ANSWER KEY

Geometry

NEXT GENERATION

Course Workbook

with Regents Questions

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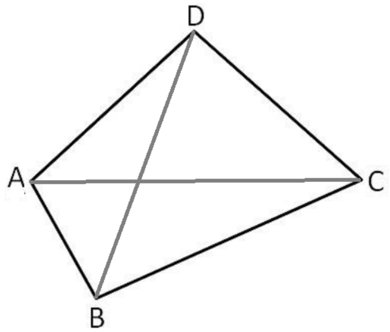
Regents Exam Notation

A code next to each Regents Question answer number indicates from which Common Core (CC) or Next Generation (NG) Regents exam or sampler the question came. For example, CC AUG '18 [25] means the question appeared on the August 2018 exam as question 25.

PRACTICE PROBLEMS

CHAPTER 1. BASIC GEOMETRY

1.1 Lines, Angles and Shapes

1. (3) line segment	2. (2) ∠CAD
3. (4) $\overline{ZYXW}$	4. (1) congruent angles
5. (4) obtuse	6. (2) m∠2 = 70°
7. (2) 90°	8. (2) The angles are adjacent.
9. (3) $\angle ECF$ and $\angle DCH$ are a pair of vertical angles	10. (4) $BC = \frac{1}{2}AC$
11. $2x - 40 = x + 10$ $x - 40 = 10$ $x = 50$ $AB = BC = 60$ $AC = 120$	12. $6x + 5 + x = 180$ $7x + 5 = 180$ $7x = 175$ $x = 25$
13. $x + 15 + x - 5 = 90$ $2x + 10 = 90$ $2x = 80$ $x = 40$ $\text{m}\angle BAC = 40 + 15 = 55°$ $\text{m}\angle DAC = 40 - 5 = 35°$	14. $2(4x + 1) = 75$ $8x + 2 = 75$ $8x = 73$ $x = 9.125$
15. $\triangle LMN$, $\triangle MNL$, $\triangle NLM$, $\triangle LNM$, $\triangle NML$, or $\triangle MLN$	16. (3) Consecutive vertices must be listed in clockwise or counterclockwise order.
17. Diagonals $\overline{AC}$ and $\overline{BD}$. 	18. a) Triangles ABE, CDE, and BCE. b) Quadrilaterals $ABCD$, $ABCE$, and $BCDE$.
19. a) Triangles TUW, RVW, RQT, and TSR. b) Quadrilaterals $QTSR$, $QTUV$, $USRV$, $QTWV$, and $SRWU$. c) Pentagons $QTUWR$ and $SRVWT$.	

1.2 Pythagorean Theorem

1. $5^2 + 7^2 = c^2$ $74 = c^2$ $c = \sqrt{74} \approx 8.6$	2. $24^2 + b^2 = 26^2$ $576 + b^2 = 676$ $b^2 = 100$ $b = 10$ cm
3. $x^2 + 19.5^2 = 20^2$ $x^2 + 380.25 = 400$ $x^2 = 19.75$ $x \approx 4.4$ ft.	4. $9^2 + b^2 = 18^2$ $b^2 = 243$ $b \approx 15.6$
5. $8^2 + x^2 = 10^2$ $64 + x^2 = 100$ $x^2 = 36$ $x = 6$ $4^2 + y^2 = 10^2$ $16 + y^2 = 100$ $y^2 = 84$ $y = \sqrt{84} \approx 9.2$ $y - x \approx 9.2 - 6 \approx 3.2\, ft.$	
6. $x^2 + 7^2 = (x+1)^2$ $x^2 + 49 = x^2 + 2x + 1$ $2x = 48$ $x = 24$ Vertical bar is $x + 1 = 25$ inches	

1.3 Perimeter and Circumference

1. $x + (x+2) + (x-3)$ Perimeter is $3x - 1$	2. $4x + (x+3) + 3x - 1 = 34$ $8x + 2 = 34$ $8x = 32$ $x = 4$ $4x = 16, x + 3 = 7, 3x - 1 = 11$
3. Length of □ = diameter = 8 in. Perimeter = $\frac{1}{2}2\pi r + l + 2w$ $= \frac{1}{2}8\pi + 8 + 14 = 4\pi + 22 \approx 34.6$ in.	4. Width of □ = radius = 2 cm. Perimeter = $\frac{1}{2}2\pi r + r + 2l + w$ $= \frac{1}{2}4\pi + 2 + 8 + 2 = 2\pi + 12$ ≈ 18.3 cm.
5. $\frac{1}{2}15\pi + 60 + 15 = 7.5\pi + 75 \approx 98.6$ ft.	6. $\frac{1}{2}4\pi + 4 \cdot 4 = 2\pi + 16 \approx 22.3$ cm.
7. $\frac{1}{2}12\pi + 8 + 14 = 6\pi + 22$ m	8. $\frac{1}{2}6\pi + 4 + 10 + 6 = 3\pi + 20$
9. All sides of the polygon are 3.5. Perimeter = $3.5 \times 7 = 24.5$ inches	10. Each side $s = 2r = 10$ Perimeter = $4\left(\frac{1}{4}2\pi r\right) + 4s = 10\pi + 40$

11. Length of arc $SBT = \frac{1}{4}2\pi r = \frac{1}{4}12\pi = 3\pi$.

Since $ABCR$ is a rectangle, its diagonals are $\cong$.

Diagonal RB is a radius, so $AC = RB = 6$.

Two methods to find $CT + AS$:

METHOD 1

Let $w = RC$, so $AR = 8 - w$.

$CT = r - RC = 6 - w$

$AS = r - AR = 6 - (8 - w) = w - 2$

$CT + AS = 6 - w + w - 2 = 4$

METHOD 2

Since RT and RS are both radii,

$RT + RS = 6 + 6 = 12.$

$CT + AS = (RT + RS) - (RC + RA) =$

$12 - 8 = 4$

Perimeter $= \widehat{SBT} + AC + (CT + AS) = 3\pi + 6 + 4 = 3\pi + 10$

1.4 Area

1. $\frac{1}{2}\pi r^2 + s^2 = \frac{1}{2}36\pi + 12^2 = 18\pi + 144$

2. $\pi r^2 + lw = 5^2\pi + (20)(10) = 25\pi + 200$

3. $\frac{3}{4}\pi r^2 + s^2 = \frac{3}{4}\pi(4)^2 + 4^2 = 12\pi + 16$

4. $\pi r^2 + s^2 = \pi x^2 + (2x)^2 = \pi x^2 + 4x^2$ or $(\pi + 4)x^2$

5. $s^2 - \pi r^2 = 8^2 - 4^2\pi = 64 - 16\pi$

6. $s = \sqrt{36} = 6 \qquad r = \frac{1}{2}s = 3$

$A = \pi r^2 = 9\pi$

7. $A = s^2 - \frac{1}{2}\pi r^2 = 6^2 - \frac{1}{2}9\pi = 36 - 4.5\pi$

8. $lw - 2\pi r^2 = (20)(10) - 2(5^2)\pi =$
$200 - 50\pi$

9. $\frac{1}{4}\pi r^2 - \frac{1}{2}bh = \frac{1}{4}(4^2)\pi - \frac{1}{2}(4)(4) =$
$4\pi - 8$

10. $lw - \frac{1}{2}bh = (12)(4) - \frac{1}{2}(9)(4) = 30$

11. $s^2 - \pi r^2 = 6^2 - 3^2\pi = 36 - 9\pi$ sq. in.

12. $lw - \pi r^2 = (36)(15) - 4^2\pi = 540 - 16\pi \approx 490$ sq. ft. $\qquad 490 \times 1.95 = \$955.50$

13. $(80 - 70)^2 + (40 - 15)^2 = c^2$

$10^2 + 25^2 = c^2$

$725 = c^2$

$c = \sqrt{725} \approx 26.93 \qquad Area \approx 26.93 \times 6 \approx 162\ sq. ft.$

14. $density = \frac{population}{area}$

$27{,}785 = \frac{x}{303}$

$x \approx 8{,}419{,}000$

15. $A = 2000 \times 500 = 1{,}000{,}000$ sq. ft.

$1{,}000{,}000\ ft^2 \times \frac{1\ acre}{43{,}560\ ft^2} \approx 22.957\ acres$

$d = \frac{6{,}887}{22.957} \approx 300$ trees per acre

16. a) $\frac{14.7}{100{,}000} = \frac{x}{1{,}632{,}000}$

$100{,}000x = 23{,}990{,}400$

$x \approx 240$

b) $d = \frac{240}{22.8} \approx 10.5$ per square mile

CHAPTER 2. COORDINATE GEOMETRY

2.1 Forms of Linear Equations

1. No. $(3) = 3(-2) + 15?$ $3 \neq 9$	2. $y = mx + b$ $y = -4x + 5$
3. $y - y_1 = m(x - x_1)$ $y + 2 = -3(x - 1)$	4. $m = \frac{-5+3}{5+2} = -\frac{2}{7}$ $y - y_1 = m(x - x_1)$ $y + 3 = -\frac{2}{7}(x + 2)$
5. $m = \frac{5-3}{8-1} = \frac{2}{7}$ $y - 3 = \frac{2}{7}(x - 1)$	6. $m = \frac{4-0}{5-(-5)} = \frac{2}{5}$ $y - 4 = \frac{2}{5}(x - 5)$
7. $-2x + y = -5$ $2x - y = 5$	8. $4y = 4\left(\frac{3}{4}x + \frac{1}{2}\right)$ $4y = 3x + 2$ $-3x + 4y = 2$ $3x - 4y = -2$

2.2 Parallel and Perpendicular Lines

1. equation (1) $2y + 2x = 6$ $2y = -2x + 6$ $y = -x + 3$	2. equation (1) $-3y = 2x + 5$ $-2x - 3y = 5$ $-2(-2x - 3y = 5)$ $4x + 6y = -10$
3. equation (2)	4. equation (1) $x - 5y = 25$ $-5y = -x + 25$ $y = \frac{1}{5}x - 5$
5. $y = -2x + 2$	6. $y = -2x$
7. $2y - x = 8 \qquad (-1) = \frac{1}{2}(4) + b$ $2y = x + 8 \qquad b = -3$ $y = \frac{1}{2}x + 4$ $m = \frac{1}{2} \qquad y = \frac{1}{2}x - 3$	8. $m = -3$ $(12) = -3(-9) + b$ $b = -15$ $y = -3x - 15$
9. $3y = 6x + 3 \qquad (4) = -\frac{1}{2}(2) + b$ $y = 2x + 1 \qquad b = 5$ $m = -\frac{1}{2} \qquad y = -\frac{1}{2}x + 5$	10. The slope of $y = 2x + 3$ is 2. The slope of $2y + x = 6$ is $-\frac{1}{2}$. Since the slopes are opp reciprocals, the lines are ⊥.

11. The slope of $x + 2y = 4$ is $-\frac{1}{2}$. The slope of $4y - 2x = 12$ is $\frac{1}{2}$. Since the slopes are neither = nor opp reciprocals, the lines are neither ∥ nor ⊥.	12. $\frac{x-1}{4} = \frac{-3}{8}$ $2(x-1) = -3$ $2x - 2 = -3$ $x = -\frac{1}{2}$

2.3 Distance Formula

1. $d = \sqrt{(6+2)^2 + (-3-3)^2}$ $= \sqrt{64+36} = \sqrt{100} = 10$	2. $d = \sqrt{(8-3)^2 + (10-5)^2}$ $= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$
3. $d = \sqrt{(7+1)^2 + (4-9)^2}$ $= \sqrt{64+25} = \sqrt{89}$	4. $d = \sqrt{(1-5)^2 + (6-3)^2}$ $= \sqrt{16+9} = \sqrt{25} = 5$
5. $d = \sqrt{(7-3)^2 + (2+4)^2}$ $= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$	6. $d = \sqrt{(6-3)^2 + (-1-8)^2}$ $= \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$
7. $d = \sqrt{(4+3)^2 + (25-1)^2}$ $= \sqrt{49+576} = \sqrt{625} = 25$	8. $d = \sqrt{(146+4)^2 + (52-2)^2}$ $= \sqrt{22{,}500 + 2{,}500} = \sqrt{25{,}000} = 50\sqrt{10}$

9. $m_\perp = -5$			
Equation of ⊥ line:	Solve system for x:	Solve for y:	Distance for $(6,-2)$ to $(5,3)$:
$y + 2 = -5(x-6)$	$-5x + 28 = \frac{1}{5}x + 2$	$y = \frac{1}{5}(5) + 2$	$d = \sqrt{(5-6)^2 + (3+2)^2}$
$y = -5x + 28$	$x = 5$	$y = 3$	$= \sqrt{1+25} = \sqrt{26}$

2.4 Midpoint Formula

1. $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+6}{2}, \frac{3-3}{2}\right) = (2,0)$	2. $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3+8}{2}, \frac{5+10}{2}\right) =$ $\left(5\frac{1}{2}, 7\frac{1}{2}\right)$
3. $\left(\frac{-5+3}{2}, \frac{1+5}{2}\right) = (-1,3)$	
4. The third terms of the arithmetic sequences x: 1, 3, ... and y: −5, 5, ... are 5 and 15, so $B(5,15)$.	5. M is the midpoint of BC. Sequences x: 4, 0, ... and y: −3, −2, ... give us third terms of −4 and −1, so $C(-4,-1)$.
6. x: 1, 3.5, 6 and y: 8, 2, −4, so $R(6,-4)$	7. x: −1, 2, 5 and y: 5, 3, 1, so $(5,1)$

2.5 Perpendicular Bisectors

1. Midpoint is $\left(\frac{3+9}{2},\frac{5+17}{2}\right)=(6,11)$ $m=\frac{17-5}{9-3}=\frac{12}{6}=2 \qquad m_{\perp}=-\frac{1}{2}$ Equation of $\perp$ bisector is $y-11=-\frac{1}{2}(x-6)$	2. Midpoint is $\left(\frac{-2+6}{2},\frac{3-3}{2}\right)=(2,0)$ $m=\frac{-3-3}{6+2}=\frac{-6}{8}=-\frac{3}{4} \qquad m_{\perp}=\frac{4}{3}$ Equation of $\perp$ bisector is $y=\frac{4}{3}(x-2)$
3. Midpoint is $\left(\frac{2+8}{2},\frac{6+12}{2}\right)=(5,9)$ $m=\frac{12-6}{8-2}=\frac{6}{6}=1 \qquad m_{\perp}=-1$ Equation of $\perp$ bisector is $y-9=-(x-5)$ $y-9=-x+5$ $y=-x+14$ y-intercept is 14	4. Midpoint is $\left(\frac{-4+2}{2},\frac{5+5}{2}\right)=(-1,5)$ The segment is horizontal with a 0 slope. Its $\perp$ bisector is vertical, with an equation of $x=-1$.
5. $M=\left(\frac{4+8}{2},\frac{2+6}{2}\right)=(6,4)$ $m=\frac{6-2}{8-4}=1 \qquad m_{\perp}=-1$ $y-4=-(x-6)$	6. $M=\left(\frac{-1+7}{2},\frac{1-5}{2}\right)=(3,-2)$ $m=\frac{1-(-5)}{-1-7}=-\frac{3}{4} \qquad m_{\perp}=\frac{4}{3}$ $y+2=\frac{4}{3}(x-3)$
7. $M=\left(\frac{3+3}{2},\frac{-1+5}{2}\right)=(3,2)$ Line segment is vertical, so its $\perp$ bisector is horizontal, with equation $y=2$.	8. $M=\left(\frac{0+6}{2},\frac{0+0}{2}\right)=(3,0)$ Line segment is horizontal, so its $\perp$ bisector is vertical, with equation $x=3$.

2.6 Directed Line Segments

1. $k=\frac{5}{6}$ $P_x=A_x+\frac{5}{6}(B_x-A_x)=3+\frac{5}{6}(9-3)=8$ $P_y=A_y+\frac{5}{6}(B_y-A_y)=5+\frac{5}{6}(17-5)=15$ $P(8,15)$	2. $k=\frac{3}{5}$ $S_x=R_x+\frac{3}{5}(T_x-R_x)=-2+\frac{3}{5}(3+2)=1$ $S_y=R_y+\frac{3}{5}(T_y-R_y)=2+\frac{3}{5}(-8-2)=-4$ $S(1,-4)$
3. $k=\frac{3}{8}$ $P_x=L_x+\frac{3}{8}(M_x-L_x)=-2+\frac{3}{8}(6+2)=1$ $P_y=L_y+\frac{3}{8}(M_y-L_y)=3+\frac{3}{8}(-3-3)=\frac{3}{4}$ $P\left(1,\frac{3}{4}\right)$	4. $k=\frac{2}{5}$ $G_x=F_x+\frac{2}{5}(H_x-F_x)=1+\frac{2}{5}(6-1)=3$ $G_y=F_y+\frac{2}{5}(H_y-F_y)=-3+\frac{2}{5}(5+3)=\frac{1}{5}$ $G\left(3,\frac{1}{5}\right)$

CHAPTER 3. POLYGONS IN THE COORDINATE PLANE

3.1 Triangles in the Coordinate Plane

1. $m_{\overline{AB}} = \frac{11-7}{5-2} = \frac{4}{3}$ $m_{\overline{BC}} = \frac{8-11}{9-5} = -\frac{3}{4}$ $\overline{AB} \perp \overline{BC}$, so $\angle B$ is a right $\angle$. Therefore, ABC is a right $\triangle$.	2. $AB = \sqrt{(5-2)^2 + (11-7)^2} = \sqrt{25} = 5$ $BC = \sqrt{(9-5)^2 + (8-11)^2} = \sqrt{25} = 5$ $AC = \sqrt{(9-2)^2 + (8-7)^2} = \sqrt{50} = 5\sqrt{2}$ Two sides are equal in length, so $\triangle ABC$ is an isosceles $\triangle$.
3. $DE = \sqrt{(5-4)^2 + (5+2)^2} = \sqrt{50}$ $EF = \sqrt{(-1-5)^2 + (3-5)^2} = \sqrt{40}$ $DF = \sqrt{(-1-4)^2 + (3+2)^2} = \sqrt{50}$ Two sides are equal in length, so $\triangle DEF$ is an isosceles $\triangle$.	4. $\overline{JK} \perp \overline{KL}$ $m_{\overline{JK}} = \frac{6-4}{6+2} = \frac{1}{4}$, so $m_{\overline{KL}} = -4$ $\frac{-2-6}{x-6} = -4$ $-8 = -4(x-6)$ $-8 = -4x + 24$ $-32 = -4x$ $x = 8$
5. Recognize that $\overline{PR}$ is horizontal (since $y = -2$ for both endpoints). $PR = 8 - 4 = 4$ An altitude may be drawn from the opp vertex, Q, to point $S(-6, -2)$. $QS = 4 - (-2) = 6$ Therefore, the area is $\frac{1}{2}(4)(6) = 12$ square units.	6. $AB = \sqrt{(5-2)^2 + (1-2)^2} = \sqrt{10}$ $DE = \sqrt{(4-1)^2 + (-5+4)^2} = \sqrt{10}$ $BC = \sqrt{(4-5)^2 + (5-1)^2} = \sqrt{17}$ $EF = \sqrt{(3-4)^2 + (-1+5)^2} = \sqrt{17}$ $AC = \sqrt{(4-2)^2 + (5-2)^2} = \sqrt{13}$ $DF = \sqrt{(3-1)^2 + (-1+4)^2} = \sqrt{13}$ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, so $\triangle ABC \cong \triangle DEF$ by SSS.
7. $m_{\overline{RS}} = \frac{-1-7}{3+1} = -2 \quad m_{\overline{ST}} = \frac{2+1}{9-3} = \frac{1}{2}$ The slopes are opp reciprocals, so they are $\perp$ lines forming a right $\angle$ at S. Since $\angle S$ is a right $\angle$, $\triangle RST$ is a right $\triangle$.	8. To prove that $\triangle JEN$ is a right $\triangle$, prove that its legs are $\perp$ by showing their slopes are opp reciprocals: $m_{\overline{JE}} = \frac{-3-1}{-2+4} = -2 \quad m_{\overline{EN}} = \frac{-1+3}{2+2} = \frac{1}{2}$ To prove that $\triangle JEN$ is an isosceles $\triangle$, prove that it legs are $\cong$ by using the distance formula: $JE = \sqrt{(-2+4)^2 + (-3-1)^2} = \sqrt{20}$ $EN = \sqrt{(2+2)^2 + (-1+3)^2} = \sqrt{20}$

3.2 Quadrilaterals in the Coordinate Plane

1. $\left(\frac{1+10}{2},\frac{3+7}{2}\right)=\left(\frac{11}{2},5\right)$
   [diagonals bisect each other, so E is the midpoint of both $\overline{AC}$ and $\overline{BD}$]

2. $m_{\overline{AB}}=\frac{-8-0}{-1+5}=-2 \quad m_{\overline{CD}}=\frac{4+4}{3-7}=-2$
   $m_{\overline{BC}}=\frac{-4+8}{7+1}=\frac{1}{2} \quad m_{\overline{AD}}=\frac{4-0}{3+5}=\frac{1}{2}$
   $\overline{AB}\perp\overline{BC}$, $\overline{BC}\perp\overline{CD}$, $\overline{CD}\perp\overline{AD}$, and $\overline{AD}\perp\overline{AB}$, so all 4 ∠'s are right ∠'s.
   Therefore, $ABCD$ is a ▭.

