22 [34]
$t = 2\pi\sqrt{\frac{67}{9.81}} \approx 16.4$;
$9.6 = 2\pi\sqrt{\frac{L}{9.81}}$
$\frac{9.6}{2\pi} = \sqrt{\frac{L}{9.81}}$
$1.528 \approx \frac{\sqrt{L}}{3.132}$
$4.785 \approx \sqrt{L}$
$L \approx 22.9$

15. AUG '22 [27]
$4x + 1 = (11 - x)^2$
$4x + 1 = 121 - 22x + x^2$
$x^2 - 26x + 120 = 0$
$(x - 6)(x - 20) = 0$
$x = \{6, \cancel{20}\}$
20 is extraneous

8.4 Graphs of Radical Functions

There are no Regents exam questions on this topic.

8.5 Negative Exponents

There are no Regents exam questions on this topic.

8.6 Rational Exponents

1. JUN '16 [1] Ans: 4
2. JAN '17 [7] Ans: 2
3. JUN '17 [16] Ans: 4
4. AUG '17 [23] Ans: 4
5. JAN '18 [11] Ans: 4
6. JUN '18 [20] Ans: 2
7. AUG '18 [12] Ans: 3
8. JUN '19 [8] Ans: 1
9. AUG '19 [14] Ans: 4
10. JAN '20 [1] Ans: 1
11. JUN '22 [1] Ans: 1
12. SPR '15 [5]
$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} = x^y$; $x^{\frac{4}{3}} = x^y$; $y = \frac{4}{3}$
13. AUG '16 [26]
$\left(3^{\frac{1}{5}}\right)^2 = \left(\sqrt[5]{3}\right)^2 = \sqrt[5]{9}$

14. JAN '17 [30]
$\left(x^{\frac{5}{3}}\right)^{\frac{6}{5}} = \left(y^{\frac{5}{6}}\right)^{\frac{6}{5}}$
$x^2 = y$

15. JUN '17 [31]
$\left(x^{\frac{1}{3}}\right)\left(x^{\frac{1}{2}}\right) = \left(x^{\frac{2}{6}}\right)\left(x^{\frac{3}{6}}\right) = x^{\frac{5}{6}}$

16. AUG '17 [25]
$(-8)^{\frac{4}{3}} = \left(\sqrt[3]{-8}\right)^4 = (-2)^4 = 16$

17. JAN '18 [32]
The denominator of the rational exponent represents the index of a root, and the 4th root of 81 is 3 and 3^3 is 27.

18. AUG '18 [26]
$\frac{2x^{\frac{3}{2}}}{2x^{\frac{2}{2}}} = x^{\frac{1}{2}} = \sqrt{x}$

19. JAN '19 [25]
$\frac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}} = \frac{x^{\frac{2}{3}}y^{\frac{5}{3}}}{x^{\frac{3}{4}}y} = \frac{x^{\frac{8}{12}}y^{\frac{5}{3}}}{x^{\frac{9}{12}}y^{\frac{3}{3}}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$

20. JUN '19 [29]
No. $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right) = \left(x^{\frac{2}{7}}\right)\left(x^{\frac{3}{5}}\right) =$
$x^{\frac{31}{35}} = \sqrt[35]{x^{31}}$.

21. AUG '19 [26]
The denominator of the rational exponent represents the index of a root, and the numerator of the rational exponent represents the power of the base.
$9^{\frac{5}{2}} = \left(\sqrt{9}\right)^5 = 243$.

22. JAN '20 [25]
Yes. $\left(p^2n^{\frac{1}{2}}\right)^8\sqrt{p^5n^4} = p^{16}n^4p^{\frac{5}{2}}n^2 =$
$p^{18\frac{1}{2}}n^6 = p^{18}n^6\sqrt{p}$

23. JUN '22 [30]
$\sqrt[3]{81} = \sqrt[3]{3^4} = 3^{\frac{4}{3}}$, so $a = \frac{4}{3}$.

24. AUG '22 [28]
$\frac{17}{8} - \frac{10}{8} = \frac{7}{8}$
$\left(y^{\frac{7}{8}}\right)^{-4} = y^{-\frac{7}{2}}$, so $n = -\frac{7}{2}$.

Chapter 9. Rational Functions

9.1 Undefined Expressions

1. AUG '17 [1] Ans: 1

9.2 Simplify Rational Expressions

1. JUN '17 [23] Ans: 4
2. JAN '18 [18] Ans: 3
3. JUN '18 [3] Ans: 3
4. JAN '19 [21] Ans: 4
5. JAN '20 [23] Ans: 1
6. AUG '22 [15] Ans: 2

9.3 Multiply and Divide Rational Expressions

There are no Regents exam questions on this topic.

9.4 Add and Subtract Rational Expressions

1. AUG '19 [7] Ans: 2

9.5 Simplify Complex Fractions

There are no Regents exam questions on this topic.

9.6 Solve Rational Equations

1. FALL '15 [1] Ans: 4
2. AUG '16 [17] Ans: 3
3. JAN '17 [17] Ans: 1
4. JUN '17 [19] Ans: 1
5. JAN'18 [12] Ans: 1
6. JUN '18 [9] Ans: 4
7. JAN '19 [15] Ans: 3
8. AUG '19 [15] Ans: 4
9. AUG '22 [18] Ans: 3
10. JUN '16 [25]
$3x\left[\frac{1}{x}-\frac{1}{3}=-\frac{1}{3x}\right]$
$3-x=-1$
$x=4$
11. AUG '17 [33]
$(p+3)(p-5)\left[\frac{3p}{p-5}-\frac{2}{p+3}=\frac{p}{p+3}\right]$
$3p(p+3)-2(p-5)=p(p-5)$
$3p^2+9p-2p+10=p^2-5p$
$2p^2+12p+10=0$
$p^2+6p+5=0$
$(p+5)(p+1)=0$
$\{-5,-1\}$
12. AUG '18 [29]
$6(x+3)\left[\frac{-3}{x+3}+\frac{1}{2}=\frac{x}{6}-\frac{1}{2}\right]$
$-18+3(x+3)=$
$x(x+3)-3(x+3)$
$-18+3x+9=x^2+3x-3x-9$
$x^2-3x=0$
$x(x-3)=0$
$\{0,3\}$

13. JUN '19 [26]

$4x(x+1)\left[\frac{7}{2x}-\frac{2}{x+1}=\frac{1}{4}\right]$

$14(x+1)-8x=x(x+1)$

$14x+14-8x=x^2+x$

$x^2-5x-14=0$

$(x+2)(x-7)=0$

$\{-2,7\}$

14. JUN '22 [27]

$n^2\left[\frac{2}{n^2}+\frac{3}{n}=\frac{4}{n^2}\right]$

$2+3n=4$

$3n=2$

$n=\frac{2}{3}$

9.7 Model Rational Expressions and Equations

1. JUN '16 [2] Ans: 3
2. JUN '17 [22] Ans: 3
3. JUN '18 [24] Ans: 3
4. JUN '19 [16] Ans: 1
5. AUG '22 [22] Ans: 2
6. JAN '18 [27]

$\frac{1}{8}+\frac{1}{6}=\frac{1}{t_b}$

$24t_b\left[\frac{1}{8}+\frac{1}{6}=\frac{1}{t_b}\right]$

$3t_b+4t_b=24$

$7t_b=24$

$t_b=\frac{24}{7}\approx 3.4$

7. JAN '20 [37]

$n(0)=\frac{1}{5}+\frac{18}{15}=1.4;\ a(0)=\frac{9}{3}=3;$ the antibiotic has a greater amount at $t=0$.