3. $AB=\sqrt{(1+6)^2+(0+3)^2}=\sqrt{58}$
   $BC=\sqrt{(4-1)^2+(7-0)^2}=\sqrt{58}$
   $CD=\sqrt{(-3-4)^2+(4-7)^2}=\sqrt{58}$
   $AD=\sqrt{(-3+6)^2+(4+3)^2}=\sqrt{58}$
   $m_{\overline{AB}}=\frac{0+3}{1+6}=\frac{3}{7} \quad m_{\overline{BC}}=\frac{7-0}{4-1}=\frac{7}{3}$
   4 ≅ sides but not 4 right ∠'s → Rhombus

4. $AB=\sqrt{(2+5)^2+(0+6)^2}=\sqrt{85}$
   $BC=\sqrt{(11-2)^2+(9-0)^2}=\sqrt{162}$
   $CD=\sqrt{(4-11)^2+(3-9)^2}=\sqrt{85}$
   $AD=\sqrt{(4+5)^2+(3+6)^2}=\sqrt{162}$
   $m_{\overline{AB}}=\frac{0+6}{2+5}=\frac{6}{7} \quad m_{\overline{BC}}=\frac{9-0}{11-2}=1$
   2 pairs of opp sides ≅ but not 4 right ∠'s → ▱

5. $AB=\sqrt{(5-1)^2+(2-1)^2}=\sqrt{17}$
   $BC=\sqrt{(6-5)^2+(-2-2)^2}=\sqrt{17}$
   $CD=\sqrt{(2-6)^2+(-3+2)^2}=\sqrt{17}$
   $AD=\sqrt{(2-1)^2+(-3-1)^2}=\sqrt{17}$
   $m_{\overline{AB}}=\frac{2-1}{5-1}=\frac{1}{4} \quad m_{\overline{BC}}=\frac{-2-2}{6-5}=-4$
   $\overline{AB}\perp\overline{BC}$, so $\angle B$ is a right ∠.
   4 ≅ sides and a right ∠ → Square

6. $m_{\overline{AB}}=\frac{4-1}{4-2}=\frac{3}{2}$
   Since $\overline{AB}\perp\overline{BC}$, $m_{\overline{BC}}=-\frac{2}{3}$.

7. $PR=\sqrt{(7+4)^2+(-5-0)^2}=\sqrt{146}$
   $QS=\sqrt{(-1-4)^2+(-8-3)^2}=\sqrt{146}$
   Diagonals are congruent.
   $m_{\overline{PR}}=\frac{-5-0}{7+4}=-\frac{5}{11} \quad m_{\overline{QS}}=\frac{-8-3}{-1-4}=\frac{11}{5}$
   Diagonals are ⊥.
   Midpoint of $\overline{PR}$ is $\left(\frac{-4+7}{2},\frac{0-5}{2}\right)=\left(\frac{3}{2},-\frac{5}{2}\right)$
   Midpoint of $\overline{QS}$ is $\left(\frac{4-1}{2},\frac{3-8}{2}\right)=\left(\frac{3}{2},-\frac{5}{2}\right)$
   Diagonals bisect each other.

8. Both pairs of opp sides of a ▱ are ∥.
   Parallel lines have the same slope.
   The slope of side $\overline{BC}$ is 3.
   For side $\overline{AD}$ to have a slope of 3, the coordinates of point D must be (1,3).
   $m_{\overline{AB}}=\frac{2-0}{5-0}=\frac{2}{5} \quad m_{\overline{CD}}=\frac{3-5}{1-6}=\frac{2}{5}$
   $m_{\overline{AD}}=\frac{3-0}{1-0}=3 \quad m_{\overline{BC}}=\frac{5-2}{6-5}=3$

9. To prove that $ABCD$ is a rhombus, show that all sides are ≅ using the distance formula:
   $AB=\sqrt{(8+1)^2+(2+5)^2}=\sqrt{130}$
   $BC=\sqrt{(11-8)^2+(13-2)^2}=\sqrt{130}$
   $CD=\sqrt{(2-11)^2+(6-13)^2}=\sqrt{130}$
   $AD=\sqrt{(2+1)^2+(6+5)^2}=\sqrt{130}$

10. To prove that $ABCD$ is a ▱, show that both pairs of opp sides of the ▱ are ∥ by showing the opp sides have the same slope:

$$m_{\overline{AB}} = \frac{5-2}{6+2} = \frac{3}{8} \qquad m_{\overline{CD}} = \frac{-3-0}{-4-4} = \frac{3}{8}$$
$$m_{\overline{AD}} = \frac{-3-2}{-4+2} = \frac{5}{2} \qquad m_{\overline{BC}} = \frac{0-5}{4-6} = \frac{5}{2}$$

Because $\frac{3}{8}$ and $\frac{5}{2}$ are not opp reciprocals, the consecutive sides of $ABCD$ are not ⊥, and $ABCD$ is not a ▭.

11. The length of each side of quad is 5. Since each side is ≅, quad $MATH$ is a rhombus. The slope of $\overline{MH}$ is 0 and the slope of $\overline{HT}$ is $-\frac{4}{3}$. Since the slopes are not opp reciprocals, the sides are not ⊥ and do not form rights ∠'s.
Since adjacent sides are not ⊥, quad $MATH$ is not a square.

12. $$m_{\overline{AB}} = \frac{6-6}{6+5} = 0 \qquad m_{\overline{CD}} = \frac{-3+3}{-3-8} = 0$$
$$m_{\overline{AD}} = \frac{-3-6}{-3+5} = -\frac{9}{2} \qquad m_{\overline{BC}} = \frac{-3-6}{8-6} = -\frac{9}{2}$$

$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ because they have equal slopes. $ABCD$ is a ▱ because opp side are ∥.
$AB = 11$ and $BC = \sqrt{85}$. $ABCD$ is not a rhombus because $AB \neq BC$.
$\overline{AB}$ and $\overline{BC}$ are not ⊥ because their slopes are not opp reciprocals. Therefore, $ABCD$ is not a ▭ because $\angle B$ is not a right ∠.

13. Use the midpoint formula to find $M(-5,5), N(0,3), P(2,-4), Q(-3,-2)$.
Use the slope formula to find

$$m_{\overline{MN}} = -\frac{2}{5} \qquad m_{\overline{PQ}} = -\frac{2}{5}$$
$$m_{\overline{MQ}} = -\frac{7}{5} \qquad m_{\overline{NP}} = -\frac{7}{5}$$

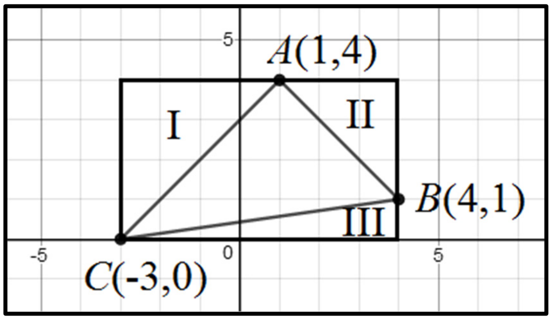
Since both opp sides have equal slopes and are ∥, $MNPQ$ is a ▱.
Use the distance formula to find $MN = \sqrt{29}$ and $NP = \sqrt{53}$.
$MN \neq NP$, so $MNPQ$ is not a rhombus since not all sides are congruent.

3.3 Perimeter and Area using Coordinates

1. $AB = \sqrt{(4-1)^2 + (1-4)^2} = \sqrt{18} = 3\sqrt{2}$
$BC = \sqrt{(-3-4)^2 + (0-1)^2} = \sqrt{50} = 5\sqrt{2}$
$AC = \sqrt{(-3-1)^2 + (0-4)^2} = \sqrt{32} = 4\sqrt{2}$
Perimeter $= 3\sqrt{2} + 5\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$

2. $AB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$
$BC = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10$
$AC = \sqrt{11^2 + (-2)^2} = \sqrt{125} = 5\sqrt{5}$
Perimeter $= 5 + 10 + 5\sqrt{5} = 15 + 5\sqrt{5}$

3. *Method 1: enclose in* ▭



Area of ▭ = 7×4 = 28

Area of I $= \frac{1}{2}(4)(4) = 8$

Area of II $= \frac{1}{2}(3)(3) = 4.5$

Area of III $= \frac{1}{2}(7)(1) = 3.5$

Area of $\triangle\, ABC = 28 - (8 + 4.5 + 3.5) = 12$ square units

Method 2: area of right △

$m_{\overline{CA}} = \frac{4}{4} = 1 \qquad m_{\overline{AB}} = -\frac{3}{3} = -1$

Slopes are opp reciprocals, so $\overline{CA} \perp \overline{AB}$ and $\angle A$ is a right $\angle$.

From problem 1, $AC = 4\sqrt{2}$ and $AB = 3\sqrt{2}$.

Area of right $\triangle\, ABC = \frac{1}{2}(AC)(AB)$

$= \frac{1}{2}(4\sqrt{2})(3\sqrt{2}) = 12$ square units.

4.

$EF = \sqrt{(6-3)^2 + (10-6)^2}$

$= \sqrt{25} = 5$

$FG = \sqrt{(18-6)^2 + (5-10)^2}$

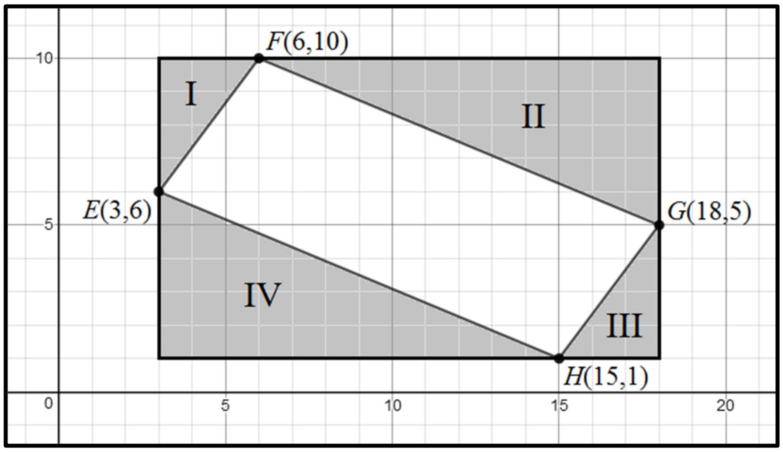
$= \sqrt{169} = 13$

Since $EFGH$ is a ▱,

$GH = EF$ and $EH = FG$.

Perimeter of $EFGH =$

$2(5) + 2(13) = 36$



Area of ▭ $= 15 \times 9 = 135$

Area of I = Area of III $= \frac{1}{2}(3)(4) = 6$

Area of II = Area of IV $= \frac{1}{2}(5)(12) = 30$

Area of ▱$EFGH = 135 - [2(6) + 2(30)] = 63$ square units

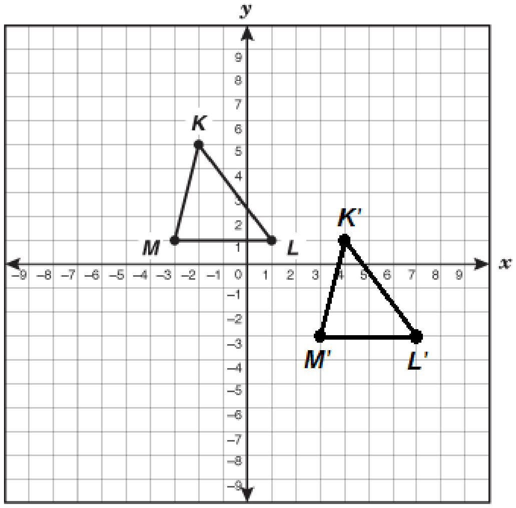
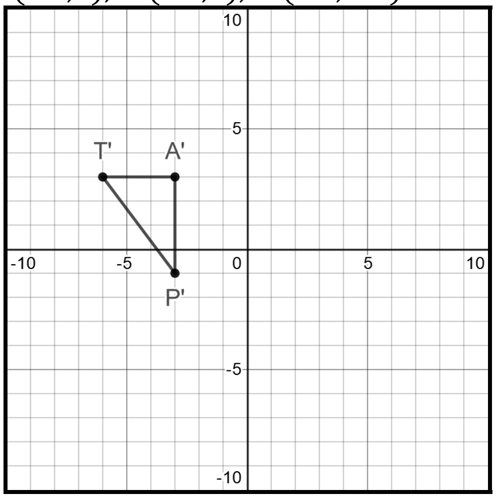
5.

vertex	*x*	*y*	*upper*	*lower*	*difference*
K	−7	−7	−14	35	−49
L	−5	2	−30	6	−36
M	3	6	−9	6	−15
N	1	−3	−7	21	−28
K	−7	−7			

$$\text{Area} = \frac{|(-49) + (-36) + (-15) + (-28)|}{2} = \frac{|128|}{2} = 64 \text{ square units.}$$

CHAPTER 4. RIGID MOTIONS

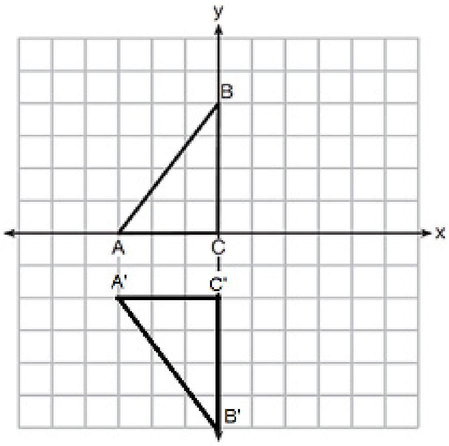
4.1 Translations

1. $(2-6, 4+1) = (-4, 5)$	2. $(-2, -2)$
3. $(x+3, y-7)$	4. $(2, -8)$
5. $(-6,6)$	6. $(x+4, y+4)$
7. $(0, -9)$	8. $T_{2,2}$ of Q is $Q'(6, -4)$
9. $R(-5,-5) \rightarrow R'(3, 0)$ is $T_{8,5}$ $U'(3, 6)$	10. $A(1,3) \rightarrow A'(4,4)$ is $T_{3,1}$ $C'(7,1)$
11. $(-5,5)$	12. $(0,1)$
13. $B(-6,4)$ and $D'(-5,-4)$	
14. 	15. $T'(-6,3)$, $A'(-3,3)$, $P'(-3,-1)$ 

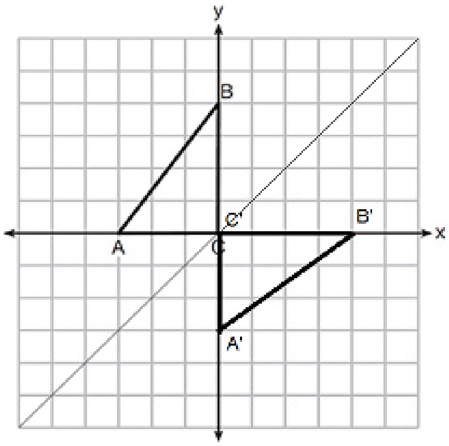
4.2 Line Reflections

1. $(2,3)$	2. $P'(-4,1)$
3. $(2,5)$	4. $(-3,4)$
5. $M'(2,8)$	6. $(-4,3)$
7. $(-2,5)$	8. $A'(0,-2)$ and $B'(4,-6)$

9.

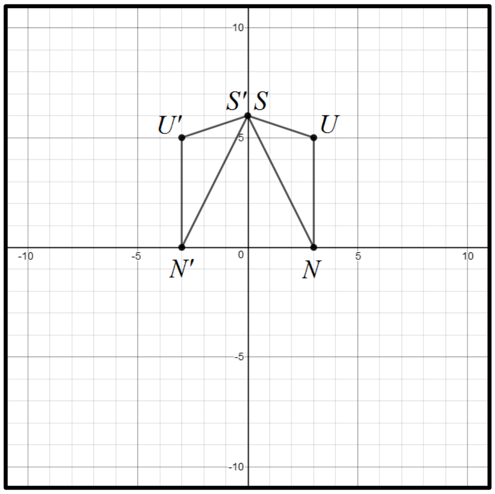


10.



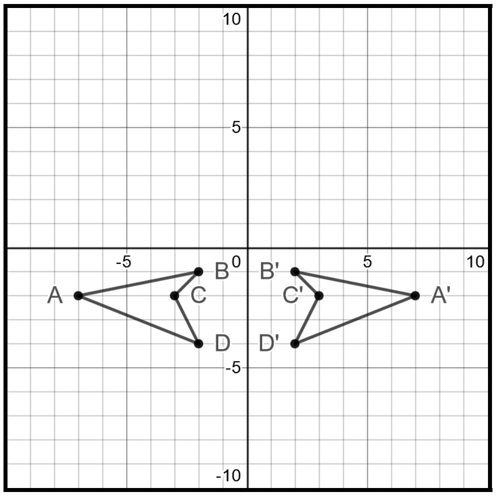
11. $X'(5,1), Y'(4,4), Z'(7,4)$

12.



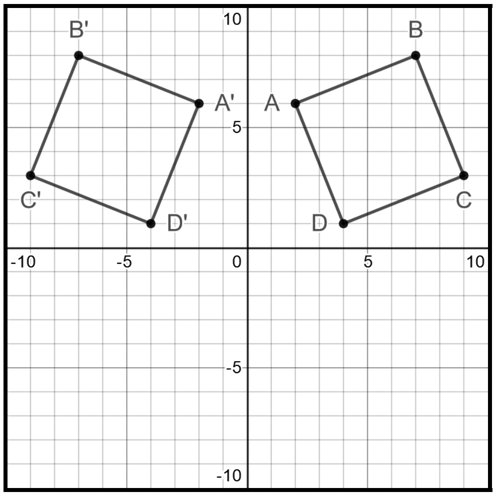
$S'(0,6), U'(-3,5), N'(-3,0)$

13.



$A'(7,-2), B'(2,-1), C'(3,-2), D'(2,-4)$

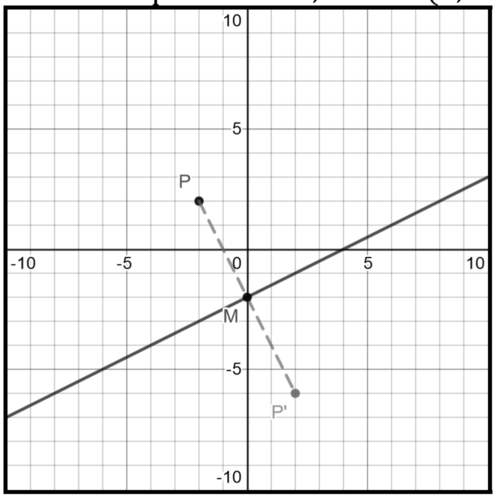
14.



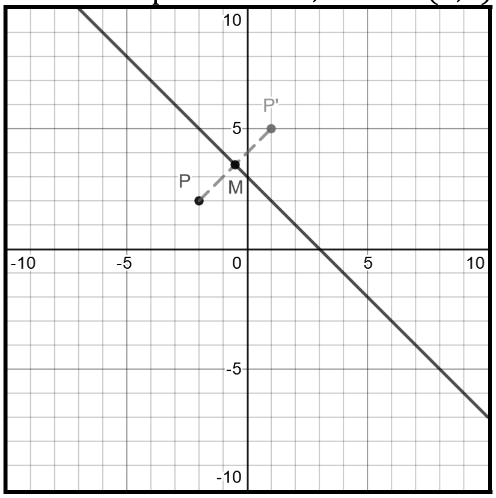
$AB = \sqrt{(7-2)^2 + (8-6)^2} = \sqrt{29}$

Area $= \left(\sqrt{29}\right)^2 = 29$

15. Equation of ⊥ line is $y - 2 = -2(x + 2)$, or $y = -2x - 2$ in slope-intercept form.
Solving $\frac{1}{2}x - 2 = -2x - 2$ gives us $x = 0$.
Substituting for x, $y = -2$, so M is $(0, -2)$.
M is the midpoint of $\overline{PP'}$, so P' is $(2, -6)$.



16. Equation of ⊥ line is $y - 2 = 1(x + 2)$, or $y = x + 4$ in slope-intercept form.
Solving $-x + 3 = x + 4$ gives us $x = -\frac{1}{2}$.
Substituting for x, $y = \frac{7}{2}$, so M is $\left(-\frac{1}{2}, \frac{7}{2}\right)$.
M is the midpoint of $\overline{PP'}$, so P' is $(1,5)$.



4.3 Rotations

1. (2)

2. (3)

3. (4)

4. $R_{(0,0),90°}: (x, y) \rightarrow (-y, x)$
$(2, 4) \rightarrow (-4, 2)$

5. A clockwise rotation of 90° is equivalent to a counterclockwise rotation of 270°.
$R_{(0,0),270°}: (x, y) \rightarrow (y, -x)$
$(-2, 5) \rightarrow (5, 2)$

6. A clockwise rotation of 180° is equivalent to a counterclockwise rotation of 180°.
$R_{(0,0),180°}: (x, y) \rightarrow (-x, -y)$
$(-2, 1) \rightarrow (2, -1)$

7. $R_{(0,0),180°}: (x, y) \rightarrow (-x, -y)$
$A'(0, 4), B'(-4, 2), C'(-5, 4), D'(-1, 6)$

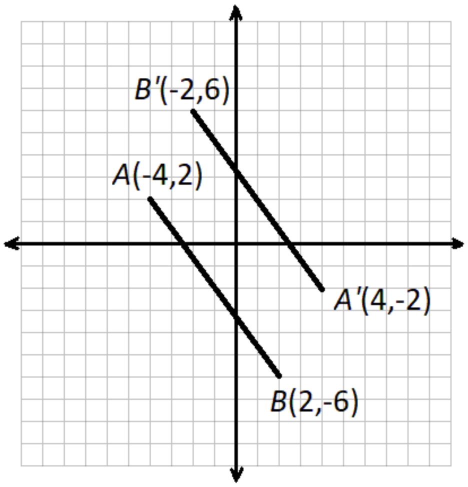
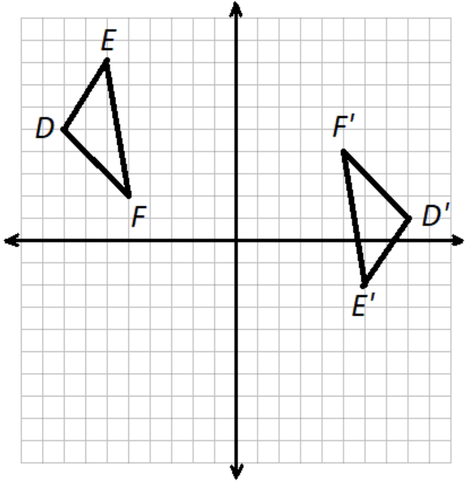
8. $R_{(0,0),90°}: (x, y) \rightarrow (-y, x)$
$A'(-2, 1), B'(-3, -4), C'(5, -3)$

9. $R_{(0,0),90°}: (x, y) \rightarrow (-y, x)$
$P(-2,5) - C(2,3) = (-4,2)$
$(-4,2) \rightarrow (-2, -4)$
$(-2, -4) + C(2,3) = P'(0, -1)$

10. $R_{(0,0),180°}: (x, y) \rightarrow (-x, -y)$
$P(3, -2) - C(2, -3) = (1,1)$
$(1,1) \rightarrow (-1, -1)$
$(-1, -1) + C(2, -3) = P'(1, -4)$

11. $R_{(0,0),90°}: (x, y) \rightarrow (-y, x)$
$A(1,2) - P(2, -1) = (-1,3) \rightarrow (-3, -1) +$
$P(2, -1) = A'(-1, -2)$
$B(-4,3) - P(2, -1) = (-6,4) \rightarrow (-4, -6) +$
$P(2, -1) = B'(-2, -7)$
$C(-3, -5) - P(2, -1) = (-5, -4) \rightarrow (4, -5) +$
$P(2, -1) = C'(6, -6)$

4.4 Point Reflections

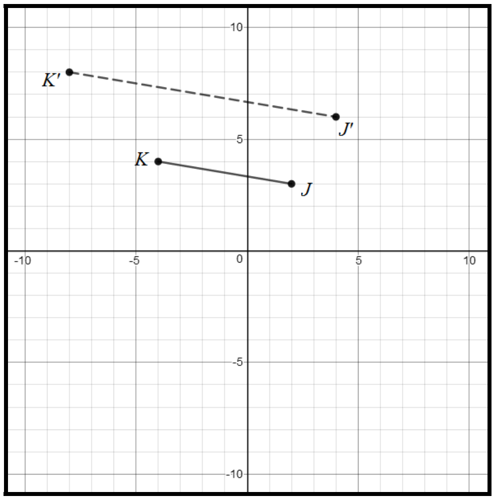
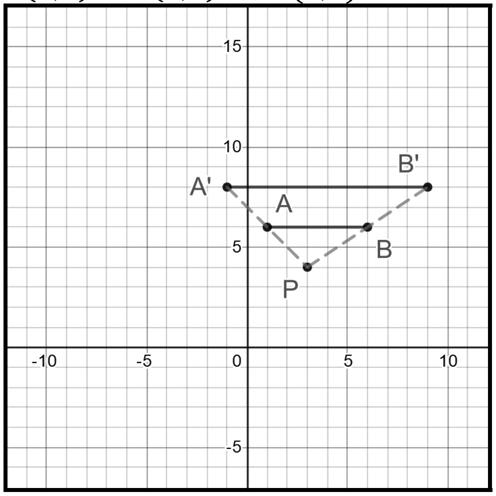
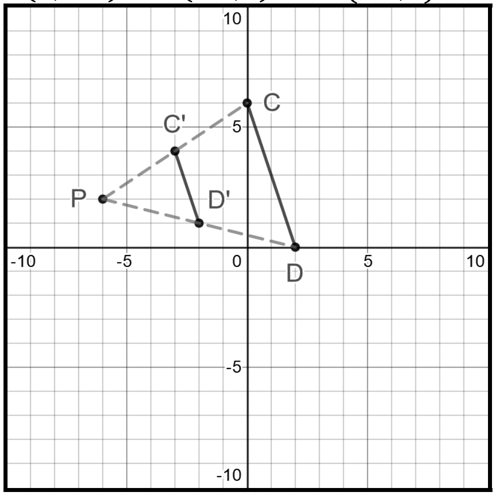
1. $K'(-4,7)$	2. (3,1)
3. $R'(-2,3), S'(-5,-1)$	4. $(2\cdot 1-5, 2\cdot(-1)-3) = N'(-3,-5)$
5. $2\times 6=12$ units	6. Use the midpoint formula: $P\left(\frac{3+7}{2},\frac{6-2}{2}\right)=P(5,2)$
7.  	8. $D'(8,1), E'(6,-2), F'(5,4)$ 

4.5 Carry a Polygon onto Itself

1. (3)	2. 180°
3. $\frac{360}{5}=72°$	4. $\frac{360}{16}=22.5°$

CHAPTER 5. DILATIONS

5.1 Dilations of Line Segments

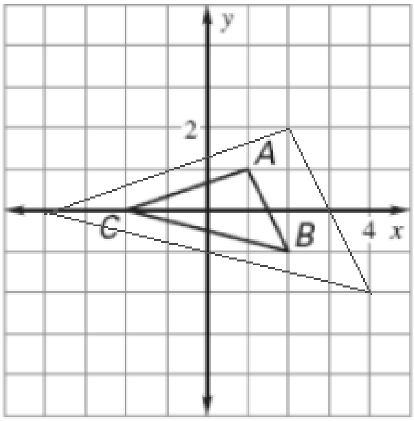
1. (1)	
2. $(15, -10)$	3. $(4, 12)$
4. $\left(\frac{3}{2}, -1\right)$	5. $(2, 5)$
6. $(12, 0)$	7. 
8. $A(1,6) - P(3,4) = (-2,2)$ $(-2,2) \rightarrow (-4,4)$ $(-4,4) + P(3,4) = A'(-1,8)$ $B(6,6) - P(3,4) = (3,2)$ $(3,2) \rightarrow (6,4)$ $(6,4) + P(3,4) = B'(9,8)$ 	9. $C(0,6) - P(-6,2) = (6,4)$ $(6,4) \rightarrow (3,2)$ $(3,2) + P(-6,2) = C'(-3,4)$ $D(2,0) - P(-6,2) = (8,-2)$ $(8,-2) \rightarrow (4,-1)$ $(4,-1) + P(-6,2) = D'(-2,1)$ 

5.2 Dilations of Polygons

1. (3) $m\angle B = m\angle B'$

2. $A'(2,2), B'(3,0), C'(1,-1)$

3.



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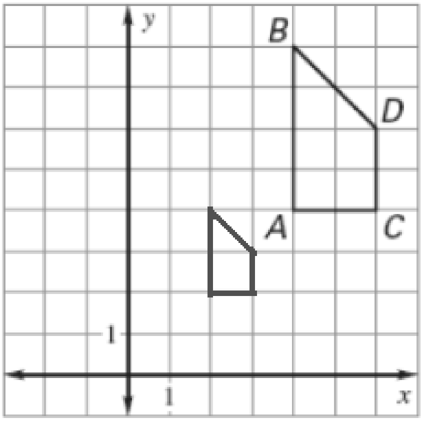

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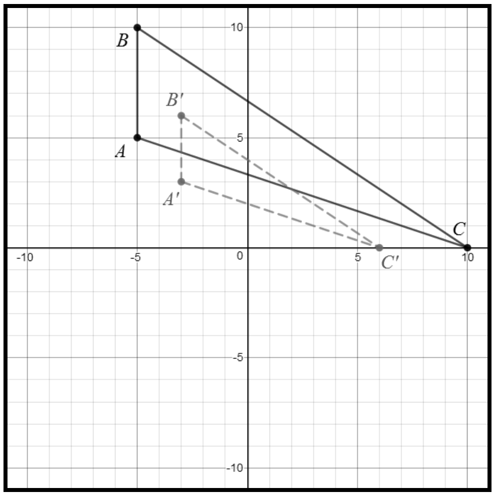

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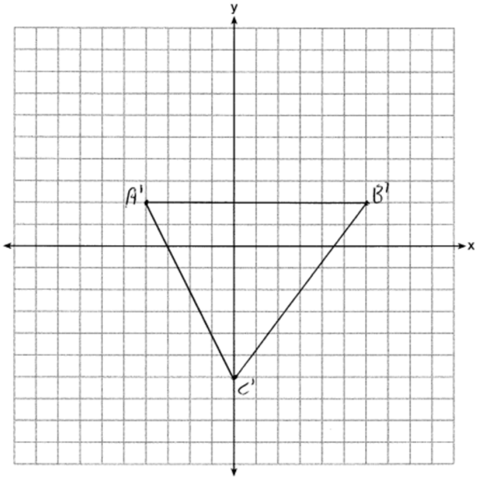

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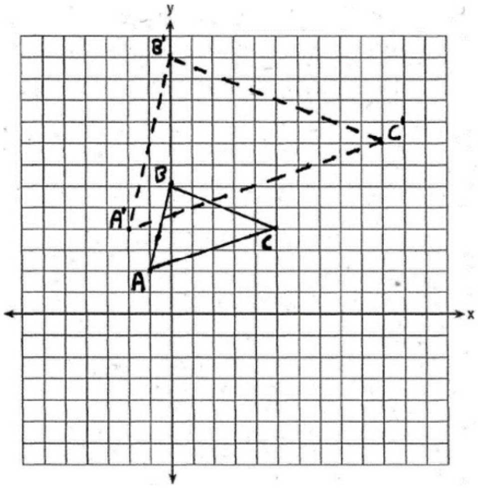

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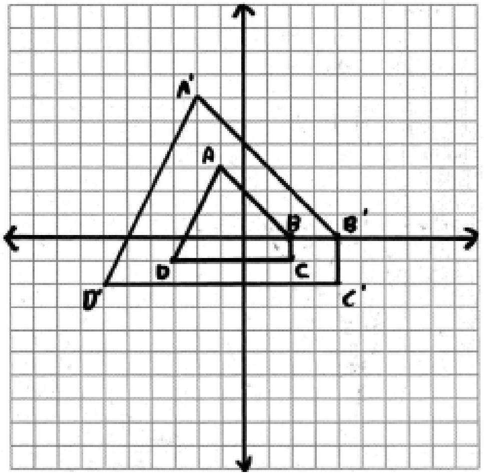
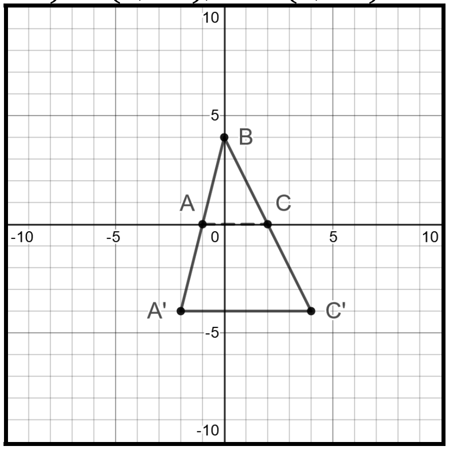
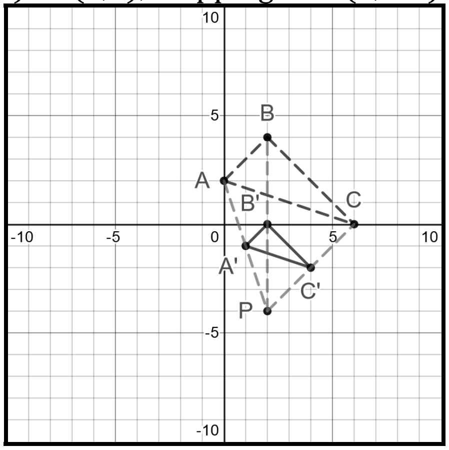


$A'(-2,6), B'(4,0), C'(4,-2), D'(-6,-2)$

9. Using $B(0,4)$ as the "new origin,"
$(-1,-4) \rightarrow (-2,-8)$, so $A'(-2,-4)$
B maps to itself, $(0,4)$
$(2,-4) \rightarrow (4,-8)$, so $C'(4,-4)$



10. Using $P(2,-4)$ as the "new origin,"
$(-2,6) \rightarrow (-1,3)$, mapping to $A'(1,-1)$
$(0,8) \rightarrow (0,4)$, mapping to $B'(2,0)$
$(4,4) \rightarrow (2,2)$, mapping to $C'(4,-2)$



11. a) $A'(-1,1), B'(4,-2), C'(3,-5), D'(-2,-2)$

b) $m_{\overline{A'B'}} = \frac{-2-1}{4+1} = -\frac{3}{5}$ $\quad m_{\overline{C'D'}} = \frac{-2+5}{-2-3} = -\frac{3}{5}$ $\quad \overline{A'B'} \parallel \overline{C'D'}$

$m_{\overline{A'D'}} = \frac{-2-1}{-2+1} = 3$ $\quad m_{\overline{B'C'}} = \frac{-5+2}{3-4} = 3$ $\quad \overline{A'D'} \parallel \overline{B'C'}$

$A'B'C'D'$ is a ▱ because both pairs of opp sides are ∥.

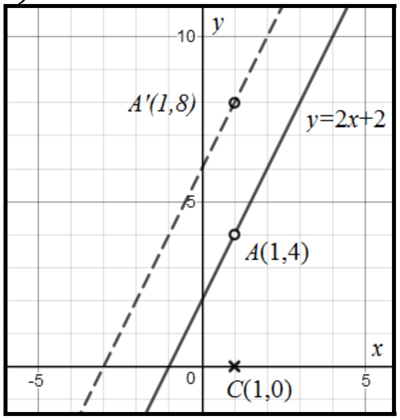
5.3 Dilations of Lines

1. $y = 3x - 20$

2. $2x + 3y = 4 \quad \rightarrow \quad y = -\frac{2}{3}x + \frac{4}{3}$
Equation of its image is $y = -\frac{2}{3}x + 4$

3. $C(1,-1)$ is a point on the line:
$(-1) = 3(1) - 4$
Therefore, the equation of the image is the same as the pre-image, $y = 3x - 4$.

4. $C(1,0)$ is not on the line.



Find the point on the line where $x = 1$:
$y = 2(1) + 2 = 4$, so $A(1,4)$.
$CA = 4$, so $CA' = 2(4) = 8$ and $A'(1,8)$
$8 = 2(1) + b \quad \rightarrow \quad b = 6$
Equation of image is $y = 2x + 6$.

CHAPTER 6. TRANSFORMATION PROOFS

6.1 Properties of Transformations

1. a) reflection b) dilation
   c) translation d) rotation
   e) dilation

2. (3) rotation

3. (2) opposite orientation; reflection

4. (4) $r_{y=x}$

5. (3) $r_{y\text{-}axis}$

6. (3) orientation

7. (4) $D_{(0,0),2}$

8. (1) dilation

9. (3) dilation by a scale factor of $\frac{1}{2}$

6.2 Sequences of Transformations

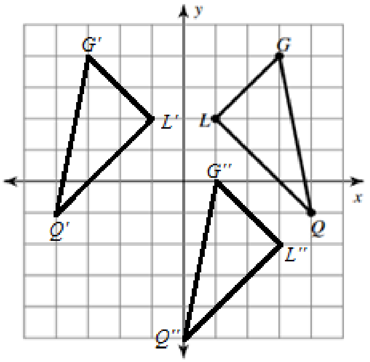
1. (4) translation followed by a reflection

2. Rotation $R_{(0,0),90^\circ}$ followed by reflection r_{x-axis}

3. Rotation $R_{A,270^\circ}$ followed by translation $T_{5,-2}$

4. Reflection $r_{y=\frac{1}{2}}$ followed by rotation $R_{(1,-3),270^\circ}$

5.



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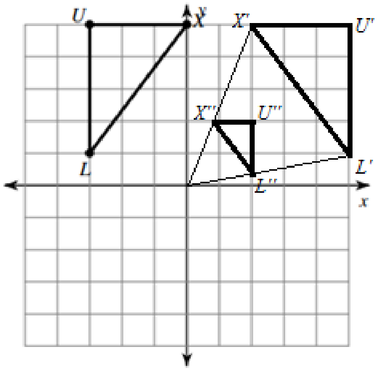

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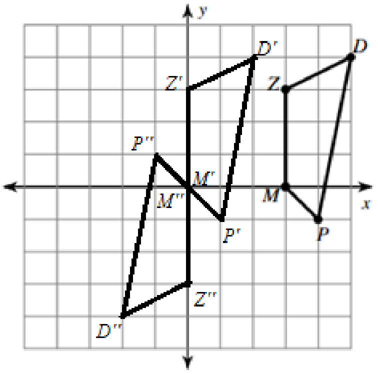
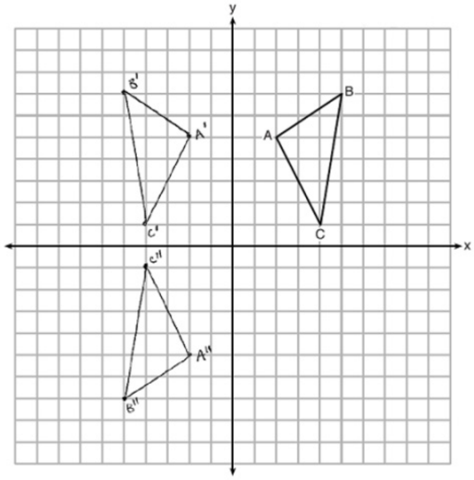
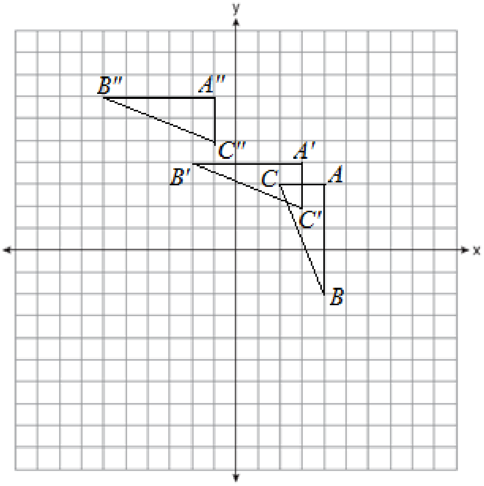


8. (3) (8,12)

<table>
<tr><td>9.