$\frac{t+1}{t+5}+\frac{18}{t^2+8t+15}=\frac{9}{t+3}$

$(t+3)(t+5)\left[\frac{t+1}{t+5}+\frac{18}{t^2+8t+15}=\frac{9}{t+3}\right]$

$(t+3)(t+1)+18=9(t+5)$

$t^2+4t+3+18=9t+45$

$t^2-5t-24=0$

$(t-8)(t+3)=0;$ (reject $t=-3$)

drugs have the same amount at $t=8$ hours.

9.8 Graphs of Rational Functions

There are no Regents exam questions on this topic.

Chapter 10. Exponential Functions

10.1 Solve Simple Exponential Equations

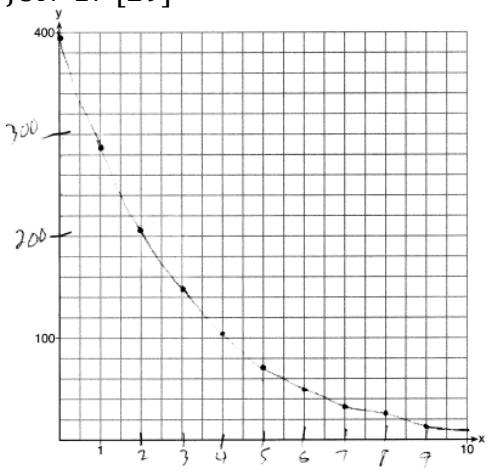
There are no Regents exam questions on this topic.

10.2 Rewrite Exponential Expressions

1. AUG '17 [10] Ans: 3
2. JAN '18 [8] Ans: 4
3. AUG '22 [24] Ans: 1

10.3 Graphs of Exponential Functions

1. JUN '18 [2] Ans: 2
2. JUN '19 [6] Ans: 3
3. JUN '19 [20] Ans: 1
4. JUN '22 [5] Ans: 3
5. AUG '22 [14] Ans: 3
6. JUN '17 [29]



7. JAN '20 [27]
   $q(x)$ is a translation of $p(x)$ by 3 units to the right and 4 units up.

10.4 Exponential Regression

1. JAN '17 [13] Ans: 3
2. JUN '18 [4] Ans: 2
3. JAN '18 [26]
   $D = 1.223(2.652)^A$
4. JUN '22 [32]
   $F(t) = 169.136(0.971)^t$

10.5 Exponential Growth or Decay

1. SPR '15 [4] Ans: 2
2. JUN '16 [15] Ans: 4
3. JUN '16 [21] Ans: 3
4. AUG '16 [13] Ans: 1
5. JAN '17 [2] Ans: 1
6. JUN '17 [13] Ans: 3
7. JAN '18 [5] Ans: 4
8. JAN '18 [23] Ans: 4

9. JUN '18 [23] Ans: 4
10. AUG '18 [8] Ans: 2
11. JAN '19 [6] Ans: 3
12. JAN '19 [20] Ans: 3
13. JUN '19 [24] Ans: 4
14. AUG '19 [3] Ans: 1
15. AUG '19 [24] Ans: 1
16. JAN '20 [2] Ans: 3
17. JUN '22 [11] Ans: 2
18. AUG '22 [11] Ans: 1

19. FALL '15 [17]
$A(t) = 100(0.5)^{\frac{t}{63}}$
t is time in years, and $A(t)$ is the amount of titanium-44 left after t years.
$\frac{A(10) - A(0)}{10 - 0} = \frac{89.58132 - 100}{10} = -1.041868$
The estimated mass at $t = 40$ is $100 - 40(-1.041868) \approx 58.3$.
The actual mass is $A(40) = 100(0.5)^{\frac{40}{63}} \approx 64.3976$.
The estimated mass is less than the actual mass.

20. JAN '20 [31]
$B(t) = 100(2)^{\frac{t}{30}}$

10.6 Periodic Compound Interest

1. AUG '16 [22] Ans: 4
2. JUN '17 [24] Ans: 2
3. AUG '19 [17] Ans: 4
4. JAN '20 [4] Ans: 1
5. JUN '22 [24] Ans: 1

10.7 Continuous Growth or Decay

1. JUN '17 [18] Ans: 2
2. JAN '19 [18] Ans: 4
3. JUN '19 [17] Ans: 2

4. AUG '19 [33]
$N(t) = 950e^{0.0475t}$; the base of e is used for continuous growth; 36 hours = 1.5 days; $N(1.5) \approx 1020$.

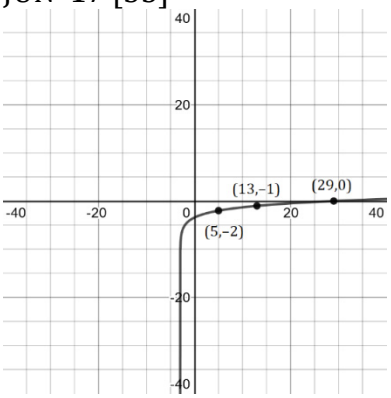
Chapter 11. Logarithms

11.1 General and Common Logarithms

1. JAN '17 [15] Ans: 3

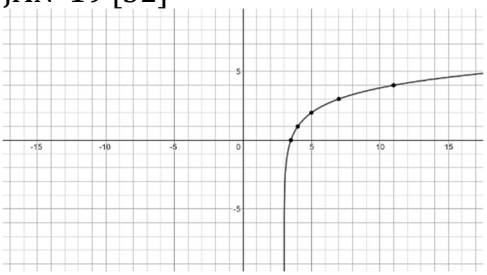
11.2 Graphs of Log Functions

1. JUN '16 [18] Ans: 1
2. JUN '18 [19] Ans: 4
3. AUG '18 [16] Ans: 2
4. JAN '19 [2] Ans: 1
5. JUN '22 [15] Ans: 4
6. AUG '22 [7] Ans: 4
7. AUG '16 [30]
$0 = \log_{10}(x - 4)$
$10^0 = x - 4$
$1 = x - 4$
$x = 5$
The x-intercept of $h(x)$ is (2,0).
f has the larger value.
8. JUN '17 [35]

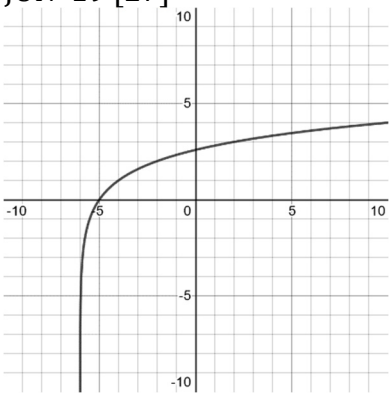


as $x \to -3, y \to -\infty$, and
as $x \to \infty, y \to \infty$.

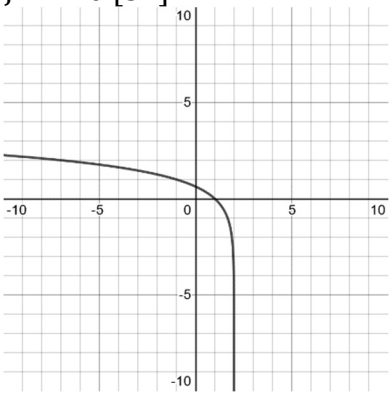
9. JAN '19 [32]



10. JUN '19 [27]



11. JAN '20 [34]



Domain is $(-\infty, 2)$;
asymptote is $x = 2$.