$R_{(0,0),180^\circ}$</td><td>10.


$A'(3,4), B'(-2,4), C'(3,2)$
$A''(-1,7), B''(-6,7), C''(-1,5)$</td></tr>
<tr><td colspan="2">11. If $\overline{AB}$ is reflected first, the coordinates are $A'(2,-6)$ and $B'(4,-2)$.
When the reflection is dilated, the coordinates are $A''(1,-3)$ and $B''(2,-1)$.
If $\overline{AB}$ is dilated first, the coordinates are $A'(1,3)$ and $B'(2,1)$.
When the dilation is reflected, the coordinates are $A''(1,-3)$ and $B''(2,-1)$.
The images are the same.</td></tr>
</table>

6.3 Transformations and Congruence

<table>
<tr><td>1. Translation so that A maps onto D.</td><td>2. Reflection over $\overline{AC}$.</td></tr>
<tr><td>3. Rotation of 180° around point J.</td><td>4. Reflection over the bisector of $\angle HLK$.
(Note that $\triangle HLK$ is isosceles.)</td></tr>
<tr><td colspan="2">5. Reflection over the vertical line passing through P, followed by the translation $T_{2,-4}$ (or vice versa). No, it is not possible because the triangles have opposite orientations and both translations and rotations preserve orientations.</td></tr>
</table>

6.4 Transformations and Similarity

<table>
<tr><td>1. $D_{A,\frac{1}{2}}$ with A as the center of dilation followed by $T_{4,0}$ mapping $A \rightarrow A'$.</td><td>2. $D_{B,\frac{1}{2}}$ with B as the center of dilation, then a translation $T_{0,-2}$ to map $B \rightarrow B'$, and finally a reflection $r_{\overline{B'C'}}$.</td></tr>
</table>

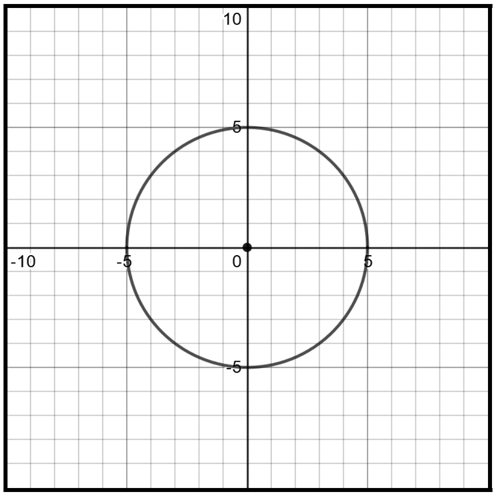
CHAPTER 7. CIRCLES IN THE COORDINATE PLANE

7.1 Equation of a Circle

1. a) $(0,0)$ and $\sqrt{10}$ b) $3^2+(-1)^2=10$ Yes.	2. $(2,-3)$ and 6
3. $(1,-3)$ and 3	4. $(0,7)$ and $4\sqrt{2}$
5. $r=4$ $x^2+y^2=16$	6. $r^2=3^2+4^2$ $r=5$ $(x+3)^2+(y-4)^2=25$
7. $r=3$ $(x-1)^2+(y+2)^2=9$	8. Center is at the midpoint of the diameter, $(2,-4)$. Radius is 4. $(x-2)^2+(y+4)^2=16$
9. $(3,-2)\rightarrow(9,-6)$	10. $(-4,2)\rightarrow(-8,4)$ and $r=2\cdot 9=18$
11. $x^2+4x+y^2=5$ $(x^2+4x+4)+y^2=5+4$ $(x+2)^2+y^2=9$ Center is $(-2,0)$, radius is 3	12. $x^2+6x+y^2-4y=12$ $(x^2+6x+9)+y^2-4y=12+9$ $(x+3)^2+y^2-4y=21$ $(x+3)^2+(y^2-4y+4)=21+4$ $(x+3)^2+(y-2)^2=25$ Center is $(-3,2)$, radius is 5
13. $x^2-16x+y^2+6y=-53$ $(x^2-16x+64)+y^2+6y=-53+64$ $(x-8)^2+y^2+6y=11$ $(x-8)^2+(y^2+6y+9)=11+9$ $(x-8)^2+(y+3)^2=20$ Center is $(8,-3)$, radius is $2\sqrt{5}$	14. $x^2-2x=-y^2+10y+1$ $x^2-2x+y^2-10y=1$ $(x^2-2x+1)+y^2-10y=1+1$ $(x-1)^2+y^2-10y=2$ $(x-1)^2+(y^2-10y+25)=2+25$ $(x-1)^2+(y-5)^2=27$ Center is $(1,5)$, radius is $3\sqrt{3}$
15. Since the center of the circle is (r,r), the equation is $(x-r)^2+(y-r)^2=r^2$. Since point $(6,3)$ is on the circle, we can substitute this point for (x,y), giving us $(6-r)^2+(3-r)^2=r^2$. Now, solve for r. $36-12r+r^2+9-6r+r^2=r^2$ $45-18r+2r^2=r^2$ $45-18r+r^2=0$ $r^2-18r+45=0$ $(r-3)(r-15)=0$ $r=\{\cancel{3},15\}$ Radius is 15.	

7.2 Graph Circles

1.



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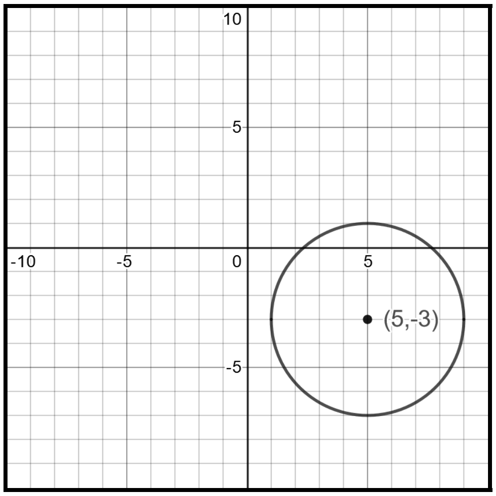

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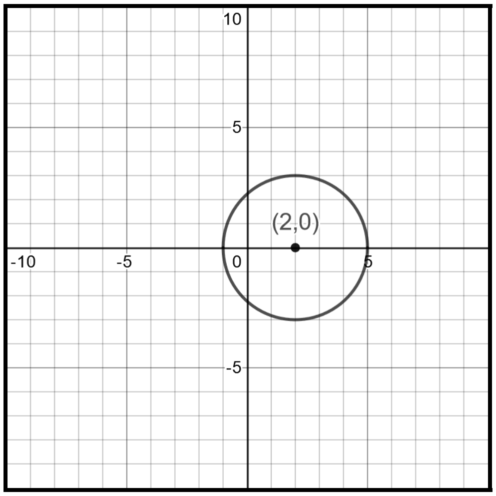

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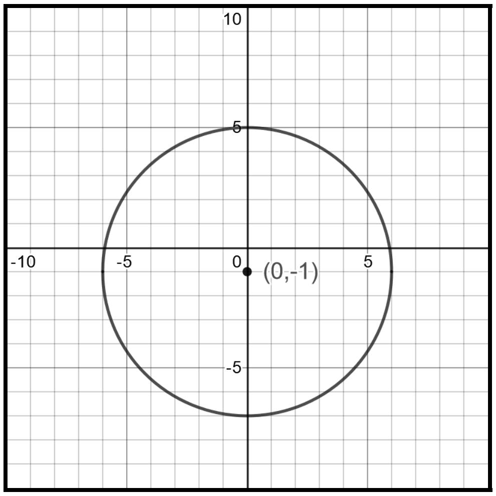


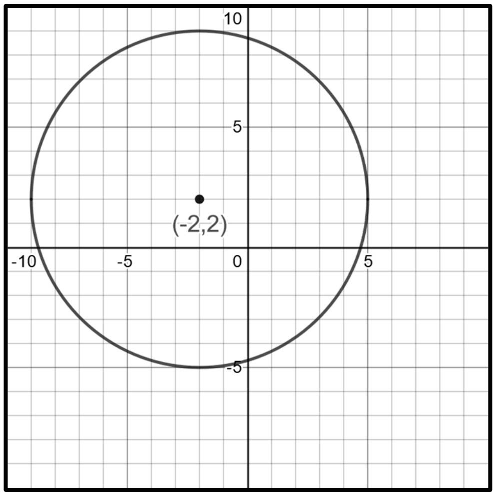
5. $x^2 + 4x + y^2 - 4y = 41$

$x^2 + 4x + 4 + y^2 - 4y = 41 + 4$

$(x + 2)^2 + y^2 - 4y = 45$

$(x + 2)^2 + y^2 - 4y + 4 = 45 + 4$

$(x + 2)^2 + (y - 2)^2 = 49$



CHAPTER 8. FOUNDATIONS OF EUCLIDEAN GEOMETRY

8.1 Postulates, Theorems and Proofs

1. (4) vertical angles
2. (1) $\overline{AC} \cong \overline{DB}$
3. (2) $AB = CD$
4. (2) reflexive prop and subtraction prop
5.

Statements	*Reasons*
∠1 and ∠2 are complementary	Given
∠2 and ∠3 are complementary	Given
$\angle 1 \cong \angle 3$	Complements of the same ∠ are ≅
$m\angle 1 = m\angle 3$	Def of ≅ ∠'s

6.

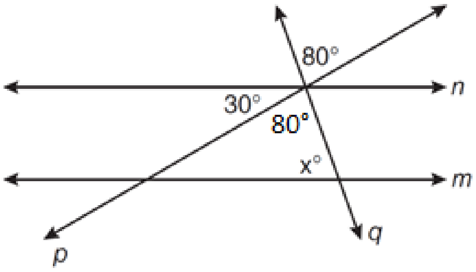
Statements	*Reasons*
$\overline{PCEG}, \overline{PC} \cong \overline{GE}$	Given
$\overline{CE} \cong \overline{CE}$	Reflexive Prop
$PC = GE,\ \ CE = CE$	Def of ≅ segments
$PC + CE = GE + CE$	Addition Prop
$PE = PC + CE, GC = GE + CE$	Segment Addition
$PE = GC$	Substitution
$\overline{PE} \cong \overline{GC}$	Def of ≅ segments

8.2 Parallel Lines and Transversals

1. (3) ∠4 and ∠8
2. (1) ∠1 and ∠8
3. (a) linear pair; supplementary
   (b) vertical angles; congruent
   (c) corresponding angles; congruent
   (d) alternate interior angles; congruent
   (e) alternate exterior angles; congruent
4. (2) consecutive interior ∠'s
5. (2) Alternate interior ∠'s are ≅.
6. (3) ∠3 and ∠6 are supplementary
7. (4) $d \parallel e$
8. $x + 20 = 2x - 10$ (alternate interior ∠'s are ≅)
   $x = 30$

9. $m\angle EYD = 180 - 123 = 57°$
(linear pair)
$m\angle AXY = m\angle EYD = 57°$
(alternate interior)

10. Vertical ∠'s are ≅, therefore:



$x = 180 - (30 + 80) = 70$

11. $a = 180 - (57 + 64) = 59°$
$b = 64°$
$c = 57°$
$d = 180 - 64 = 116°$

12. $15x - 5 = 180 - 125$
$15x - 5 = 55$
$15x = 60$
$x = 4$

$7y + 27 = 125$
$7y = 98$
$y = 14$

13.

Statements	*Reasons*
∠1 and ∠3 are supplementary	Given
∠1 and ∠2 are a linear pair	Def of linear pair
∠1 and ∠2 are supplementary	Linear pairs are supplementary
∠2 ≅ ∠3	Supplements of the same ∠ are ≅
∠2 and ∠3 are alternate exterior ∠'s	Def of alternate exterior ∠'s
$m \parallel n$	If a transversal intersects two lines to form ≅ alternate exterior ∠'s, then the lines are ∥

CHAPTER 9. TRIANGLES AND CONGRUENCE

9.1 Angles of Triangles

1. $180 - (60 + 40) = 80$

2. $50 + 70 = 120°$

3. $\mathrm{m}\angle ABC = 180 - 125 = 55°$
$\mathrm{m}\angle ACR = 60 + 55 = 115°$

4. $60 + 60 = 120°$

5. $6x + 5 = 3x + 65$
$3x = 60$
$x = 20$

6. $5x + x + 12 + x = 180$
$7x + 12 = 180$
$7x = 168$
$x = 24$
∠'s are 24°, 36°, and 120°, so △ is obtuse.

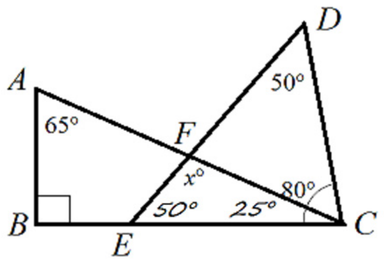
7. $\mathrm{m}\angle J = 180 - (90 + 48) = 42°$
$\mathrm{m}\angle JMS = 180 - 59 = 121°$
$\mathrm{m}\angle JSM = 180 - (42 + 121) = 17°$

8. $\mathrm{m}\angle EHI = 20°$ (vertical ∠'s)
$\mathrm{m}\angle EIH = 60°$ (vertical ∠'s)
$\mathrm{m}\angle HEI = 180 - (20 + 60) = 100°$

9. $\mathrm{m}\angle RTQ = 63°$ (alternate interior)
$\mathrm{m}\angle 2 = 180 - (90 + 63) = 27°$

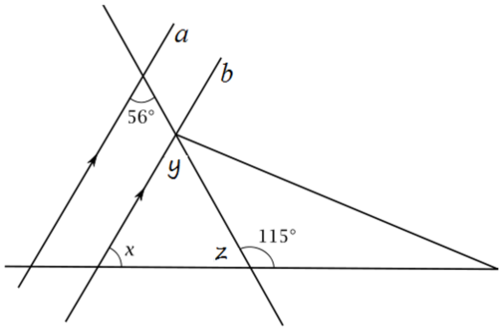
10. $\mathrm{m}\angle A = 180 - (90 + 52) = 38°$
$\mathrm{m}\angle EBC = 90 + 38 = 128°$
$\mathrm{m}\angle EBD = \frac{1}{2}\mathrm{m}\angle EBC = 64°$
$\mathrm{m}\angle ABD = 52 + 64 = 116°$
$\mathrm{m}\angle D = 180 - (38 + 116) = 26°$

11.



$\mathrm{m}\angle \mathrm{ACB} = 180 - (90 + 65) = 25°$
$\mathrm{m}\angle \mathrm{DEC} = 180 - (80 + 50) = 50°$
$x = 180 - (50 + 25) = 105°$

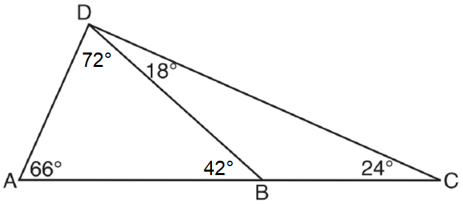
12.



$y = 56°$ (corresponding ∠'s)
$z = 65°$ (linear pair)
$x = 180 - (56 + 65) = 59°$

13. $\mathrm{m}\angle ABD = 180 - (93 + 43) = 44$
$x + 19 + 2x + 6 + 3x + 5 = 180$
$6x + 30 = 180$
$x = 25$
$\mathrm{m}\angle BDC = x + 19 = 44$
Yes, because alternate interior ∠'s $\angle ABD$ and $\angle BDC$ are ≅, $\overline{AB} \parallel \overline{DC}$.

9.2 Triangle Inequality Theorem

1. (2)	2. (4)
3. (1)	4. The sum of any two sides of a triangle must be greater than the third side. 7 + 8 < 16.
5. (2)	6. $4x + 3x - 1 + x + 3 = 34$ $8x + 2 = 34$ $x = 4$ 16, 11, and 7. Yes, because 16 < 11 + 7.
7. $\angle B, \angle A, \angle C$	8. $\overline{EF}, \overline{DE}, \overline{DF}$
9. $m\angle ABD = 42°$ (exterior $\angle$ of $\triangle CBD$) $m\angle ADB = 72°$ $(180 - 66 - 42)$ $\overline{AB}$ is the longest side and $\overline{AD}$ is the shortest side of $\triangle ABD$.  	10. $m\angle ADB = 36°$ and $m\angle CDB = 93°$. $\overline{BD}$ is the shortest side of $\triangle BCD$, but $\overline{BD}$ is not the shortest side of $\triangle ABD$. Therefore, $\overline{AB}$ is the shortest segment.

9.3 Segments in Triangles

1. (a) $\overline{BE}$ (b) $\overline{AD}$ (c) $\overline{CF}$	2. (4)
3. $6x - 6 = 90$ $6x = 96$ $x = 16$	4. $4x - 17 = 3x - 4$ $x = 13$ $m\angle XYW = 4(13) - 17 = 35$ So, $m\angle XYZ = 2(35) = 70°$
5. $4x - 8 + 6x + 13 = 90$ $10x + 5 = 90$ $x = 8.5$	
6. $x^2 + 3x = 6x + 10$ $x^2 - 3x - 10 = 0$ $(x - 5)(x + 2) = 0$ $x = 5$ (reject $x = -2$) $12y + 24 = 2(2y + 60)$ $12y + 24 = 4y + 120$ $8y = 96$ $y = 12$	
7. a) $2x + 3 = 7x - 47$ $50 = 5x$ $10 = x$ $DH = 2(10) + 3 = 23$ $DG = 2DH = 46$ b) $y^2 + 9 = 90$ $y^2 = 81$ $y = 9$ $m\angle EFH = 12y = 108°$	

9.4 Isosceles and Equilateral Triangles

1. $\frac{180-120}{2} = 30°$

2. $\frac{180-46}{2} = 67°$

3. (a) $\overline{CE}$ (b) $\overline{AF}$ (c) $\overline{BD}$

4. (3) an obtuse angle

5. $m\angle A = \frac{180-80}{2} = 50°$
   $m\angle BCD = 80 + 50 = 130°$

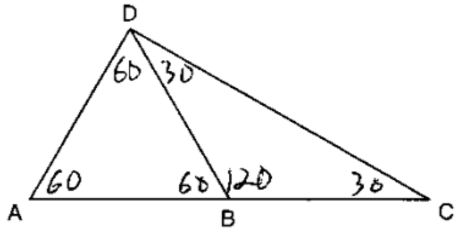
6. $m\angle LMO = 55°$
   $m\angle NMO = 180 - 55 = 125°$
   $m\angle N = 180 - (125 + 28) = 27°$

7. a) $60°$ (equilateral △)
   b) $\frac{180-50}{2} = 65°$ (base ∠ of isosceles △)
   c) $180 - (60 + 65) = 55°$ (parts of a straight ∠)
   d) $65 + 50 = 115°$ (exterior ∠)
   e) $60 + 60 = 120°$ (exterior ∠)
   f) $60°$ (vertical ∠'s with $\angle BAC$)

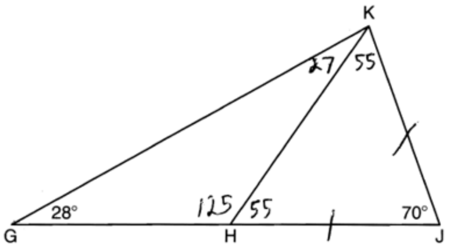
8. $m\angle QRP = \frac{180-54}{2} = 63°$
   $m\angle QRS = 180 - 63 = 117°$
   $x = \frac{180-117}{2} = 31.5°$

9. $m\angle EFG = 90 - 60 = 30°$
   $x = 180 - (90 + 30) = 60°$

10. 30



11. No, $m\angle KGH \neq m\angle GKH$



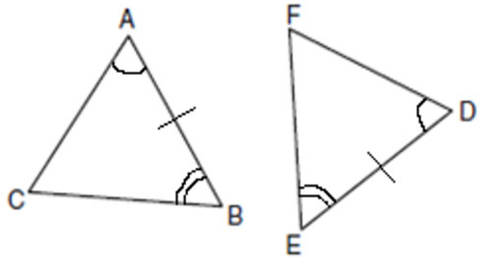
12. The altitude is also the median.
    $x^2 + (x+7)^2 = 13^2$ *[Pythagorean Thm]*
    $x^2 + x^2 + 14x + 49 = 169$
    $2x^2 + 14x - 120 = 0$
    $2(x+12)(x-5) = 0$
    $x = 5$ (reject $x = -12$)
    Base $= 2x = 10$

13.

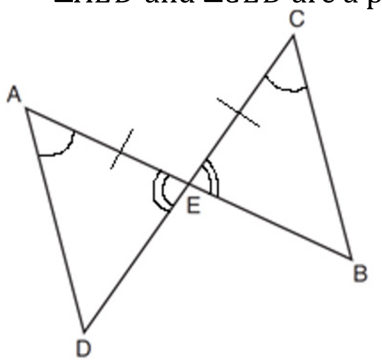
Statements	*Reasons*
$m\angle K = 70^\circ, m\angle MLN = 55^\circ$	Given
$m\angle JLK = m\angle MLN$	Vertical ∠'s are equal in measure
$m\angle JLK = 55^\circ$	Substitution
$m\angle J + m\angle K + m\angle JLK = 180^\circ$	Sum of the ∠'s of a △ is 180°
$m\angle J + 70^\circ + 55^\circ = 180^\circ$	Substitution
$m\angle J = 55^\circ$	Subtraction
$m\angle JLK = m\angle J$	Transitive prop
$\triangle JKL$ is isosceles	If two ∠'s of a △ are equal in measure, then the △ is isosceles

9.5 Triangle Congruence Methods

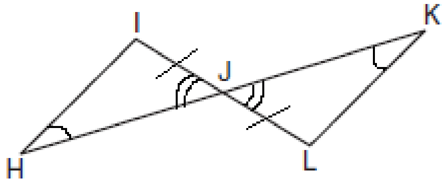
<table>
<tr><td>1. (2) by SSS</td><td>2. (1) by SAS</td></tr>
<tr><td>3. (2) $\angle A \cong \angle X$</td><td>4. (3) $\overline{JL} \cong \overline{MO}$</td></tr>
<tr><td>5. (1) $\angle A \cong \angle L$</td><td>6. $\overline{AG} \cong \overline{OL}$</td></tr>
<tr><td>7. ASA



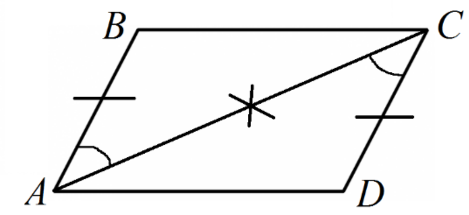
</td><td>8. ASA
$\angle AED$ and $\angle CEB$ are a pair of vertical ∠'s.



</td></tr>
<tr><td>9. AAS
$\overline{IJ} \cong \overline{LJ}$ by def of bisector
$\angle IJH \cong \angle LJK$ by vertical ∠'s



</td><td>10. SAS
$\overline{CA} \cong \overline{AC}$ by Reflexive Prop



</td></tr>
<tr><td>11. AAS
$\angle BAC \cong \angle DAC$ by def. of ∠ bisector
$\overline{AC} \cong \overline{AC}$ by Reflexive Prop</td><td>12. AAS
$\angle BAC \cong \angle DCA$ by alternate interior ∠'s
$\overline{AC} \cong \overline{CA}$ by Reflexive Prop</td></tr>
</table>

9.6 Prove Triangles Congruent

1. 3. Def of ⊥
   6. Alternate Interior ∠'s Thm
   8. AAS
   9. CPCTC

2.

Statements	*Reasons*
$\overline{BE}$ and $\overline{AD}$ intersect at C, $\overline{BC} \cong \overline{EC}, \overline{AC} \cong \overline{DC}$	Given
$\angle BCA \cong \angle ECD$	Vertical ∠'s are ≅
$\triangle ABC \cong \triangle DEC$	SAS

3.

Statements	*Reasons*
$\overline{FH} \cong \overline{FI}, \overline{SH} \cong \overline{SI}$	Given
$\overline{FS} \cong \overline{FS}$	Refelexive Prop
$\triangle FHS \cong \triangle FIS$	SSS
$\angle H \cong \angle I$	CPCTC

4.

Statements	*Reasons*
$\overline{AD}$ bisects $\overline{BC}$ at E, $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$	Given
$\angle B$ and $\angle C$ are right ∠'s	Def. of ⊥
$\overline{BE} \cong \overline{CE}$	Def. of segment bisector
$\angle B \cong \angle C$	Right ∠'s are ≅
$\angle AEB \cong \angle DEC$	Vertical ∠'s are ≅
$\triangle ABE \cong \triangle DCE$	ASA
$\overline{AB} \cong \overline{DC}$	CPCTC

5.

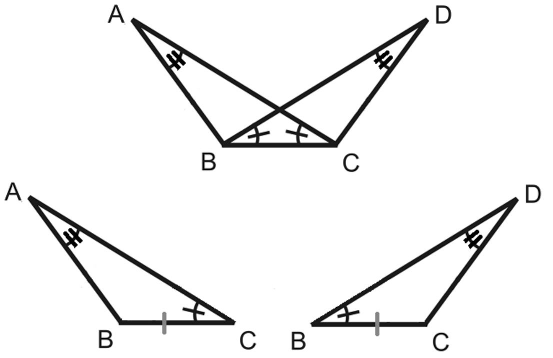
Statements	*Reasons*
$\triangle ABC, \overline{BD}$ bisects $\angle ABC, \overline{BD} \perp \overline{AC}$	Given
$\angle CBD \cong \angle ABD$	Def. of ∠ bisector
$\overline{BD} \cong \overline{BD}$	Reflexive Prop
$\angle CDB$ and $\angle ADB$ are right ∠'s	Def. of ⊥
$\angle CDB \cong \angle ADB$	Right ∠'s are ≅
$\triangle CDB \cong \triangle ADB$	ASA
$\overline{AB} \cong \overline{CB}$	CPCTC

6.

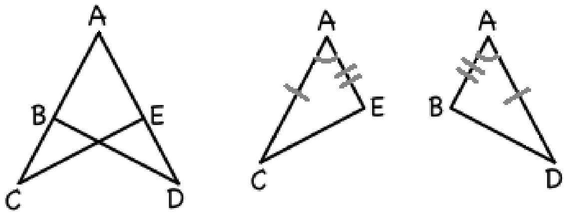
Statements	*Reasons*
$\overline{AFEC}, \overline{AF} \cong \overline{EC}, \angle 1 \cong \angle 2, \angle 3 \cong \angle 4$	Given
$\angle 1$ is supplementary to $\angle DFC$ and $\angle 2$ is supplementary to $\angle BEA$	Linear pairs are supplementary
$\angle DFC \cong \angle BEA$	$\cong$ Supplements Thm
$\overline{EF} \cong \overline{EF}$	Refelexive Prop
$AF + EF = EC + EF$ $\overline{AE} \cong \overline{FC}$	Addition Prop
$\triangle ABE \cong \triangle CDF$	ASA

9.7 Overlapping Triangles

1. AAS
Separate out the overlapping triangles and mark the given pairs of congruent angles. Note also that the two triangles share the same side, $\overline{BC}$, which must be congruent to itself by the Reflexive Prop.



2. SAS
Separate out the overlapping triangles and mark the given pairs of congruent sides. Note also that the two triangles share the same angle, $\angle A$, which must be congruent to itself by the Reflexive Prop.



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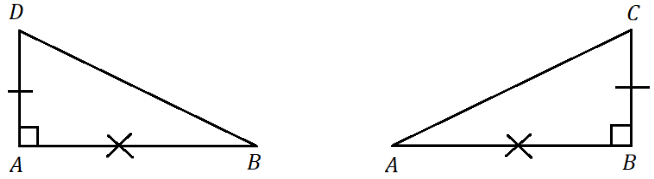
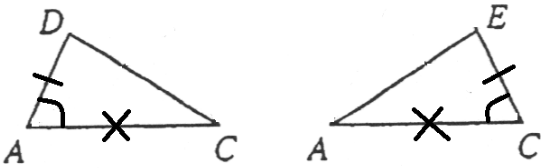


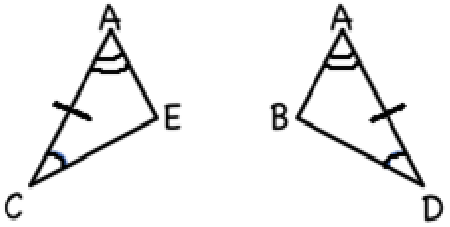
Statements	*Reasons*
$\overline{DA} \cong \overline{CB}$ (S)	Given
$\overline{DA} \perp \overline{AB}, \overline{CB} \perp \overline{AB}$	Given
$\angle DAB$ and $\angle CBA$ are right $\angle$'s	Def. of $\perp$
$\angle DAB \cong \angle CBA$ (A)	Right $\angle$'s are $\cong$
$\overline{AB} \cong \overline{AB}$ (S)	Reflexive Prop
$\triangle DAB \cong \triangle CBA$	SAS

4.



Statements	*Reasons*
$\overline{AB} \cong \overline{CB}$	Given
$\overline{AD} \cong \overline{CE}$ (S)	Given
$\angle DAC \cong \angle ECA$ (A)	Isosceles △ Thm
$\overline{AC} \cong \overline{AC}$ (S)	Reflexive Prop
$\triangle DAC \cong \triangle ECA$	SAS
$\angle ADC \cong \angle CEA$	CPCTC

5.



Statements	*Reasons*
$\angle C \cong \angle D$ (A)	Given
$\overline{AC} \cong \overline{AD}$ (S)	Given
$\angle A \cong \angle A$ (A)	Reflexive Prop
$\triangle ACE \cong \triangle ADB$	ASA
$\overline{CE} \cong \overline{DB}$	CPCTC

CHAPTER 10. TRIANGLES AND SIMILARITY

10.1 Properties of Similar Triangles

1. $\text{m}\angle P = 25^\circ$ and $m\angle R = 45^\circ$, so $\text{m}\angle Q = 180 - (25 + 45) = 110^\circ$

2. $\text{m}\angle B = 180 - (50 + 30) = 100$ So, $\text{m}\angle X = 100$

3. $\frac{25}{10} = \frac{AC}{6}$
$10AC = 150$
$AC = 15$

4. $\frac{50}{XY} = \frac{40}{20}$
$40XY = 1000$
$XY = 25$

5. $\frac{30}{21} = \frac{ML}{7}$
$ML = 10$

6. $\frac{9}{36} = \frac{4}{x}$
$9x = 144$
$x = 16$

7. $5x + 8x + 11x = 60$
$24x = 60$
$x = 2.5$
$5x = 5(2.5) = 12.5$

8. $\frac{15}{18} = \frac{5}{6}$

10.2 Triangle Similarity Methods

1. $\triangle QRS \sim \triangle UTS$
$\angle Q \cong \angle U$ and $\angle R \cong \angle T$ by alternate interior ∠'s formed by ∥ lines.
The △s are similar by AA~.
(Also, $\angle RSQ \cong \angle TSU$ by vertical ∠'s)

2. $\triangle AEB \sim \triangle CED$
$\angle A \cong \angle DCE$ and $\angle B \cong \angle CDE$ by corresponding ∠'s formed by ∥ lines. The △s are similar by AA~.
(Also, $\angle E \cong \angle E$ by Reflexive Prop.)

3. (3) $\angle ACB \cong \angle DFE$ (the included ∠'s for SAS similarity)

4. SAS

5. AA

6. (4)

10.3 Prove Triangles Similar

1. $\angle ACB \cong \angle AED$ (Given)
$\angle A \cong \angle A$ (Reflexive Prop)
$\triangle ABC \sim \triangle ADE$ (AA~)

2. $\overline{AB} \perp \overline{BE}, \overline{DE} \perp \overline{BE}$, and $\angle BFD \cong \angle ECA$	(Given)
$\angle B$ and $\angle E$ are right $\angle$'s	(def of $\perp$ lines)
$\angle B \cong \angle E$	(right $\angle$'s are $\cong$)
$\angle BFD$ and $\angle DFE$ are supplementary and $\angle ECA$ and $\angle ACB$ are supplementary	(linear pairs)
$\angle DFE \cong \angle ACB$	($\angle$'s supplementary to $\cong$ $\angle$'s are $\cong$)
$\triangle ABC \sim \triangle DEF$	(AA~)

3. right $\angle Q$, right $\angle T$, $PQ = 6, QR = 8, ST = 3, TU = 4$	(Given)
$\angle Q \cong \angle T$	(right $\angle$'s are $\cong$)
$\frac{6}{3} = \frac{8}{4}$ *[scale of 2]*	(def of proportion)
$\frac{PQ}{ST} = \frac{QR}{TU}$	(substitution)
$\triangle PQR \sim \triangle STU$	(SAS~)
$\angle R \cong \angle U$	(CASTC)

[Alternatively, use Pythagorean Thm to find $PR = 10$ and $SU = 5$, then $\triangle PQR \sim \triangle STU$ by SSS~]

10.4 Triangle Angle Bisector Theorem

1. $\frac{x}{4} = \frac{9}{6}$ $6x = 36$ $x = 6$	2. $\frac{2}{x-2} = \frac{5}{9}$ $5(x-2) = 18$ $5x - 10 = 18$ $5x = 28$ $x = 5.6$
3. Let $x = AB$. So, $BC = 30 - x$. $\frac{x}{30 - x} = \frac{21}{24}$ $24x = 21(30 - x)$ $24x = 630 - 21x$ $45x = 630$ $x = 14$	4. $\frac{x-2}{x+1} = \frac{x}{x+4}$ $(x-2)(x+4) = x(x+1)$ $x^2 + 2x - 8 = x^2 + x$ $2x - 8 = x$ $x = 8$ $P = x + (x-2) + (x+1) + (x+4)$ $= 8 + 6 + 9 + 12 = 35$
5. $6^2 + 8^2 = (DF)^2$ $100 = (DF)^2$ $DF = 10$ Let $x = EG$. By angle bisector thm, $\frac{x}{8 - x} = \frac{6}{10}$ $6(8 - x) = 10x$ $\quad$ $3^2 + 6^2 = (DG)^2$ $48 - 6x = 10x$ $\quad$ $45 = (DG)^2$ $48 = 16x$ $\quad$ $DG = \sqrt{45} \approx 6.7$ $x = EG = 3$	

10.5 Side Splitter Theorem

1. $\frac{CB}{BA}=\frac{CE}{ET}$ $\frac{3}{10-3}=\frac{6}{x}$ $3x=42$ $x=14$	2. $\frac{12}{x}=\frac{16}{4}$ $16x=48$ $x=3$
3. $\frac{3}{5}=\frac{6}{BC}$ $3BC=30$ $BC=10$	4. $\frac{AD}{DB}=\frac{AE}{EC}$ $\frac{8}{10-8}=\frac{12}{x}$ $8x=24$ $x=3$
5. $\frac{3}{9}=\frac{x}{15}$ $9x=45$ $x=5$	6. $\frac{EB}{25}=\frac{12}{18}$ $18EB=300$ $EB\approx 16.7$
7. $\frac{MN}{NP}=\frac{MR}{RQ}$ $\frac{13}{8}=\frac{x}{42-x}$ $13(42-x)=8x$ $546-13x=8x$ $546=21x$ $x=26$ $MR=26, RQ=42-26=16$	8. $\frac{CD}{DA}=\frac{CE}{EB}$ $\frac{4}{10-4}=\frac{x+2}{4x-7}$ $4(4x-7)=6(x+2)$ $16x-28=6x+12$ $10x-40$ $x=4$ $CE=(4)+2=6$
9. $CD=AD-AC=(2x+2)-(x-3)$ $=x+5$ $\frac{AB}{BE}=\frac{AC}{CD}$ $\frac{16}{20}=\frac{x-3}{x+5}$ $16x+80=20x-60$ $140=4x$ $35=x$ $AC=x-3=32$	10. By the side splitter thm, $\frac{9}{6}=\frac{EL}{BE}$, or $\frac{6}{9}=\frac{BE}{EL}$. By the ∠ bisector thm, $\frac{BE}{x}=\frac{EL}{15}$ $x(EL)=15(BE)$ $x=15\cdot\frac{BE}{EL}$ By substitution, $x=15\cdot\frac{6}{9}=10$, so $BW=10$
11. $\frac{5}{10}=\frac{12}{x}$ 24 miles	

10.6 Triangle Midsegment Theorem

1. $x = \frac{1}{2}(24) = 12$

2. $5x = 2(2x + 2)$
   $5x = 4x + 4$
   $x = 4$
   $DE = 2(4) + 2 = 10$
   $AC = 5(4) = 20$

3. $AB = 36 \div 3 = 12$
   $EF = \frac{1}{2}AB = \frac{1}{2} \cdot 12 = 6$ cm
   (triangle midsegment thm)

4. $BE = \frac{1}{2}BC = 6$ (def. of midpoint)
   $EF = \frac{1}{2}AB = 10$ ($\triangle$ midseg. thm)
   $AF = \frac{1}{2}AC = 8$ (def. of midpoint)
   Perimeter = $AB + BE + EF + AF = 44$

5. $BC = 2MN = 16$
   $AC = 2ML = 10$, so $NC = \frac{1}{2}AC = 5$
   $AB = 2NL = 12$, so $MB = \frac{1}{2}AB = 6$
   Perimeter = $BC + NC + MN + MB = 35$

6. $ST = 2(3.5) = 7$ (midseg. thm)
   Let $x = RS = RT$
   Perimeter $2x + 7 = 25$, so $x = 9$.
   $NT = \frac{1}{2}x = 4.5$ (def. of midpoint)

7. $PR = 2SU = 36$
   $ST = \frac{1}{2}QR = 11$
   $\angle STP$ and $\angle TSU$ are $\cong$ alternate interior $\angle$'s, so m$\angle STP = 48°$
   $\angle SUR$ and $\angle TSU$ are supplementary consecutive interior $\angle$'s, so m$\angle SUR = 132°$

Chapter 11. Points of Concurrency

11.1 Incenter and Circumcenter

1. (4) $\angle DBG \cong \angle EBG$	2. $m\angle REC = \frac{1}{2}(84) = 42$ $m\angle BRC = 180 - (42 + 28) = 110^\circ$
3. a) $r = TS = 3$, so $C = 2\pi r = 6\pi$. b) $m\angle QPR = 2 \cdot 18 = 36^\circ$ $m\angle QRP = 180 - (39 + 36) = 105^\circ$ $m\angle TRP = \frac{1}{2} m\angle QRP = 52.5^\circ$	4. a) $BD = 15, AF = 16$, and $AE = 17$ b) $AG = CG = BG = 19$ *[The circumcenter is equidistant from the three vertices of the triangle.]* c) $r = BG = 19$, so $A = \pi r^2 = 361\pi$
5. $9.4^2 + (PQ)^2 = 13.4^2$ $(PQ)^2 = 13.4^2 - 9.4^2 = 91.2$ $A = \pi r^2 = \pi \cdot (PQ)^2 = 91.2\pi \approx 286.5$ sq. units	6. Draw $\overline{CR}$. $(CR)^2 = \left(\sqrt{53}\right)^2 + 26^2 = 729.$ $CR = \sqrt{729} = 27$ Circumference $= 2\pi r = 2\pi \cdot (CR) = 54\pi$