11.3 Properties of Logarithms

There are no Regents exam questions on this topic.

11.4 Use Logarithms to Solve Equations

1. JUN '18 [14] Ans: 1

2. JAN '20 [9] Ans: 4

3. AUG '16 [34]

$7 = 20(0.5)^{\frac{t}{8.02}}$

$0.35 = (0.5)^{\frac{t}{8.02}}$

$\log 0.35 = \frac{t \log 0.5}{8.02}$

$t = \frac{8.02 \log 0.35}{\log 0.5} \approx 12$

4. AUG '16 [37]

(a) $A(n) = 5{,}000(1.045)^n$

$B(n) = 5{,}000\left(1 + \frac{0.046}{4}\right)^{4n}$

(b) $B(6) - A(6) \approx$

$6578.87 - 6511.30 \approx 67.57$

$10{,}000 = 5{,}000\left(1 + \frac{0.046}{4}\right)^{4n}$

(c) $2 = 1.0115^{4n}$

$\log 2 = 4n \log 1.0115$

$n = \frac{\log 2}{4 \log 1.0115} \approx 15.2$ yrs

5. JUN '17 [37]

(a) $100 = 140\left(\frac{1}{2}\right)^{\frac{5}{h}}$

(b) $\log\left(\frac{100}{140}\right) = \log\left(\frac{1}{2}\right)^{\frac{5}{h}}$

$\log\frac{5}{7} = \frac{5}{h}\log\frac{1}{2}$

$h = \frac{5 \log\frac{1}{2}}{\log\frac{5}{7}} \approx 10.3002$

(c) $40 = 140\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$

$\log\frac{2}{7} = \log\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$

$\log\frac{2}{7} = \frac{t \log\frac{1}{2}}{10.3002}$

$t = \frac{10.3002 \log\frac{2}{7}}{\log\frac{1}{2}} \approx 18.6$

6. AUG '17 [36]

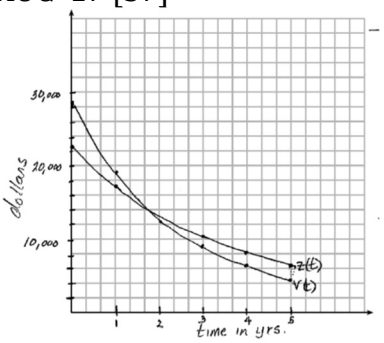
(a) $y = 4.168(3.981)^x$

(b) $100 = 4.168(3.981)^x$

$\log\frac{100}{4.168} = x \log 3.981$

$x = \frac{\log\frac{100}{4.168}}{\log 3.981} \approx 2.25$

7. AUG '17 [37]



$28482.698(0.684)^t = 22151.327(0.778)^t$

$\frac{28482.698}{22151.327} = \left(\frac{0.778}{0.684}\right)^t$

$\log\frac{28482.698}{22151.327} = t \log\left(\frac{0.778}{0.684}\right)$

$V(t) = Z(t)$ when $t \approx 1.95$.

At 1.95 years, the value of the car equals the loan balance. Zach can cancel the policy after 6 years because at that point the value of the car is less than \$3,000.

8. JUN '18 [35]

$C(t) = 63{,}000\left(1 + \frac{0.0255}{12}\right)^{12t}$

$63{,}000\left(1 + \frac{0.0255}{12}\right)^{12t} = 100{,}000$

$(1.002125)^{12t} = \frac{100}{63}$

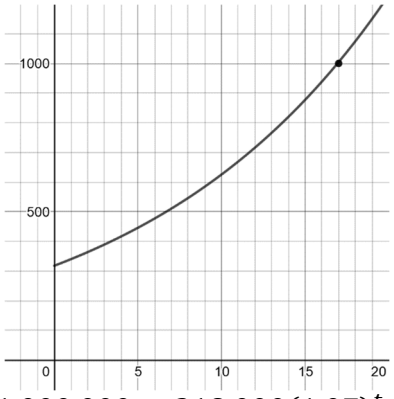
$12t \log(1.002125) = \log\frac{100}{63}$

$t = \frac{\log\frac{100}{63}}{12 \log 1.002125} \approx 18.14$

9. JAN '19 [37]

(a) $A(t) = 318{,}000(1.07)^t$

(b)



(c) $1{,}000{,}000 = 318{,}000(1.07)^t$

$\frac{1000}{318} = 1.07^t$

$\log\frac{1000}{318} = t\log 1.07$

$t = \frac{\log\frac{1000}{318}}{\log 1.07} \approx 17$ yrs

(c) The graph nearly intersects the point (17,1000000)

10. JUN '19 [34]

$s(t) = 200(0.5)^{\frac{t}{15}}$

$\frac{1}{10} = (0.5)^{\frac{t}{15}}$

$\log\frac{1}{10} = \log(0.5)^{\frac{t}{15}}$

$-1 = \frac{t}{15}\log(0.5)$

$t = -\frac{15}{\log(0.5)} \approx 50$

11. JAN '20 [36]

$y = 101.523(0.883)^x$;

$29 = 101.523(0.883)^x$

$.28565 = (0.883)^x$

$\log(.28565) = x\log(0.883)$

$x \approx 10.07$ km

12. AUG '22 [33]

a) $p(t) = 11{,}000(2)^{\frac{t}{20}}$

b) $1{,}000{,}000 = 11{,}000(2)^{\frac{t}{20}}$

$\frac{1000}{11} = 2^{\frac{t}{20}}$

$\log\left(\frac{1000}{11}\right) = \log 2^{\frac{t}{20}}$

$\log\left(\frac{1000}{11}\right) = \frac{t}{20}\log 2$

$t = \frac{20\log\left(\frac{1000}{11}\right)}{\log 2} \approx 130.13$

11.5 Natural Logarithms

1.	JAN '17 [23]	Ans: 1
2.	JUN '17 [2]	Ans: 1
3.	JAN '18 [13]	Ans: 3
4.	JAN '18 [10]	Ans: 1
5.	JUN '18 [18]	Ans: 1
6.	AUG '18 [1]	Ans: 4
7.	AUG '18 [19]	Ans: 3
8.	AUG '19 [18]	Ans: 4
9.	JUN '22 [7]	Ans: 1
10.	AUG '22 [9]	Ans: 4

11. FALL '15 [13]

(a) $100 = 325 + (68 - 325)e^{-2k}$

$-225 = -257e^{-2k}$

$\ln\frac{225}{257} = -2k$

$k = \frac{\ln\frac{225}{257}}{-2} \approx 0.066$

(b) $T(7) = 325 - 257e^{-0.066(7)} \approx 163$

12. JUN '16 [32]

$A = Pe^{rt}$

$135{,}000 = 100{,}000e^{5r}$

$1.35 = e^{5r}$

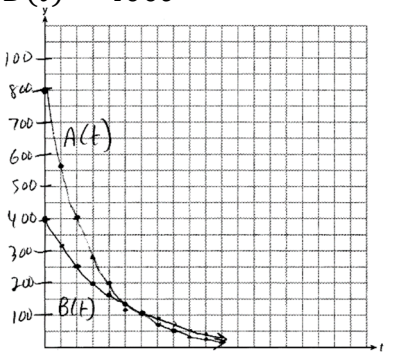
$\ln 1.35 = 5r$

$r = \frac{\ln 1.35}{5} \approx 0.06$ or 6%

13. JUN '16 [37]

(a) $A(t) = 800e^{-0.347t}$

$B(t) = 400e^{-0.231t}$



(b) $800e^{-0.347t} = 400e^{-0.231t}$

$\frac{800}{400} = \frac{e^{-0.231t}}{e^{-0.347t}}$

$2 = e^{0.116t}$

$\ln 2 = 0.116t$

$t \approx 6$

(c) $0.15 = e^{-0.347t}$

$\ln 0.15 = \ln e^{-0.347t}$

$\ln 0.15 = -0.347t$

$t = \frac{\ln 0.15}{-0.347} \approx 5.5$

14. JAN '17 [28]

$\frac{\ln\frac{1}{2}}{1590}$ is negative, so $M(t)$ represents decay.

15. JAN '18 [29]

$20e^{0.05t} = 30e^{0.03t}$

$\frac{2}{3}e^{0.05t} = e^{0.03t}$

$\frac{2}{3} = \frac{e^{0.03t}}{e^{0.05t}}$

$\frac{2}{3} = e^{-0.02t}$

$\ln\frac{2}{3} = -0.02t$

$t = \frac{\ln\frac{2}{3}}{-0.02} \approx 20.3$ months

16. AUG '18 [35]

$2 = e^{0.0375t}$

$\ln 2 = 0.0375t$

$t = \frac{\ln 2}{0.0375} \approx 18.5$

11.6 Evaluate Loan Formulas

1. JUN '22 [9] Ans: 3

2. SPR '15 [9]

$720 = \frac{120{,}000\left(\frac{0.048}{12}\right)\left(1+\frac{0.048}{12}\right)^n}{\left(1+\frac{0.048}{12}\right)^n - 1}$

$720 = \frac{480(1.004)^n}{(1.004)^n - 1}$

$1.5 = \frac{(1.004)^n}{(1.004)^n - 1}$

$1.5(1.004)^n - 1.5 = (1.004)^n$

$0.5(1.004)^n = 1.5$

$1.004^n = 3$

$n \log 1.004 = \log 3$

$n = \frac{\log 3}{\log 1.004} \approx 275.2$ months

$\frac{275.2}{12} \approx 23$ years

3. JAN '17 [36]

(a) $20{,}000 = PMT\left(\frac{1-1.00625^{-60}}{0.00625}\right)$

$PMT = 20{,}000\left(\frac{0.00625}{1-1.00625^{-60}}\right)$

$PMT \approx \$\,400.76$

(b) $21{,}000 - x = 300\left(\frac{1-1.00625^{-60}}{0.00625}\right)$

$x = -300\left(\frac{1-1.00625^{-60}}{0.00625}\right) + 21{,}000$

$x \approx \$\,6{,}028$

4. JUN '17 [34]

$N = 12 \times 15 = 180$

(a) $M = 172{,}600 \cdot \frac{0.00305(1.00305)^{180}}{1.00305^{180} - 1}$

$M \approx \$\,1{,}247$

(b) $1100 = (172{,}600 - x) \cdot \frac{0.00305(1.00305)^{180}}{1.00305^{180} - 1}$

$1100\left(\frac{1.00305^{180} - 1}{0.00305(1.00305)^{180}}\right) = 172{,}600 - x$

$x = -1100\left(\frac{1.00305^{180} - 1}{0.00305(1.00305)^{180}}\right) + 172{,}600$

$x \approx \$\,20{,}407$

5. JUN '18 [31]

$M = \frac{137{,}250\left(\frac{0.036}{12}\right)\left(1 + \frac{0.036}{12}\right)^{360}}{\left(1 + \frac{0.036}{12}\right)^{360} - 1} \approx \$624$

Chapter 12. Trigonometric Functions

12.1 Trigonometric Ratios

There are no Regents exam questions on this topic.

12.2 Radians

There are no Regents exam questions on this topic.

12.3 Unit Circle

1. AUG '16 [16] Ans: 1
2. AUG '17 [7] Ans: 4
3. JAN '18 [15] Ans: 1
4. JUN '22 [19] Ans: 2
5. AUG '22 [5] Ans: 4
6. JAN '17 [27]
   $\csc\theta = \frac{1}{\sin\theta}$, and $\sin\theta$ on a unit circle represents the y value of a point on the unit circle.
   Since $y = \sin\theta$. $\csc\theta = \frac{1}{y}$.

12.4 Solve Simple Trigonometric Equations

There are no Regents exam questions on this topic.

12.5 Circles of Any Radius

1. SPR '15 [3] Ans: 1
2. JUN '16 [17] Ans: 1
3. AUG '19 [5] Ans: 1
4. JUN '18 [32]
   $r = \sqrt{2^2 + 1^2} = \sqrt{5}$
   $\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{5}}$
5. JUN '19 [28]
   $r = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$
   $\cos\theta = -\frac{x}{r} = -\frac{24}{25}$

12.6 Pythagorean Identity

1. JAN '17 [4] Ans: 1
2. JUN '17 [12] Ans: 2
3. AUG '18 [11] Ans: 2
4. AUG '16 [28]
   $\sin^2\theta + (-0.7)^2 = 1$
   $\sin^2\theta = 0.51$
   $\sin\theta = \pm\sqrt{0.51}$
   Since θ is in Quadrant II, $\sin\theta = \sqrt{0.51}$
   and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{0.51}}{-0.7} \approx -1.02$
5. JAN '19 [31]
   $\sin^2\theta + \cos^2\theta = 1$
   $\left(\frac{4}{7}\right)^2 + t^2 = 1$
   $\frac{16}{49} + t^2 = 1$
   $t^2 = \frac{33}{49}$
   $t = \pm\frac{\sqrt{33}}{7}$
   Since θ is in Quadrant II, $t = -\frac{\sqrt{33}}{7}$

12.7 Simplify Trigonometric Expressions

1. AUG '22 [29]

$\cot A = \frac{\cos A}{\sin A}$

$-3 = \frac{\frac{3}{\sqrt{10}}}{\sin A}$

$\sin A = \frac{\frac{3}{\sqrt{10}}}{-3}$

$\sin A = \left(\frac{3}{\sqrt{10}}\right)\left(-\frac{1}{3}\right) = -\frac{1}{\sqrt{10}}$ or $-\frac{\sqrt{10}}{10}$

12.8 Graphs of Parent Trig Functions

1. JAN '20 [16] Ans: 4

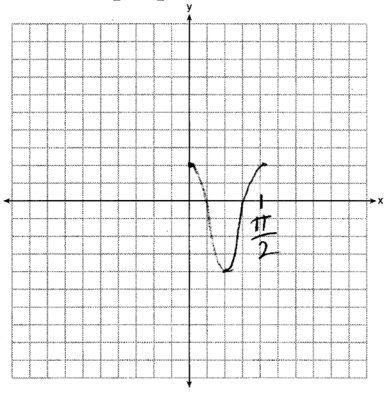
12.9 Trigonometric Transformations

1. FALL '15 [6] Ans: 4
2. JAN '17 [1] Ans: 2
3. JAN '17 [19] Ans: 2
4. JAN '17 [22] Ans: 3
5. JUN '17 [6] Ans: 4
6. JUN '17 [8] Ans: 1
7. AUG '17 [5] Ans: 3
8. AUG '17 [18] Ans: 4
9. AUG '18 [20] Ans: 1
10. JAN '19 [13] Ans: 1
11. JUN '22 [22] Ans: 2
12. AUG '22 [3] Ans: 2
13. AUG '22 [20] Ans: 4
14. JAN '18 [30]

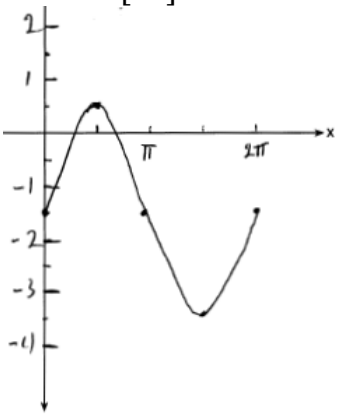
q has the smaller minimum value for the domain $[-2,2]$.
h's minimum is $2(-1) + 1 = -1$ and q's minimum is -8.