11.2 Orthocenter and Centroid

1. a) circumcenter c) orthocenter b) centroid d) incenter	2. (3) an obtuse triangle
3. (4) circumcenter and orthocenter	4. (1) centroid
5. a) 48 b) 16 c) 24	
6. $SP = 2PR = 24$ $AT = 2AR = 40$ $TM = PT + \frac{1}{2}PT = 42$ $PY = \frac{1}{3}AY = 9$	
7. $GC = 2(FG) = 24$ cm	8. $FD = \frac{1}{2}(AF) = 3$
9. $TA = TO + OA = 10 + 2(10) = 30$	10. $BP = \frac{2}{3}(BF) = \frac{2}{3}(18) = 12$
11. $FG = \frac{1}{3}(CF) = \frac{1}{3}(24) = 8$	12. $CR = 2(RF)$ $24 = 2(2x - 6)$ $x = 9$
13. $BP = 2(PM)$ $7x + 4 = 2(2x + 5)$ $x = 2$ $PM = 2(2) + 5 = 9$	14. $QC = 2(CM)$ $5x = 2(x + 12)$ $x = 8$ $QM = QC + CM = 5(8) + (8 + 12) = 60$

CHAPTER 12. RIGHT TRIANGLES AND TRIGONOMETRY

12.1 Congruent Right Triangles

<table>
<tr><td>1. A</td><td>2. (3)</td></tr>
<tr><td colspan="2">3. Right $\triangle$s MAT and HTA (Given)
$\overline{MT} \cong \overline{AH}$ (H) (Given)
$\overline{AT} \cong \overline{AT}$ (L) (Reflexive Prop)
$\triangle MAT \cong \triangle HTA$ (HL)
$\angle M \cong \angle H$ (CPCTC)</td></tr>
<tr><td colspan="2">4. $\overline{CA} \perp \overline{AB}, \overline{ED} \perp \overline{DF}, \overline{CE} \cong \overline{BF}$ (Given)
$\overline{AB} \cong \overline{ED}$ (L) (Given)
$\angle A$ and $\angle D$ are right $\angle$'s (Def. of $\perp$)
$\triangle ABC$ and $\triangle DEF$ are right $\triangle$s (Def. of right $\triangle$s)
$\overline{EB} \cong \overline{EB}$ (Reflexive Prop)
$CE + EB = BF + EB$, $\overline{CB} \cong \overline{FE}$ (H) (Addition Prop)
$\triangle ABC \cong \triangle DEF$ (HL)
$\overline{AC} \cong \overline{DF}$ (CPCTC)</td></tr>
</table>

12.2 Equidistance Theorems

<table>
<tr><td>1. 12</td><td>2. 26°</td></tr>
<tr><td>3. $RS = 15$. $\triangle PSR$ is isosceles, so m$\angle R = 30°$. Therefore, m$\angle RSQ = 60°$.</td><td>4. $2x + 10 = 5x - 17$
$10 = 3x - 17$
$27 = 3x$
$x = 9$</td></tr>
<tr><td>5. $2x + 5 = 4x - 25$
$5 = 2x - 25$
$30 = 2x$
$x = 15$</td><td>6. $5x - 3 = 3x + 1$
$2x = 4$
$x = 2$
m$\angle BAC = 41(2) = 82°$
The $\perp$ bisector of the base of an isosceles $\triangle$ is also the bisector of the vertex $\angle$. Therefore, m$\angle BAD = 41°$ and m$\angle B = 49°$.</td></tr>
<tr><td>7. P is the incenter, so $\overline{PQ} \cong \overline{PR} \cong \overline{PS}$.</td><td>8. P is the circumcenter (external to $\triangle ABC$), so $\overline{PA} \cong \overline{PB} \cong \overline{PC}$.</td></tr>
</table>

12.3 Geometric Mean Theorems

1. $x^2 = 3 \cdot 9 = 27$
$x = \sqrt{27} = 3\sqrt{3}$

2. $x^2 = 12 \cdot 3 = 36$
$x = \sqrt{36} = 6$

3. $AD = 16 - 7 = 9$
$x^2 = 7 \cdot 9 = 63$
$x = \sqrt{63} = 3\sqrt{7}$

4. $x^2 = 12 \cdot 2 = 24$
$x = \sqrt{24} = 2\sqrt{6}$

5. $x^2 = 4 \cdot 7 = 28$
$x = \sqrt{28} = 2\sqrt{7}$

6. $x^2 = 8 \cdot 18 = 144$
$x = 12$

7. $6^2 = x(x+5)$
$36 = x^2 + 5x$
$x^2 + 5x - 36 = 0$
$(x+9)(x-4) = 0$
$x = 4$ (reject $x = -9$)

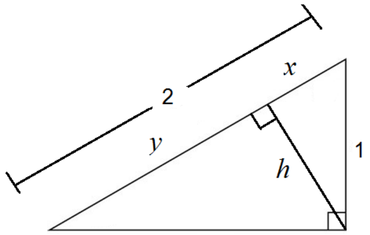
8. $10^2 = x(x+21)$
$100 = x^2 + 21x$
$x^2 + 21x - 100 = 0$
$(x+25)(x-4) = 0$
$x = 4$ (reject $x = -25$)

9. Let $x = AD$
$4^2 = x(x+6)$
$x^2 + 6x - 16 = 0$
$(x+8)(x-2) = 0$
$x = 2$ (reject $x = -8$)
$AC = x + (x+6) = 10$

10. $x^2 = 4 \cdot 5 = 20$
$x = \sqrt{20} = 2\sqrt{5}$
$y^2 = 9 \cdot 4 = 36$
$y = \sqrt{36} = 6$
$z^2 = 9 \cdot 5 = 45$
$z = \sqrt{45} = 3\sqrt{5}$

11. $x^2 = 24 \cdot 6 = 144$
$x = \sqrt{144} = 12$
$y^2 = 30 \cdot 6 = 180$
$y = \sqrt{180} = 6\sqrt{5}$
$z^2 = 24 \cdot 30 = 720$
$z = \sqrt{720} = 12\sqrt{5}$

12.

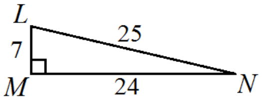
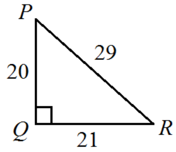


(a)
$1^2 = 2x$
$1 = 2x$
$x = \frac{1}{2}$

(b)
$y = 2 - \frac{1}{2} = \frac{3}{2}$
$h^2 = xy = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$
$h = \sqrt{\frac{3}{4}}$

CHAPTER 13. TRIGONOMETRY

13.1 Trigonometric Ratios

1. $\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \tan A = \frac{3}{4}$	2. $\sin B = \frac{15}{17}, \cos B = \frac{8}{17}, \tan B = \frac{15}{8}$
3. $\sin S$ and $\cos R$	4. $\tan B = \frac{8}{15} \approx 0.533$
5. $\sin x = \frac{28}{53} \approx 0.528$	6. $\cos A = \frac{16}{20} = 0.8$
7. $\sin N = \frac{7}{25}$  	8. $\tan P = \frac{21}{20}$ 
9. $30^2 + 40^2 = c^2$ $2500 = c^2$ $c = 50$ $\quad$ $\sin B = \frac{30}{50} = \frac{3}{5}$	

13.2 Use Trigonometry to Find a Side

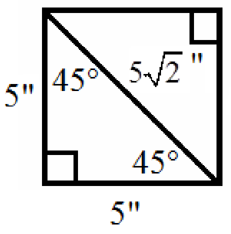
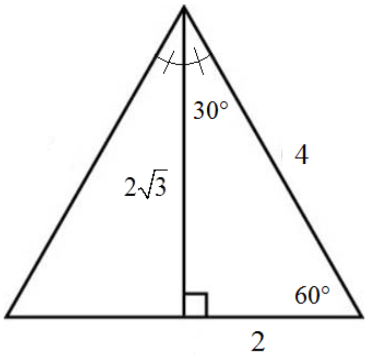
1. (3)	2. $\sin 15° = \frac{w}{10}$ $w = 10 \sin 15° \approx 2.6$
3. $\sin 32° = \frac{x}{30}$ $x = 30 \sin 32° \approx 15.9$ ft.	4. $\tan 62° = \frac{x}{15}$ $x = 15 \tan 62° \approx 28.2$ ft.
5. $\sin 57° = \frac{x}{8}$ $x = 8 \sin 57° \approx 6.7$ ft.	6. $\tan 32° = \frac{x}{25}$ $x = 25 \tan 32° \approx 15.6$ ft.
7. $\cos 65° = \frac{x}{5}$ $x = 5 \cos 65° \approx 2.1$ ft.	8. $\sin 48° = \frac{9}{x}$ $x = \frac{9}{\sin 48°} \approx 12$ ft.
9. $\tan 11° = \frac{400}{x}$ $x = \frac{400}{\tan 11°} \approx 2{,}058$ ft.	10. $\tan 58° = \frac{x}{6}$ $x = 6 \tan 58° \approx 9.60$ ft. $A = \frac{1}{2}bh \approx \frac{1}{2}(6)(9.60) \approx 28.8$ ft^2
11. $\tan 52° = \frac{50}{x}$ $x = \frac{50}{\tan 52°} \approx 39$ ft. from base to stake $\sin 52° = \frac{50}{x}$ $x = \frac{50}{\sin 52°} \approx 63$ ft. wire	12. $\sin 50° = \frac{x}{110}$ $x = 110 \sin 50° \approx 84$ ft. high $\cos 50° = \frac{x}{110}$ $x = 110 \cos 50° \approx 71$ ft. between the ropes

13. $\cos 72° = \frac{x}{10}$ $x = 10\cos 72° \approx 3.09$ ft. ≈ 37 in. from base $\sin 72° = \frac{y}{10}$ $x = 10\sin 72° \approx 9.51$ ft. ≈ 114 in. up wall	14. $\tan 28° = \frac{h}{200}$ (*h = cliff + lighthouse*) $h = 200\tan 28° \approx 106.34$ ft. $\tan 18° = \frac{c}{200}$ (*c = cliff height alone*) $c = 200\tan 18° \approx 64.98$ ft. $x = h - c \approx 106.34 - 64.98 \approx 41.4$ ft.

13.3 Use Trigonometry to Find an Angle

1. $\sin x = \frac{3}{7}$ $x = \sin^{-1}\left(\frac{3}{7}\right) \approx 25.4°$	2. $\tan A = \frac{11.2}{18.3}$ $A = \tan^{-1}\left(\frac{11.2}{18.3}\right) \approx 31.5°$
3. $\sin x = \frac{30}{50}$ $x \approx 37°$	4. $\sin A = \frac{8}{12}$ $A \approx 42°$
5. $\cos x = \frac{6}{28}$ $x \approx 78°$	6. $\sin A = \frac{10}{16}$ $m\angle A \approx 38.7°$ $m\angle B = 90 - m\angle A \approx 51.3°$
7. $\tan x = \frac{420}{2000}$ $x \approx 12°$	8. $\tan x = \frac{350}{1000}$ $x \approx 19°$
9. $12^2 + 16^2 = r^2$ $r = 20$ $s = 50 - 20 = 30$ $\sin x = \frac{16}{30}$ $x \approx 32°$	
10. 36 – 28 = 8 ft $\sin x = \frac{8}{12}$ $x \approx 41.8°$	
11. $\tan x = \frac{6}{4}$ $x \approx 56°$ $\sin 56° = \frac{b}{15}$ $b = 15\sin 56° \approx 12$ ft.	

13.4 Special Triangles

1. 30-60-90 △ with a factor of 12.	$x = 12\sqrt{3} \approx 21$
2. $5\sqrt{2}$ inches Each side is 5 inches. 45-45-90 △ with factor of 5. 	3. 4 units The altitude of an equilateral △ is also the ∠ bisector. 30-60-90 △ with factor of 2. 

13.5 Cofunctions

1. $m\angle B = 90 - 25 = 65°$	2. $72 + x = 90$ $x = 18°$
3. $x + 15 + x - 5 = 90$ $2x + 10 = 90$ $2x = 80$ $x = 40$	4. $2x - 1 + 3x + 6 = 90$ $5x + 5 = 90$ $5x = 85$ $x = 17$
5. $x + 20 + x = 90$ $2x + 20 = 90$ $2x = 70$ $x = 35$	6. $x - 3 + 2x + 6 = 90$ $3x + 3 = 90$ $3x = 87$ $x = 29$
7. $2x + 20 + 40 = 90$ $2x + 60 = 90$ $2x = 30$ $x = 15$	8. $2x - 25 + 55 = 90$ $2x + 30 = 90$ $2x = 60$ $x = 30$

13.6 SAS Sine Formula for Area of a Triangle

1. $A = \frac{1}{2} \cdot 6 \cdot 8 \cdot \frac{1}{4} = 6$	2. $A = \frac{1}{2} \cdot 12 \cdot 15 \cdot \sin 150 = 45$
3. $A = \frac{1}{2} \cdot 7 \cdot 10 \cdot \sin 25 \approx 14.8$	4. $m\angle C = 180 - 2 \cdot 75 = 30$ $A = \frac{1}{2} \cdot 10 \cdot 10 \cdot \sin 30 = 25$
5. $A = \frac{1}{2} \cdot 14 \cdot 16 \cdot \sin 30 = 56$	6. $A = \frac{1}{2} \cdot 11 \cdot 13 \cdot \sin 70 \approx 67$
7. $A = \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 30° = 4$	8. $A = \frac{1}{2} \cdot 4 \cdot 5 \cdot \sin 59° \approx 8.6$
9. $A = \frac{1}{2} \cdot 16 \cdot 21 \cdot \sin 58 \approx 142.5$	10. $A = \frac{1}{2} \cdot 12 \cdot 8 \cdot \sin 40° \approx 30.9$
11. $A = \frac{1}{2} \cdot 12 \cdot 31 \cdot \sin 62 \approx 164.2$	12. Vertex $\angle = 180 - 2(50) = 80°$ $A = \frac{1}{2}(20.4)(20.4) \sin 80 \approx 204.9$
13. $m\angle R = 180 - (38 + 17) = 125°$ $A = \frac{1}{2}(15)(31.6) \sin 125 \approx 194$	14. $12 = \frac{1}{2} \cdot 8 \cdot b \cdot \sin 30°$ $12 = 4b \cdot \sin 30°$ $3 = b \cdot \sin 30°$ $b = \frac{3}{\sin 30°} = 6$

15. $42 = \frac{1}{2} \cdot 24 \cdot b \cdot \sin 30°$ $42 = 12b \cdot \sin 30°$ $3.5 = b \cdot \sin 30°$ $b = \frac{3.5}{\sin 30°} = 7$	16. $12 = \frac{1}{2} \cdot 6 \cdot c \cdot \sin 30°$ $12 = 3c \cdot \sin 30°$ $4 = c \cdot \sin 30°$ $c = \frac{4}{\sin 30°} = 8$

CHAPTER 14. QUADRILATERALS

14.1 Angles of Polygons

<table>
<tr><td>1. $\frac{5}{2}(5-3)=5$</td><td>2. $\frac{10}{2}(10-3)=35$</td></tr>
<tr><td>3. $(10-2)\cdot 180° = 1{,}440°$</td><td>4. $1{,}080° \div 8 = 135°$</td></tr>
<tr><td colspan="2">5. Regular Polygons
<table>
<tr><th>Number of sides</th><th>Measure of an exterior angle</th></tr>
<tr><td>3</td><td>120°</td></tr>
<tr><td>4</td><td>90°</td></tr>
<tr><td>5</td><td>72°</td></tr>
<tr><td>6</td><td>60°</td></tr>
<tr><td>7</td><td>≈ 51.4°</td></tr>
<tr><td>8</td><td>45°</td></tr>
<tr><td>9</td><td>40°</td></tr>
<tr><td>10</td><td>36°</td></tr>
</table>
</td></tr>
</table>

14.2 Properties of Quadrilaterals

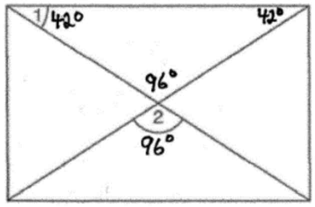
1. (4) trapezoid	2. (1) rhombus
3. (3) rhombus	4. (3) the rhombus and square
5. (2) An isosceles trapezoid has a pair of ∥ bases and a pair of ≅ legs, but may not be a ▱.	6. In a ▱, diagonals bisect each other, so $AM = 10$.
7. In a ▱, opp ∠'s are ≅, so $6x - 30 = 4x + 10$ $2x = 40$ $x = 20$ $m\angle A = 6(20) - 30 = 90$	8. In a ▱, diagonals bisect each other, so $AE = 5$. In a rhombus, diagonals are ⊥, so $\triangle AEB$ is a right △ with a right ∠ at E. $(BE)^2 = (AB)^2 - (AE)^2$ $(BE)^2 = 8^2 - 5^2 = 39$ $BE = \sqrt{39} \approx 6.24$ $BD = 2BE \approx 12.5$
9. $AE = \frac{1}{2}AC = 9 \quad BE = \frac{1}{2}BD = 12$ (diagonals bisect each other) $m\angle AEB = 90°$ (⊥ diagonals) $(AB)^2 = 9^2 + 12^2 = 225$ $AB = \sqrt{225} = 15$ (Pythagorean Thm.)	10. $AE = \frac{1}{2}AC = 6$ $(DE)^2 + 6^2 = 10^2$ $(DE)^2 = 64$ $DE = 8$ $DB = 2 \cdot DE = 16$

11. $m\angle ABC = 180 - 100 = 80°$
(consecutive ∠'s are supplementary)
$m\angle DBC = \frac{1}{2}80 = 40°$
(diagonals are ∠ bisectors)

12. $m\angle 1 + m\angle 2 = 180 - 120 = 60°$
(consecutive ∠'s are supplementary)
$m\angle 2 = 60 - 45 = 15°$

13. Label E at the intersection of diagonals.
$m\angle AEM = 90°$ (⊥ diagonals)
$m\angle AMT = 90 - 12 = 78°$

14. ∠1 is a base ∠ of an isosceles △.



$m\angle 2 = 96$

15. Since $ABCD$ is a rhombus, all sides are ≅, so AB = AD = 24.
Diagonals of a rhombus are ⊥, so $\triangle AEB$ is a right △ with a right ∠ at E.
$\sin 30 = \frac{BE}{24}$
$BE = 24 \sin 30 = 12$
Diagonals bisect each other, so
DE = BE = 12

16. Since $ABCD$ is a rhombus, the diagonals are ∠ bisectors, so
$m\angle ABD = \frac{1}{2} m\angle ABC = 30.$
Diagonals of a rhombus are ⊥, so $\triangle AEB$ is a right △ with a right ∠ at E.
$\sin 30 = \frac{18}{AB}$
$AB = \frac{18}{\sin 30} = 36$
All sides of a rhombus are ≅,
so DC = AB = 36.

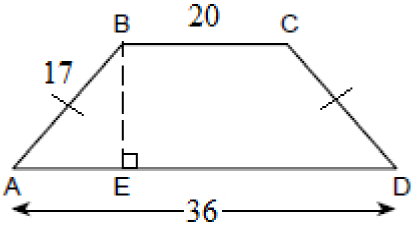
14.3 Trapezoids

1. a) Diagonals are ≅, so BD = 25.
b) $m\angle D = m\angle C = 180 - 105 = 75°$.

2. $GT = \frac{40-24}{2} = 8$
$(AG)^2 = 10^2 - 8^2 = 36$
$AG = \sqrt{36} = 6$
$LF = AG = 6$ (altitudes are ≅)

3. Draw altitude $\overline{CE}$.
$ED = \frac{26-12}{2} = 7$
$(CE)^2 = 25^2 - 7^2$
$CE = \sqrt{576} = 24$

4. $AE = \frac{36-20}{2} = 8$
$(BE)^2 = 17^2 - 8^2 = 225$
$BE = \sqrt{225} = 15$



5. Draw altitude $\overline{AT}$. $\triangle RAT$ is an isosceles right △ with legs of 6.
$(RA)^2 = 6^2 + 6^2$
$RA = \sqrt{72} = 6\sqrt{2}$

6. $\triangle RST$
They share the same base, $\overline{RS}$, and congruent altitudes (since $\overline{VT} \parallel \overline{RS}$).