12.10 Graph Trigonometric Functions

1. JUN '16 [28]

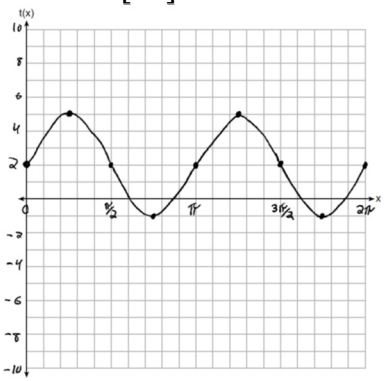


2. AUG '17 [35]



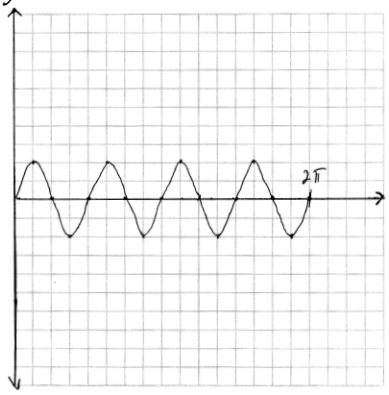
Part a sketch is shifted $\frac{\pi}{3}$ units right.

3. AUG ’18 [30]

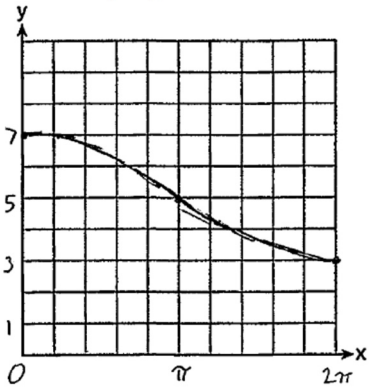


4. AUG ’19 [34]

$y = 2 \sin 4x$



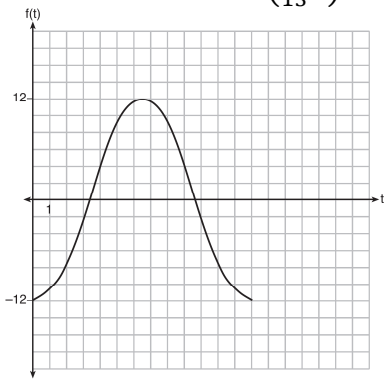
5. JUN ’22 [31]



12.11 Model Trigonometric Functions

1.	JUN ’16 [13]	Ans: 3
2.	JUN ’16 [24]	Ans: 4
3.	AUG ’16 [10]	Ans: 2
4.	JUN ’17 [15]	Ans: 4
5.	JAN ’18 [4]	Ans: 2
6.	JUN ’18 [10]	Ans: 4
7.	AUG ’18 [22]	Ans: 2
8.	JAN ’19 [7]	Ans: 1
9.	JUN ’19 [22]	Ans: 3
10.	AUG ’19 [12]	Ans: 4
11.	JUN ’22 [20]	Ans: 4

12. SPR '15 [14]
Since the water level has a minimum of –12 and a maximum of 12, the amplitude is 12. The value of A is –12 since at 8:30 it is low tide. The time from low to high tide is 6.5 hours, so the period of the function is 13 hours.
$\frac{2\pi}{B} = 13$, so $B = \frac{2\pi}{13}$.
So, $f(t) = -12\cos\left(\frac{2\pi}{13}t\right)$.

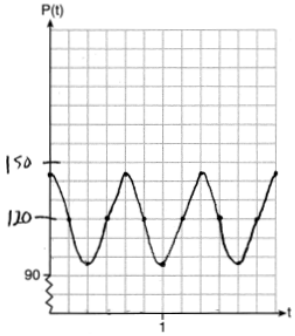


Since the function is increasing from $t = 13$ to $t = 19.5$ (which corresponds to 9:30 pm to 4:00 am), fishing at 10:30 pm is recommended.

13. AUG '16 [25]
Amplitude, because the height of the graph shows the volume of the air.

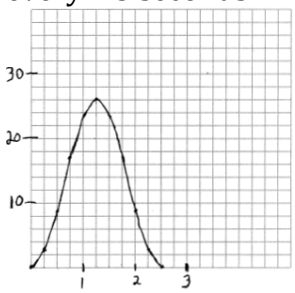
14. JUN '17 [28]
Period is $\frac{2}{3}$. The wheel rotates once every $\frac{2}{3}$ second.

15. JAN '18 [37]



The period of P is $\frac{2}{3}$, which means the patient's blood pressure reaches a high every $\frac{2}{3}$ second and a low every $\frac{2}{3}$ second. The patient's blood pressure is high because 144 over 96 is greater than 120 over 80.

16. JUN '19 [37]
Period is $\frac{2\pi}{0.8\pi} = 2.5$.
This means the wheel rotates once every 2.5 seconds.



No, because the maximum of $f(t) = 26$.

17. AUG '19 [28]
$250(1) + 2450 = 2700$.
The maximum lung capacity of a person is 2700 mL.

18. AUG '19 [30]
$\frac{10.1+2}{2} = 6.05$; $\frac{2.5+0.1}{2} = 1.3$;
$6.05 - 1.3 = 4.75$

19. JAN '20 [30]
The period for A is $2(340 - 60) = 560$; for B, it is $2(400 - 180) = 440$; and for C, it is $2(380 - 60) = 640$. Therefore, C's period is the longest.

Chapter 13. Properties of Functions

13.1 Even and Odd Function Graphs

1. FALL '15 [2] Ans: 3
2. AUG '16 [14] Ans: 1
3. JUN '18 [6] Ans: 2
4. AUG '19 [11] Ans: 2

13.2 Algebraically Determine Even or Odd [CC]

1. AUG '17 [31]
   $j(-x) = (-x)^4 - 3(-x)^2 - 4 = x^4 - 3x^2 - 4$
   Since $j(x) = j(-x)$, the function is even.

13.3 Inverse Functions

1. JUN '16 [16] Ans: 2
2. JAN '17 [8] Ans: 3
3. AUG '17 [14] Ans: 2
4. JAN '18 [21] Ans: 2
5. JUN '18 [15] Ans: 3
6. AUG '18 [6] Ans: 2
7. JAN '19 [17] Ans: 3
8. JUN '19 [9] Ans: 2
9. AUG '19 [23] Ans: 2
10. JUN '22 [17] Ans: 3
11. AUG '22 [23] Ans: 3
12. FALL '15 [9]
    $x = (y-3)^3 + 1$
    $x - 1 = (y-3)^3$
    $\sqrt[3]{x-1} = y - 3$
    $\sqrt[3]{x-1} + 3 = y$
    $f^{-1}(x) = \sqrt[3]{x-1} + 3$

13.4 Average Rate of Change

1. JAN '17 [21] Ans: 4
2. JAN '17 [24] Ans: 1
3. JUN '17 [21] Ans: 3
4. AUG '17 [9] Ans: 3
5. JUN '18 [7] Ans: 1
6. JUN '19 [4] Ans: 1
7. AUG '19 [22] Ans: 3
8. JAN '20 [12] Ans: 1
9. JUN '22 [12] Ans: 4
10. JUN '16 [36]
    $\frac{f(4)-f(-2)}{4-(-2)} = \frac{80-1.25}{6} = 13.125$
    $\frac{g(4)-g(-2)}{4-(-2)} = \frac{179-(-49)}{6} = 38$
    $g(x)$ has a greater rate of change
11. AUG '16 [31]
    $\frac{156.25-56.25}{70-50} = \frac{150}{20} = 7.5$
    Between 50-70 mph, each additional mph in speed requires 7.5 more feet to stop.

12. JUN '18 [36]
$\frac{h(2)-h(1)}{2-1} = -12$
Use the calculator's Calc → zero feature to find where $h(t) = 0$, or:
$12\cos\left(\frac{\pi}{3}t\right) + 8 = 0$
$\cos\left(\frac{\pi}{3}t\right) = -\frac{2}{3}$
Reference ∠ is $\cos^{-1}\left(\frac{2}{3}\right) \approx 0.8411$
cos is negative in Q2 and Q3, so
$\frac{\pi}{3}t = \pi - 0.8411 \approx 2.30$ and
$\frac{\pi}{3}t = \pi + 0.8411 \approx 3.98$
$\frac{\pi}{3}t = 2.30$ gives us $t = \frac{3(2.30)}{\pi} \approx 2.2$
$\frac{\pi}{3}t = 3.98$ gives us $t = \frac{3(3.98)}{\pi} \approx 3.8$
$h(t) = 0$ at $t \approx \{2.2, 3.8\}$.

13. AUG '18 [27]
$\frac{p(8)-p(4)}{8-4} \approx 48.78$

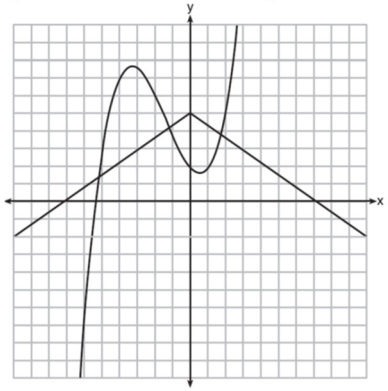
14. JAN '19 [30]
$\frac{B(11)-B(8)}{11-8} \approx -10.1$
The average monthly high temperature decreases 10.1 degrees each month from August to November.

15. JUN '19 [26]
$\frac{13.9-9.4}{4-1} = 1.5$
The average rate of change in the number of hours of daylight from January 1 to April 1 is 1.5.

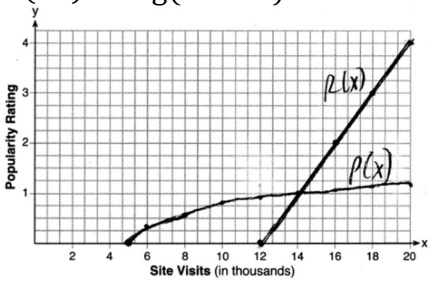
16. AUG '22 [25]
$\frac{60-20}{4-2} = 20$

13.5 Solutions to Equation of Two Functions

1. AUG '16 [3] Ans: 2
2. JAN '17 [12] Ans: 2
3. JAN '17 [16] Ans: 2
4. JUN '17 [5] Ans: 2
5. JAN '18 [14] Ans: 1
6. JAN '19 [24] Ans: 4
7. JUN '19 [14] Ans: 4
8. AUG '19 [20] Ans: 2
9. JAN '20 [21] Ans: 2
10. AUG '22 [10] Ans: 1
11. FALL '15 [10]
$\{-5.17, -1.13, 1.75\}$

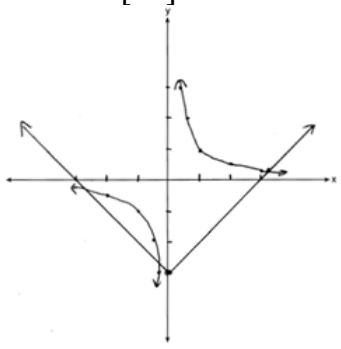


12. JUN '18 [37]
$P(16) = \log(16-4) \approx 1.1$



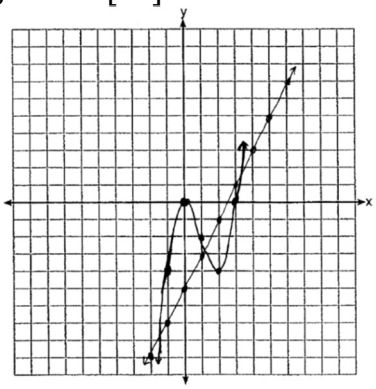
14000

13. AUG '19 [32]



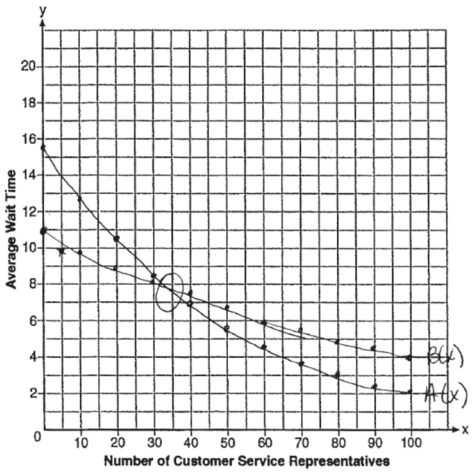
$x = 3.3$

14. JUN '22 [31]



3 solutions

15. AUG '22 [37]



$x \approx 35$

$4.0264 - 2.0812 \approx 2$ min, which is the difference in average wait time for companies with 100 representatives

13.6 Solutions to Inequality of Two Functions [NG]

There are no Regents exam questions on this topic.