14.4 Use Quadrilateral Properties in Proofs

1.	$ABCD$ is a ▱	(Given)
	$\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$	(opp sides of ▱s are ≅)
	$\overline{BD} \cong \overline{BD}$	(Reflexive Prop)
	$\triangle ABD \cong \triangle CDB$	(SSS)
2.		(Givens omitted)
	$\overline{BO}$ and $\overline{NR}$ bisect each other	(diagonals of a ▱ bisect each other)
	$\overline{NX} \cong \overline{RX}$ and $\overline{BX} \cong \overline{OX}$	(def of bisector)
	$\angle BXN \cong \angle OXR$	(vertical $\angle$'s are ≅)
	$\triangle BNX \cong \triangle ORX$	(SAS)
3.		(Givens omitted)
	$\overline{AM} \cong \overline{DM}$	(def of midpoint)
	$\angle A \cong \angle D$	(all $\angle$'s in a ▭ are ≅)
	$\overline{AB} \cong \overline{CD}$	(opp sides of a ▭ are ≅)
	$\triangle ABM \cong \triangle DCM$	(SAS)
	$\overline{BM} \cong \overline{CM}$	(CPCTC)
4.		(Givens omitted)
	$\overline{FH} \cong \overline{SL}$	(opp sides of ▱s are ≅)
	$\overline{FH} \parallel \overline{SL}$	(opp sides of ▱s are ∥)
	$\angle AFH \cong \angle LSG$	(alternate interior $\angle$'s thm)
	$\triangle LGS \cong \triangle HAF$	(AAS)
5.		(Givens omitted)
	$\overline{AB} \cong \overline{CD}$	(all sides of a square are ≅)
	$\angle B \cong \angle C$	(all $\angle$'s of a square are ≅)
	$\overline{EF} \cong \overline{EF}$	(Reflexive Prop)
	$\overline{BE} + \overline{EF} \cong \overline{FC} + \overline{EF}$,	
	so $\overline{BF} \cong \overline{EC}$	(Addition Prop)
	$\triangle ABF \cong \triangle DCE$	(SAS)
	$\overline{AF} \cong \overline{DE}$	(CPCTC)
6.		(Givens omitted)
	$\overline{AD} \cong \overline{BC}$	(opp sides of a ▭ are ≅)
	$\angle A \cong \angle B$	(all $\angle$'s of a ▭ are ≅)
	$\triangle ADF \cong \triangle BCE$	(ASA)
	$\overline{AF} \cong \overline{BE}$	(CPCTC)
	$\overline{EF} \cong \overline{EF}$	(Reflexive Prop)
	$\overline{AF} - \overline{EF} \cong \overline{BE} - \overline{EF}$,	
	so $\overline{AE} \cong \overline{BF}$	(Subtraction Prop)

7.	(Givens omitted)
$\overline{PE} \cong \overline{OE}$	(all sides of a rhombus are ≅)
$\angle EPR \cong \angle EOR$	(opp ∠'s of a rhombus are ≅)
$\angle SPR - \angle EPR = \angle VOR - \angle EOR$, so $\angle SPE \cong \angle VOE$	(Subtraction Prop)
$\angle SEP \cong \angle VEO$	(vertical ∠'s are ≅)
$\triangle SEP \cong \triangle VEO$	(ASA)
$\overline{SE} \cong \overline{EV}$	(CPCTC)

14.5 Prove Types of Quadrilaterals

1. Yes, one pair of opp sides are both ∥ and ≅.	2. Yes, both pairs of opp sides are ≅.

3. (3) Diagonals are ⊥.

4. $\triangle AOB \cong \triangle COD$	(Given)
$\overline{AB} \cong \overline{CD}$	(CPCTC)
$\angle OAB \cong \angle OCD$	(CPCTC)
$\overline{AB} \parallel \overline{CD}$	(alternate interior ∠'s converse)
$ABCD$ is a ▱	(quad with pair of opp sides both ∥ and ≅ → ▱)

5. $ABCD$ is a ▱, $DF = EB$	(Given)
$\overline{AEB} \parallel \overline{DFC}$	(opp sides of a ▱ are ∥)
$\overline{AB} \cong \overline{DC}$	(opp sides of a ▱ are ≅)
$\overline{AB} - \overline{EB} \cong \overline{DC} - \overline{DF}$, so $\overline{AE} \cong \overline{FC}$	(Subtraction Prop)
$AECF$ is a ▱	(quad with pair of opp sides both ∥ and ≅ → ▱)

6. $ABCD$ is a ▱, $CEBF$ is a rhombus	(Given)
$\overline{BE} \cong \overline{CE}$	(all sides of a rhombus are ≅)
$2 \cdot \overline{BE} \cong 2 \cdot \overline{CE}$	(Multiplication Prop)
$\overline{BD} \cong 2 \cdot \overline{BE}$, $\overline{AC} \cong 2 \cdot \overline{CE}$	(diagonals of a ▱ bisect each other)
$\overline{BD} \cong \overline{AC}$	(Substitution)
$ABCD$ is a ▭	(if a ▱ has ≅ diagonals, then it is a ▭)

7.	(Givens omitted)
$\angle BFA, \angle BFC, \angle DEC, \angle DEA$ are right ∠'s	(def of ⊥)
$\angle BFA \cong \angle DEC$	(right ∠'s are ≅)
$\overline{FE} \cong \overline{FE}$	(Reflexive Prop)
$\overline{AE} - \overline{FE} \cong \overline{FC} - \overline{FE}$, so $\overline{AF} \cong \overline{EC}$	(Subtraction Prop)
$\triangle BFA \cong \triangle DEC$	(AAS)
$\overline{AB} \cong \overline{CD}$ and $\overline{BF} \cong \overline{DE}$	(CPCTC)
$\angle BFC \cong \angle DEA$	(right ∠'s are ≅)
$\triangle BFC \cong \triangle DEA$	(SAS)
$\overline{AD} \cong \overline{CB}$	(CPCTC)
ABCD is a ▱	(quad with both pairs of opp sides ≅ → ▱)

CHAPTER 15. CIRCLES

15.1 Circumference and Rotation

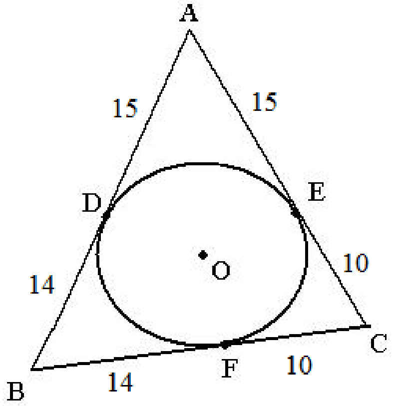
1. $D = 1000$ ft. $C = 2 \cdot 5\pi = 10\pi$ ft. $R = \frac{D}{C} = \frac{1000}{10\pi}$ ft ≈ 31.8 It must make at least 32 revolutions.	2. D = 100 ft. = 1200 in. $C = 8\pi$ in. $R = \frac{D}{C} = \frac{1200}{8\pi}$ in ≈ 47.7 47 clocks can be framed.
3. $C = 2\pi$ $D = C \cdot R = 2\pi \cdot 1100.5 \approx 6{,}914.65$ ft. $6{,}914.65\ ft\ \times \frac{1\ mile}{5{,}280\ ft} \approx 1.3\ miles$	4. D = 2 mi. = 10,560 ft. $C = 394\pi$ ft. $R = \frac{10{,}560}{394\pi}$ ft. ≈ 8.5
5. $D = 1$ mile = 5,280 ft. $C = 2\pi$ $R = \frac{5280}{3 \cdot 2\pi}$ ft. ≈ 280.1 281 rotations are needed.	

15.2 Arcs and Chords

1. (4) supplementary	2. (4) right
3. $\frac{2}{12} = \frac{1}{6}$ $\qquad$ $\frac{1}{6}(360°) = 60°$	4. $\text{m}\angle ABC = 30°$
5. $\text{m}\angle AOC = 48°$	6. $\text{m}\,\overset{\frown}{BC} = 140°$ $\text{m}\,\overset{\frown}{AC} = 180 - 140 = 40°$
7. $\parallel$ chords intercept $\cong$ arcs, so $\text{m}\,\overset{\frown}{DB} = \frac{180 - 110}{2} = 35°$	8. $\frac{180 - 80}{2} = 50°$
9. $\text{m}\angle A = 90°$, so $\text{m}\angle C = 35°$	10. $\text{m}\,\overset{\frown}{GFE} = 86 \times 2 = 172°$ $\text{m}\angle F = 180 - 86 = 94°$
11. $\text{m}\angle ADC = \frac{132 + 82}{2} = 107°$	12. $15x = 360°$ $x = 24°$ $\text{m}\,\overset{\frown}{FE} = 48°$ and $\text{m}\,\overset{\frown}{GD} = 168°$ $\text{m}\angle D = \text{m}\angle G = \frac{1}{2}\text{m}\,\overset{\frown}{FE} = 24°$ *or* $\text{m}\angle E = \text{m}\angle F = \frac{1}{2}\text{m}\,\overset{\frown}{GD} = 84°$

15.3 Tangents

1. Perimeter = 78.



2. $\triangle ABC$ is a right $\triangle$.
$(AC)^2 = 8^2 + 15^2$
$(AC)^2 = 289$
$AC = 17$

AD is a radius.
$CD = AC - AD = 17 - 8 = 9$

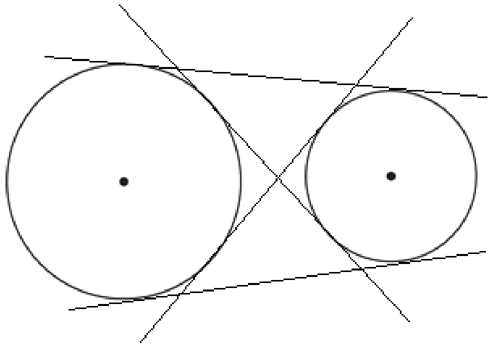
3. $\overline{AB} \cong \overline{AC}$, so $\triangle ABC$ is an isosceles $\triangle$.
m$\angle A = 180 - 2(66) = 48°$

4. m $\widehat{AB} = 180 - 38 = 142°$,
so m$\angle AOB = 142°$

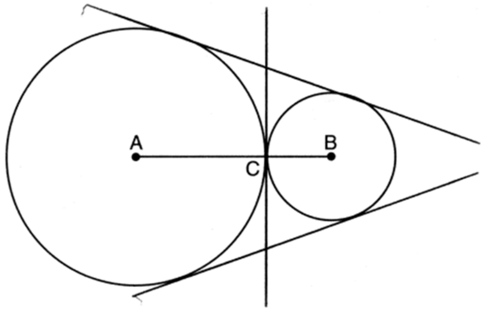
5. m $\widehat{RS} = 180 - 54 = 126°$.

6. The measure of minor arc DC is
$360 - 246 = 114°$.
m$\angle DEC = 180 - 114 = 66°$.

7.



8.



15.4 Secants

1. a) tangent b) secant
c) diameter d) chord

2. $x = \dfrac{86 - 44}{2} = 21°$

3. m$\angle P = \dfrac{70 - 20}{2} = 25°$

4. $30 = \dfrac{140 - \text{m}\widehat{CD}}{2}$, so m $\widehat{CD} = 80°$.
m $\widehat{DE} = 360 - (140 + 80) = 140°$.

5. $(PA)^2 = (PB)(PC)$
$(PA)^2 = (4)(16)$
$(PA)^2 = 64$
$PA = 8$

6. $(PQ)(PR) = (PS)(PT)$
$(6)(24) = (8)(PT)$
$PT = 18$

7. $(BC)(AC) = (EC)(DC)$
$(3)(12) = (EC)(9)$
$EC = 4$

8. $x^2 = 3(x + 18)$
$x^2 = 3x + 54$
$x^2 - 3x - 54 = 0$
$(x - 9)(x + 6) = 0$
$x = 9$

9. a) Let $x = TM$.
$x(x + 2) = (12)(2)$
$x^2 + 2x - 24 = 0$
$(x + 6)(x - 4) = 0$
$x = 4$
$RT = 6 + 4 = 10$

b) $(PS)^2 = (8)(18)$
$(PS)^2 = 144$
$PS = 12$

10. a) m $\overset{\frown}{ACB} = 360 - (56 + 112) = 192$
m $\overset{\frown}{CB} = \frac{1}{4}(192) = 48$
m$\angle CEB = \dfrac{56 + 48}{2} = 52$
b) m$\angle F = \dfrac{192 - 112}{2} = 40$
c) m$\angle DAC = \dfrac{112 + 48}{2} = 80$

11. Because they are radii of the same circle, $BP = a$ and $BQ = a$.
By the Corollary to the Intersecting Secants Thm, $(AC)^2 = AP \cdot AQ$.
$b^2 = (c - a)(c + a)$ [substitute b for AC, $(c - a)$ for AP, and $(c + a)$ for AQ]
$b^2 = c^2 - a^2$ [multiply the binomials]
$a^2 + b^2 = c^2$ [add a^2 to both sides]

15.5 Circle Proofs

1.

Statements	Reasons
Circle O, $\overset{\frown}{AB} \cong \overset{\frown}{AC}$	Given
$\overline{AB} \cong \overline{AC}$ (S)	If two arcs are ≅, their chords are ≅
$\overline{AO} \cong \overline{AO}$ (S)	Reflexive Prop
$\overline{OC} \cong \overline{OB}$ (S)	All radii in a circle are ≅
$\triangle AOC \cong \triangle AOB$	SSS

Alternately, $\overline{AB} \cong \overline{AC}$ could be replaced with $\angle AOC \cong \angle AOB$ (central ∠'s of ≅ arcs are ≅), and then $\triangle AOC \cong \triangle AOB$ by SAS.

2.

Statements	Reasons
Circle Q, $\overline{PQR} \perp \overline{ST}$	Given
$\overline{PQR} \cong \overline{PQR}$ (S)	Reflexive Prop
$\angle PRS$, $\angle PRT$ are right ∠'s	⊥ segments form right ∠'s
$\angle PRS \cong \angle PRT$ (A)	Right ∠'s are ≅
$\overline{QR}$ bisects $\overline{ST}$	If a radius is ⊥ to a chord, it bisects the chord
$\overline{RS} \cong \overline{RT}$ (S)	Def of bisector
$\triangle PRS \cong \triangle PRT$	SAS
$\overline{PS} \cong \overline{PT}$	CPCTC

3.

Statements	Reasons
Tangents $\overline{PA}$ and $\overline{PB}$, radii $\overline{OA}$ and $\overline{OB}$, and $\overline{OP}$ intersects the circle at C.	Given
$\overline{OA} \perp \overline{PA}$ and $\overline{OB} \perp \overline{PB}$	a tangent is ⊥ to a radius at the point of tangency
$\angle PAO$ and $\angle PBO$ are right ∠'s	⊥ lines form right ∠'s
$\overline{OP} \cong \overline{OP}$ (H)	Reflexive Prop
$\overline{OA} \cong \overline{OB}$ (L)	all radii in a circle are ≅
$\triangle AOP \cong \triangle BOP$	HL
$\angle AOP \cong \angle BOP$	CPCTC

4.

Statements	Reasons
Diameter $\overline{BOD}$, m $\overset{\frown}{BR} = 70$, m $\overset{\frown}{YD} = 70$	Given
$\text{m}\angle RDB = \text{m}\angle YBD = 35$ (A)	The measure of an inscribed ∠ is half its intercepted arc
$\angle BRD$ and $\angle DYB$ are right ∠'s	an inscribed ∠ of a semicircle is a right ∠
$\angle BRD \cong \angle DYB$ (A)	right ∠'s are ≅
$\overline{BD} \cong \overline{DB}$ (S)	Reflexive Prop
$\triangle RBD \cong \triangle YDB$	AAS

Alternately, $\overline{BR} \cong \overline{YD}$ (if two arcs are ≅, their chords are ≅), which leads to the △s ≅ by HL

5.

Statements	Reasons
Quad $ABCD$ inscribed in circle O, $\overline{AB} \parallel \overline{DC}$, diagonals $\overline{AC}$ and $\overline{BD}$	Given
$\overset{\frown}{AD} \cong \overset{\frown}{BC}$	∥ chords intercept ≅ arcs
$\angle BDC \cong \angle ACD$ (A)	inscribed ∠'s that intercept ≅ arcs are ≅
$\angle DAC \cong \angle DBC$ (A)	inscribed ∠'s that intercept the same arc are ≅
$\overline{AD} \cong \overline{BC}$ (S)	≅ chords intercept ≅ arcs
$\triangle ACD \cong \triangle BDC$	AAS

6.

Statements	Reasons
$\overline{AD}$ is a diameter, $\overline{AD} \parallel \overline{BC}$	Given
$\angle BEA \cong \angle CED$ (A)	vertical ∠'s are ≅
$\angle BAC \cong \angle CDB$ (A)	inscribed ∠'s that intercept the same arc are ≅
$\overset{\frown}{AB} \cong \overset{\frown}{DC}$	∥ chords intersect ≅ arcs
$\overline{AB} \cong \overline{DC}$ (S)	≅ chords intersect ≅ arcs
$\triangle BAE \cong \triangle CDE$	AAS
$\overline{BE} \cong \overline{CE}$	CPCTC

15.6 Arc Lengths and Sectors

1. $\frac{90}{360} = \frac{L}{30\pi}$ $L = 7.5\pi$ feet	2. $\frac{40}{360} = \frac{L}{12\pi}$ $L = \frac{4}{3}\pi$ m
3. Central ∠ = 180 – 150 = 30° $\frac{30}{360} = \frac{L}{34\pi}$ $L \approx 9$ cm	4. $\frac{\theta}{360} = \frac{14\pi}{36\pi}$ $\theta = 140°$
5. $\frac{\theta}{360} = \frac{247}{2\pi \cdot 150}$ $\theta \approx 94°$	6. $\frac{165}{360} = \frac{L}{2\pi \cdot 2.4}$ $L \approx 6.9$ meters
7. $\frac{40}{360} = \frac{S}{36\pi}$ $S = 4\pi$ sq. units	8. Central ∠ = 360 – 45 = 315° $\frac{315}{360} = \frac{S}{16\pi}$ $S = 14\pi$ sq. cm
9. Area of sector: $\frac{90}{360} = \frac{S}{100\pi}$ $S = 25\pi$ Area of △: $\frac{1}{2}(10)(10) = 50$ Area of segment = $25\pi - 50$ sq. units	10. Area of sector: $\frac{120}{360} = \frac{S}{144\pi}$ $S = 48\pi$ sq. units Area of △: $\frac{1}{2}(12)(12)sin120° = 36\sqrt{3}$ Area of segment = $48\pi - 36\sqrt{3}$ sq. units
11. $\frac{9}{360} = \frac{L}{2\pi \cdot 3954}$ $L \approx 621.1$ miles	

12. a) $\triangle LSB$ is a right triangle with $LB = 6$ and $SB = 3$

$\sin\angle SLB = \frac{3}{6} = \frac{1}{2}$, so m$\angle SLB = 30°$ m$\angle ALB = 2$m$\angle SLB = 60°$

b) $A_{sector} = \frac{60}{360} \cdot 36\pi = 6\pi$

$A_{\triangle ALB} = \frac{1}{2}(6)(6)\sin 60° = 9\sqrt{3}$ *[or $LS = 3\sqrt{3}$ by Pyth. Thm, $A_{\triangle ALB} = \frac{1}{2}(6)3\sqrt{3} = 9\sqrt{3}$]*

$A_{segment} = A_{sector} - A_{\triangle ALB} = 6\pi - 9\sqrt{3}$

c) $A_{shaded} = \frac{1}{2} \cdot A_{circleS} - A_{segment} = \frac{9}{2}\pi - \left(6\pi - 9\sqrt{3}\right) = 9\sqrt{3} - \frac{3}{2}\pi$