Chapter 14. Sequences and Series

14.1 Arithmetic Sequences

1. JUN '17 [20] Ans: 3
2. AUG '17 [34]
$\frac{6.25 - 2.25}{21 - 5} = \frac{4}{16} = \$0.25$ fine per day.
$2.25 - 5(0.25) = \$1$ replacement fee.
$a_n = 1.25 + (n - 1)(0.25)$ or
$a_n = 0.25n + 1.00$
$a_{60} = \$16$

14.2 Geometric Sequences

1. JAN '17 [14] Ans: 1
2. JAN '19 [4] Ans: 2
3. JUN '19 [10] Ans: 3
4. JAN '20 [17] Ans: 1
5. AUG '17 [30]
$a_{1966} = 1.25$ and $a_{2015} = 8.75$
$r^{2015-1966} = \frac{8.75}{1.25}$
$r^{49} = 7$
$r = \sqrt[49]{7} \approx 1.04$
4 % growth

14.3 Recursively Defines Sequences

1. JUN '16 [10] Ans: 4
2. JUN '16 [23] Ans: 3
3. AUG '16 [8] Ans: 1
4. AUG '16 [18] Ans: 3
5. AUG '16 [24] Ans: 4
6. AUG '17 [24] Ans: 3
7. JAN '18 [24] Ans: 3
8. AUG '18 [10] Ans: 4
9. JAN '19 [16] Ans: 4
10. AUG '19 [9] Ans: 3
11. JAN '20 [13] Ans: 4
12. JUN '22 [21] Ans: 2
13. AUG '22 [16] Ans: 4
14. SPR '15 [11]
$a_1 = x + 1$
$a_2 = x(x + 1)$
$a_3 = x^2(x + 1)$
$a_n = x^{n-1}(x + 1)$
$x^{n-1} = 0$ or $x + 1 = 0$
$x = \{0, -1\}$
15. JAN '17 [34]
Jillian's plan, because distance increases by one mile each week.
$a_1 = 10$
$a_n = a_{n-1} + 1$
$a_n = n + 12$
16. AUG '17 [29] :
$a_1 = 4$
$a_n = 2a_{n-1} + 1$
$a_8 = 639$
17. JUN '18 [30]
$a_1 = 3, a_2 = 7, a_3 = 15, a_4 = 31$
No, because there is no common ratio:
$\frac{7}{3} \neq \frac{15}{7}$

18. JUN '19 [31]
Common ratio is $\frac{9}{6} = 1.5$
$a_1 = 6$
$a_n = 1.5a_{n-1}$

19. AUG '19 [31]
$a_1 = 4$
$a_n = 3a_{n-1}$

20. JUN '22 [37]
1.5%; $P(t) = 92.2(1.015)^t$;
$300 = 92.2(1.015)^t$
$\frac{300}{92.2} = 1.015^t$
$\log\frac{300}{92.2} = t\log 1.015$
$t = \frac{\log\frac{300}{92.2}}{\log 1.015} \approx 79$ years
[another appropriate equation is $P(t) = 92.2e^{0.015t}$, solve using ln]

14.4 Sigma Notation

There are no Regents exam questions on this topic.

14.5 Arithmetic Series

There are no Regents exam questions on this topic.

14.6 Geometric Series

1. AUG '16 [9] Ans: 1
2. AUG '17 [21] Ans: 4
3. JAN '18 [22] Ans: 2
4. AUG '18 [13] Ans: 1
5. JUN '19 [5] Ans: 4
6. AUG '19 [2] Ans: 3
7. AUG '22 [21] Ans: 1
8. JUN '16 [34]
$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$
$S_{15} = \frac{33{,}000 - 33{,}000(1.04)^{15}}{1 - 1.04} \approx$ 660,778.39
9. JAN '19 [29]
$S_{10} = \frac{15 - 15(1.03)^{10}}{1 - 1.03} \approx 171.958$
10. JAN '20 [29]
$S_n = \frac{300 - 300(1.2)^n}{1 - 1.2}$
$S_{10} = \frac{300 - 300(1.2)^{10}}{1 - 1.2} \approx 7787.6$
11. JAN '20 [33]
$a_n = 100(0.8)^{n-1}$
$S_{20} = \frac{100 - 100(0.8)^{20}}{1 - 0.8} \approx 494.2$
Sonja has $40 \times 12 = 480$ in of wire, so she does not have enough
12. JUN '22 [26]
$S_5 = \frac{6 - 6(0.8)^5}{1 - 0.8} \approx 20.17$

Chapter 15. Probability

15.1 Theoretical and Empirical Probability

1. JAN '20 [24] Ans: 3

2. AUG '17 [28]
Since there are six flavors, each flavor can be assigned a number, 1-6. Use the simulation to see the number of times the same number is rolled 4 times in a row.

15.2 Probability Involving And or Or

1. JUN '22 [4] Ans: 2

2. JUN '16 [29]
$P(S\ or\ M) = P(S) + P(M) - P(S\ and\ M)$
$\frac{974}{1376} = \frac{649}{1376} + \frac{433}{1376} - x$
$x = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376}$

3. AUG '19 [25]
$P(W\ or\ E) = P(W) + P(E) - P(W\ and\ E)$
$P(W\ or\ E) = \frac{165}{825} + \frac{66}{825} - \frac{33}{825} = \frac{198}{825}$

15.3 Two-Way Frequency Tables

1. AUG '16 [7] Ans: 1

2. JAN '20 [11] Ans: 4

3. FALL '15 [8]
Based on these data, the two events do not appear to be independent.
$P(F) = \frac{106}{200} = 0.53$, while
$P(F|T) = \frac{54}{90} = 0.6$,
$P(F|R) = \frac{25}{65} = 0.39$, and
$P(F|C) = \frac{27}{45} = 0.6$.
The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.

4. JAN '17 [31]
No, because $P(M|R) \neq P(M)$
$\frac{70}{180} \neq \frac{230}{490}$ $0.38 \neq 0.47$

5. JUN '17 [32]
A student is more likely to jog if both siblings jog.
one jogs: $\frac{416}{2239} \approx 0.19$.
both jog: $\frac{400}{1780} \approx 0.22$.

6. JUN '18 [25]
$\frac{103}{110+103} = \frac{103}{213}$

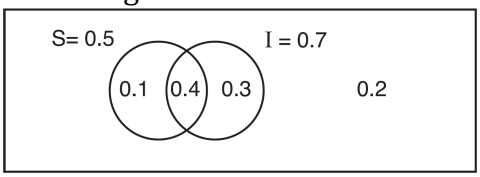
7. JUN '19 [36]
$P(F|L) = \frac{12}{27}$; $P(F) = \frac{22}{45}$;
Since $P(F|L) \neq P(F)$, the events are not independent.

8. JUN '22 [29]
$P(bl|gl) = \frac{0.14}{0.35} = 0.4$;
$P(bl) = 0.14 + 0.26 = 0.4$;
Yes, because $P(bl|gl) = P(bl)$

9. AUG '22 [31]
$P(F|CR) = \frac{36}{78} \approx 0.462$;
$P(F) = \frac{105}{215} \approx 0.488$;
No, because $P(F|CR) \neq P(F)$

15.4 Series of Events [CC]

1. JUN '16 [11] Ans: 1
2. JUN '17 [14] Ans: 2
3. JUN '18 [11] Ans: 4
4. AUG '18 [18] Ans: 2
5. AUG '18 [24] Ans: 4
6. JAN '19 [12] Ans: 2
7. AUG '19 [13] Ans: 2
8. JAN '20 [8] Ans: 4
9. SPR '15 [13]
   This scenario can be modeled with a Venn Diagram:



   Since $P(\overline{S\ or\ I}) = 0.2$, $P(S\ or\ I) = 0.8$.
   $P(S\ and\ I) = P(S) + P(I) -$
   $P(S\ or\ I) = 0.5 + 0.7 - 0.8 = 0.4$
   If S and I are independent,
   $P(S\ and\ I) = P(S) \cdot P(I)$.
   However, $0.4 \neq (0.5)(0.7)$.
   Therefore, salary and insurance have not been treated independently.

10. AUG '16 [32]
    $P(A \text{ or } B) = P(A) + P(B) -$
    $P(A \text{ and } B)$
    $0.8 = 0.6 + 0.5 - P(A \text{ and } B)$
    $P(A \text{ and } B) = 0.3$
    A and B are independent since
    $P(A \text{ and } B) = P(A) \times P(B)$
    $0.3 = 0.6 \times 0.5$
11. JAN '17 [35]
    $P(P|K) = \frac{P(P\ and\ K)}{P(K)} = \frac{0.019}{0.023} \approx 82.6\%$
    A key club member has an 82.6% probability of being enrolled in AP Physics.
12. AUG '17 [26]
    $P(W|D) = \frac{P(W \text{ and } D)}{P(D)} = \frac{0.4}{0.5} \approx 0.8$
13. JAN '18 [34]
    $\frac{47}{108} = \frac{1}{4} + \frac{116}{459} - P(M \text{ and } J)$
    $P(M \text{ and } J) = \frac{31}{459}$
    No, because $\frac{31}{459} \neq \frac{1}{4} \times \frac{116}{459}$
14. JAN '19 [28]
    $P(A\ and\ B) = P(A) \cdot P(B|A) =$
    $(0.8)(0.85) = 0.68$

Chapter 16. Statistics

16.1 Data Collection

1. JAN '17 [6] Ans: 3
2. AUG '17 [17] Ans: 2
3. AUG '18 [2] Ans: 2
4. JUN '19 [1] Ans: 3
5. AUG '19 [6] Ans: 4
6. JAN '20 [15] Ans: 3
7. JUN '22 [16] Ans: 4
8. AUG '22 [4] Ans: 2
9. JUN '16 [26]
   Randomly assign participants to two groups. One group uses the toothpaste with ingredient *X* and the other group uses the toothpaste without ingredient *X*.
10. JAN '17 [26]
    sample: pails of oranges; population: truckload of oranges. It is likely that about 5% of all the oranges are unsatisfactory.

16.2 Bias

1. AUG '16 [2] Ans: 1
2. JUN '17 [3] Ans: 3
3. JAN '18 [1] Ans: 4
4. JAN '19 [10] Ans: 2
5. JUN '22 [2] Ans: 3
6. AUG '22 [1] Ans: 3
7. JUN '18 [28]
   Self selection is a cause of bias because people with more free time are more likely to respond.

16.3 Normal Distribution

1. JUN '16 [9] Ans: 2
2. JAN '17 [18] Ans: 4
3. JAN '19 [1] Ans: 2
4. JAN '19 [23] Ans: 1
5. JUN '19 [18] Ans: 4
6. AUG '19 [19] Ans: 1

16.4 Areas Under Normal Curves

1. AUG '16 [4] Ans: 3
2. AUG '17 [11] Ans: 1
3. JAN '18 [7] Ans: 3
4. JUN '18 [17] Ans: 2
5. JUN '22 [14] Ans: 1

6. FALL '15 [16]
normalcdf(510, 540, 480, 24) ≈ 0.0994.
Use z-scores to compare the two sets of data. Joanne's score range corresponds to 1.25 to 2.5 SD above the mean:
$z = \frac{510-480}{24} = 1.25$
$z = \frac{540-480}{24} = 2.5$
Calculating equivalent scores,
$1.25 = \frac{x-510}{20}$
$x = 535$
$2.5 = \frac{x-510}{20}$
$x = 560$
Maria must score in the interval 535–560.

7. JUN '17 [26]
normalcdf(0, 8.25, 8, 0.5) ≈ 69%

8. AUG '18 [28]
normalcdf(200, 245, 225, 18) ≈ 0.784
1200 × 0.784 ≈ 941

9. AUG '22 [30]
normalcdf(4, 1E99, 3.39, 0.55) ≈ 0.1337.
0.1337 × 9256 ≈ 1237

16.5 Plausible Outcomes

1. JUN '16 [7] Ans: 3
2. JUN '17 [10] Ans: 3
3. JAN '18 [20] Ans: 2
4. JUN '19 [13] Ans: 2
5. JAN '20 [14] Ans: 4
6. SPR '15 [12]
a) Yes. The margin of error = $2SD = 2(0.062) \approx 0.124$.
This indicates that 95% of the observations fall within ± 0.124 of 0.247. So, plausible values would fall into the interval $(0.123, 0.371)$. 9 out of 50, or 0.18, falls within this interval.
b) The company has evidence that the population proportion could be at least 25%. As seen in the dot plot, it can be expected to obtain a sample proportion of 0.18 or less several times, even when the population proportion is 0.25, due to sampling variability. The results do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue.

7. JUN '16 [35]
$0.602 \pm 2(0.066) \approx (0.47, 0.73)$
Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

8. AUG '16 [29]
Using a 95% level of confidence, $\mu \pm 2$ standard deviations, or $226 \pm 2(38)$, sets the usual wait time as 150 to 302 seconds. 360 seconds is unusual.

9. JUN '17 [36]
$0.506 \pm 2(0.078) = (0.35, 0.66)$
The 32.5% value falls below (outside of) the 95% interval.

10. JAN '18 [35]
$138.905 \pm 2(7.95) = (123, 155)$
No, since 125 (50% of 250) falls within the 95% interval.

11. AUG '18 [32]
$2(0.042) \approx 0.08$
The percent of users making in-app purchases will be within 8 percentage points of 35%; that is, between 27% and 43%.

12. JAN '19 [35]
$29.101 \pm 2(0.934) = (27.23, 30.97)$
Yes, since 30 falls within the 95% interval.

13. JUN '19 [32]
$0.499 \pm 2(0.049) = (0.401, 0.597)$
No, since 0.43 falls within this interval, Robin's coin is likely not unfair.

14. AUG '19 [35]
$0.301 \pm 2(0.058) = (0.185, 0.417)$
$\frac{14}{60} \approx 0.23$, which is not unusual because it falls within this interval.

15. JUN '22 [35]
$0.651 \pm 2(0.034) = (0.58, 0.72)$
No, because $\frac{122}{200} = 0.61$ falls within this interval.

16. AUG '22 [36]
$0.819 \pm 2(0.053) = (0.713, 0.925)$
Because 0.7 does not fall within this interval.

16.6 Difference in Means [CC]

1. JAN '17 [9] Ans: 2

2. FALL '15 [14] :
a) The mean difference between the students' final grades in group 1 and group 2 is –3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription.
b) If the observed difference –3.64 is the result of the assignment of students to groups alone, then a difference of –3.64 or less should be observed fairly regularly in the simulation output. However, a difference of –3 or less occurs in only about 2% of the results. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups' mean final grades.

3. AUG '16 [36]
Some of the students who did not drink energy drinks read faster than those who did drink energy drinks.
$17.7 - 19.1 = -1.4$. Differences of -1.4 and less occur $\frac{25}{232}$ or about 10% of the time, so the difference is not unusual.

4. JUN '18 [34]
$23 - 18 = 5$
$\bar{x} \pm 2\sigma = 0.030 \pm 2(1.548) = (-3.066, 3.126)$
Yes, 5 is outside of the interval of the middle 95%, so it is statistically significant.

5. AUG '18 [36]
John found the means of the scores of the two rooms and subtracted the means. The mean score for the classical room was 7 higher than the rap room $(82 - 75)$. Yes, only 4 of the 250 results (1.6%) had a mean difference of 7 or more, which is less than 5% and therefore statistically significant. It is likely the difference was due to the music.

16.7 Estimate Population Parameters

1. AUG ’16 [12] Ans: 2
2. AUG ’17 [16] Ans: 2
3. AUG ’17 [22] Ans: 1