13. $\triangle LSB$ is an isosceles right triangle with $LB = 3\sqrt{2}$ and $SB = SL = 3$

m$\angle SLB = 45°$ m$\angle ALB = 2$m$\angle SLB = 90°$

$A_{sector} = \frac{90}{360} \cdot 18\pi = \frac{9}{2}\pi$

$A_{\triangle ALB} = \frac{1}{2}(6)(3) = 9$

$A_{segment} = A_{sector} - A_{\triangle ALB} = \frac{9}{2}\pi - 9$

$A_{shaded} = \frac{1}{2} \cdot A_{circleS} - A_{segment} = \frac{9}{2}\pi - \left(\frac{9}{2}\pi - 9\right) = 9$

$A_{shaded} = A_{\triangle ALB} = 9$

CHAPTER 16. SOLIDS

16.1 Volume of a Sphere

1. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 = 972\pi\ \text{m}^3$	2. $r = \frac{1}{2} \cdot \frac{29.5}{\pi} = \frac{14.75}{\pi}$ $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{14.75}{\pi}\right)^3 \approx 433.5$ cu in.
3. $V_{before} = \frac{4}{3}\pi(3)^3 = 36\pi$ $\quad V_{after} = \frac{4}{3}\pi(6)^3 = 288\pi$ $\quad 288\pi - 36\pi = 252\pi\ \text{in}^3$	

16.2 Volume of a Prism or Cylinder

1. $V = Bh = 6 \cdot 8 \cdot 4 = 192\ \text{ft}^3$	2. $V = e^3 = (1.5)^3 = 3.375\ \text{cm}^3$
3. $V = \pi r^2 h = \pi(4)^2(10) \approx 502.65\ \text{in}^3$	4. $V = \pi r^2 h = \pi(6)^2(15) \approx 1696.5\ \text{in}^3$
5. $V = \pi r^2 h$ $32\pi = \pi r^2(2)$ $r^2 = 16$ $\quad r = \sqrt{16} = 4$ in.	6. $(x-2)(x+1)(2x) =$ $(x^2 - x - 2)(2x) =$ $2x^3 - 2x^2 - 4x$
7. $V = 5^3 = 125$ $\quad V = 10^3 = 1000$ $1000 \div 125 = 8$	8. $V = \pi r^2 h$ $342 = 9\pi h$ $\quad h = \frac{38}{\pi}$ $36 \div \frac{38}{\pi} = \frac{36\pi}{38} \approx 2.97$ $\quad$ 2 cans
9. $V_{larger} = Bh = 12 \cdot 30 \cdot 16 = 5760$ $\quad V_{smaller} = Bh = 6 \cdot 12 \cdot 9 = 648$ $5760 - 648 = 5{,}112\ \text{in}^3$	
10. $V_{container} = (20)(15)(10) = 3{,}000\ \text{in}^3$ $\quad V_{cup} = \pi(2)^2(5) = 20\pi\ \text{in}^3$ $3{,}000 \div 20\pi \approx 47.7$ $\quad$ 47 cups	
11. $V_{prism} = Bh$ $\quad V_{cylinder} = \pi r^2 h$ $V = (5)(3.5)(7) = 122.5$ $\quad V = \pi(2.5)^2(7) \approx 137.4$ $\quad$ Cylinder by 14.9 in^3	

16.3 Volume of a Pyramid or Cone

1. $V = \frac{1}{3}Bh = \frac{1}{3}(10)(8)(6) = 160\ \text{in}^3$	2. $V = 3 \times 8 = 24\ \text{cm}^3$
3. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(8) = 24\pi\ \text{in}^3$	4. $V = \frac{1}{3}Bh$ $256 = \frac{1}{3}B(12)$ $B = 64$ $\quad s = \sqrt{64} = 8$
5. $V = 5^3 + \frac{1}{3}(25)(6) = 175\ \text{cm}^3$	

6. a) $V_{cylinder} = \pi r^2 h = \pi \cdot 5^2 \cdot 17.8 \approx 1398.0087$ cu. units

$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 5^2 \cdot 6.2 \approx 162.3156$ cu. units

$V_{container} = 1398.0087 + 162.3156 \approx 1560.324$ cu. units

b) Half the volume of the container $= \frac{1}{2} \cdot 1560.324 \approx 780.162$ cu. units

The cone is completely filled, so subtract $780.162 - 162.316 = 617.846$.

The remaining water fills $\frac{617.846}{1398.009} \approx 0.442$ of the cylinder.

The water reaches .0442 of the cylinder's height, $0.442 \times 17.8 \approx 7.9$ units.

From the apex of the cone, the water reaches $6.2 + 7.9 \approx 14.1$ units.

16.4 Density

1. $W = VD = 100\ cm^3 \cdot \frac{11.34\ g}{1\ cm^3} = 1{,}134\ g$

2. $D = \frac{W}{V} = \frac{88.6\ g}{10\ cm^3} = 8.86\ g/cm^3$

3. $D = \frac{W}{V} = \frac{94.44\ g}{12\ cm^3} = 7.87\ g/cm^3$

4. $V = \frac{W}{D} = \frac{9.8}{0.7} = 14\ mL$

5. $V = \frac{1}{3}Bh = \frac{1}{3}(24)(10) = 80\ in^3$

$W = 80\ \cancel{in^3} \cdot \frac{2\ \cancel{g}}{1\ \cancel{cm^3}} \cdot \frac{(2.54)^3\ \cancel{cm^3}}{1\ \cancel{in^3}} \cdot \frac{1\ \cancel{kg}}{1000\ \cancel{g}} \cdot \frac{2.2\ lbs}{1\ \cancel{kg}} \approx 5.8\ lbs$

16.5 Lateral Area and Surface Area

1. $SA = 2lw + 2hl + 2hw$
$= 2 \cdot 8 \cdot 6 + 2 \cdot 4 \cdot 8 + 2 \cdot 4 \cdot 6$
$= 96 + 64 + 48 = 208$ ft^2

2. $SA = 2lw + 2hl + 2hw$
$= 2(3)(1.5) + 2(2)(3) + 2(2)(1.5)$
$= 9 + 12 + 6 = 27$ ft^2

3. $SA = 2lw + 2hl + 2hw$
$= 2(3.0)(2.2) + 2(7.5)(3.0) + 2(7.5)(2.2)$
$= 13.2 + 45 + 33 = 91.2$ cm^2

4. $SA = 2lw + 2hl + 2hw$
$= 2(5.5)(3) + 2(5.5)(6.75) + 2(3)(6.75)$
$= 33 + 74.25 + 40.5 = 147.75$ cm^2

5. $V = (10)(2)(4) = 80$ cm^3
$SA = 2(10)(2) + 2(10)(4) + 2(2)(4)$
$= 136$ cm^2

6. $s = \sqrt[3]{64} = 4$
$SA = (6)(4)^2 = 96$ in^2

7. $LA = 2\pi rh = 2\pi(6)(9) = 108\pi$ ft^2
$SA = 2\pi r^2 + LA = 72\pi + 108\pi = 180\pi$ ft^2

8. $LA = 2\pi rh = 2\pi(5)(11) = 110\pi$ ft^2
$SA = 2\pi r^2 + LA = 50\pi + 110\pi = 160\pi$
≈ 502.7 ft^2

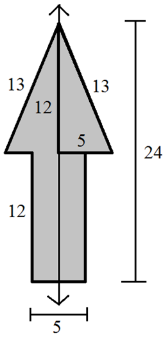
9.
$$\begin{array}{ll} 2(x+3)(x-4) & \quad 2x^2 - 2x - 24 \\ +2(x+3)(5) & \qquad\quad 10x + 30 \\ +2(x-4)(5) & \underline{+ \quad 10x - 40} \\ & \quad 2x^2 + 18x - 34 \end{array}$$

10. $V = \pi r^2 h$, so $h = \frac{V}{\pi r^2}$

Radius r	Height h	Surface Area SA
2	$h = \frac{V}{\pi r^2} = \frac{144\pi}{4\pi} = 36$	$SA = 2\pi r^2 + 2\pi rh = 8\pi + 144\pi = 152\pi$
4	$h = \frac{V}{\pi r^2} = \frac{144\pi}{16\pi} = 9$	$SA = 2\pi r^2 + 2\pi rh = 32\pi + 72\pi = 104\pi$
6	$h = \frac{V}{\pi r^2} = \frac{144\pi}{36\pi} = 4$	$SA = 2\pi r^2 + 2\pi rh = 72\pi + 48\pi = 120\pi$

The cylinder with the radius of 4 has the least surface area.

16.6 Rotations of Two-Dimensional Objects

1. sphere	2. cone
3. cylinder diameter = 10 in and height = 5 in $V = \pi r^2 h = \pi(5)^2(5) = 125\pi$ in^3	4. cylinder with a hemisphere on top $V_{cylinder} + V_{hemisphere} =$ $\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = 27\pi + 18\pi = 45\pi$
5. 12-inch radius sphere with a 6-inch radius spherical cutout in its center $V_{outer\ sphere} - V_{cutout} =$ $\frac{4}{3}\pi(12)^3 - \frac{4}{3}\pi(6)^3 =$ $2{,}304\pi - 288\pi = 2{,}016\pi$	6. cylinder with a cone on top Cylinder has radius of 2.5 and height of 12. $V_{cylinder} = \pi(2.5)^2(12) = 75\pi$  Cone has a height of $24 - 12 = 12$. Use the Pythagorean Thm to find the radius of the cone: $r^2 + 12^2 = 13^2$, so $r = 5$ $V_{cone} = \frac{1}{3}\pi(5)^2(12)$ $= 100\pi$ $V_{object} = V_{cone} + V_{cylinder} =$ 175π.

16.7 Cross Sections

1. (4)	2. (2)
3. (1)	4. (4)
5. (1)	6. (a) pentagon (b) rectangle

CHAPTER 17. CONSTRUCTIONS

17.1 Copy Segments, Angles, and Triangles

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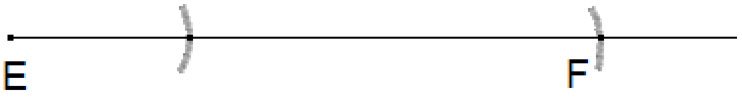 </td></tr>
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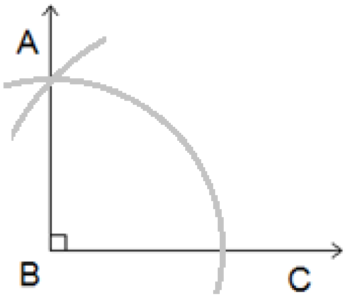
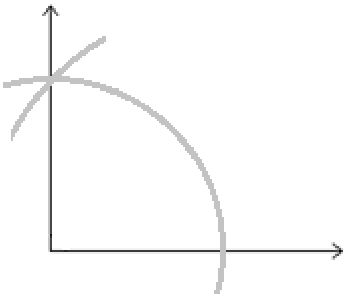</td></tr>
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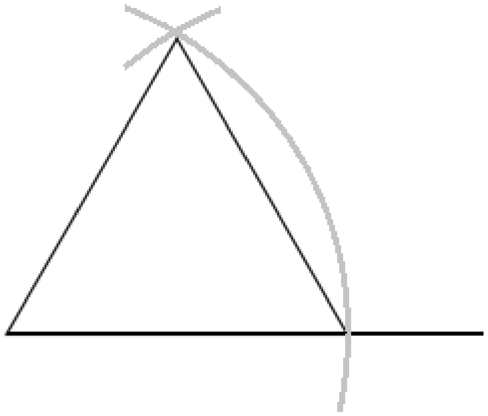</td></tr>
<tr><td>4. SAS</td><td>5. AAS</td></tr>
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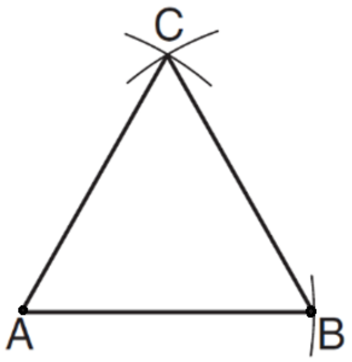
17.2 Construct an Equilateral Triangle

1.



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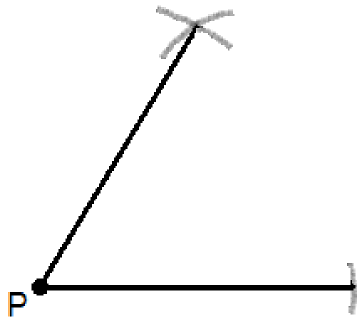

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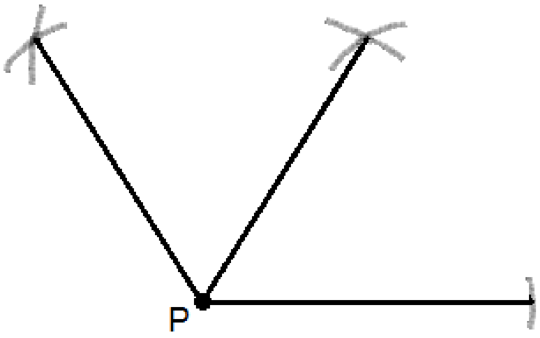
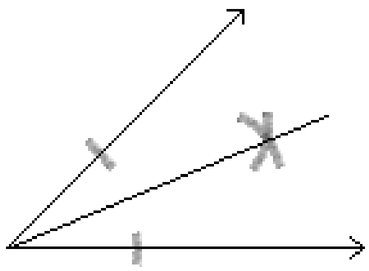


17.3 Construct an Angle Bisector

1.



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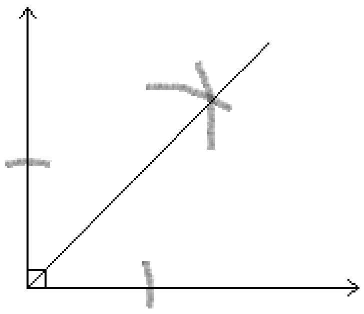

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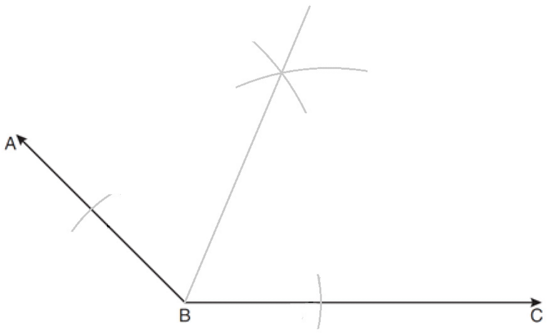

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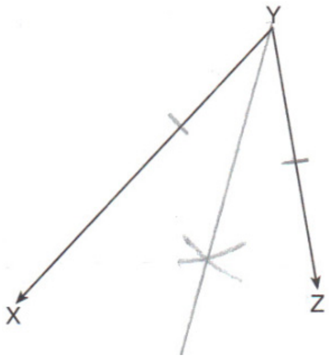

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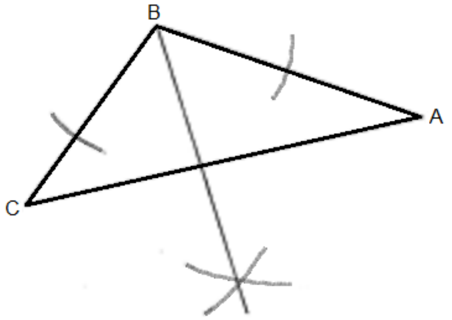

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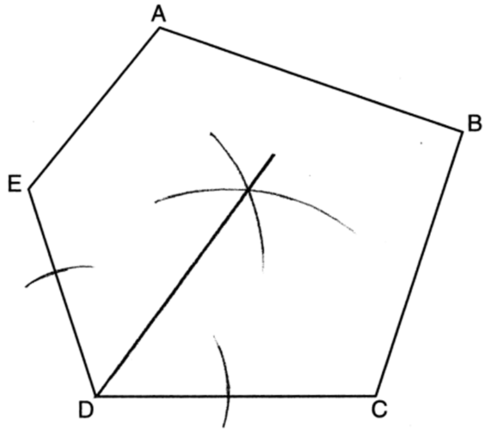

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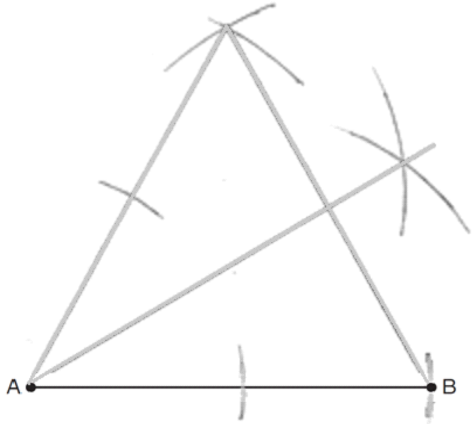
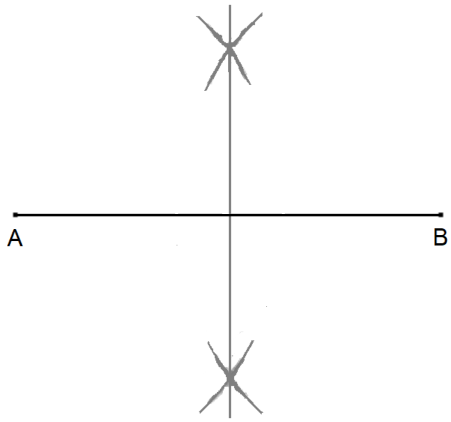


17.4 Construct a Perpendicular Bisector

1.



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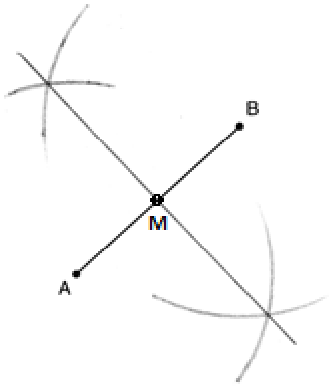

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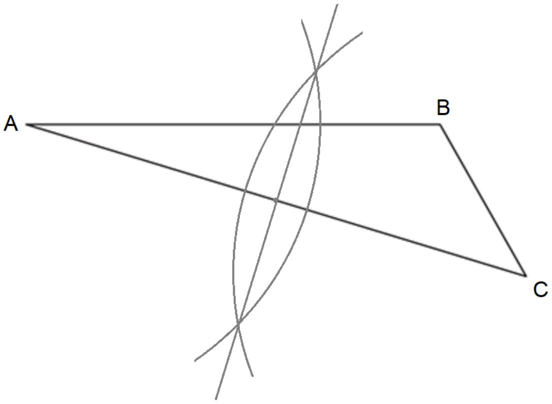

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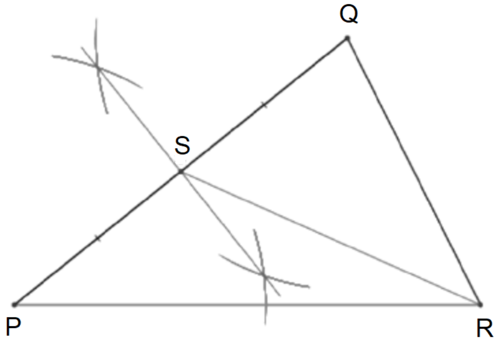
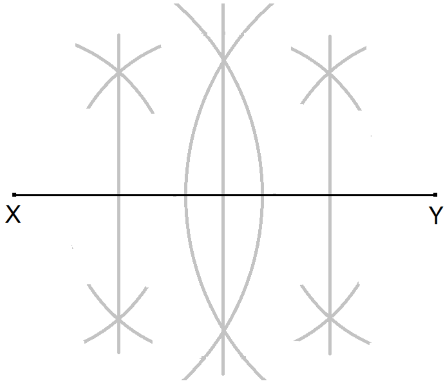


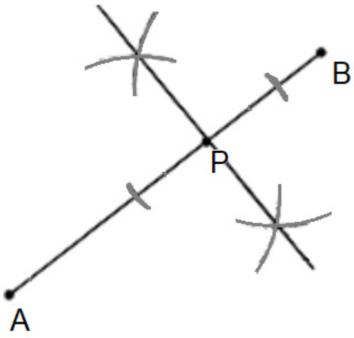
5. Construct the perpendicular bisector of $\overline{XY}$. Then, construct the bisector of each half.



17.5 Construct Lines Through a Point

1. (2)

2.



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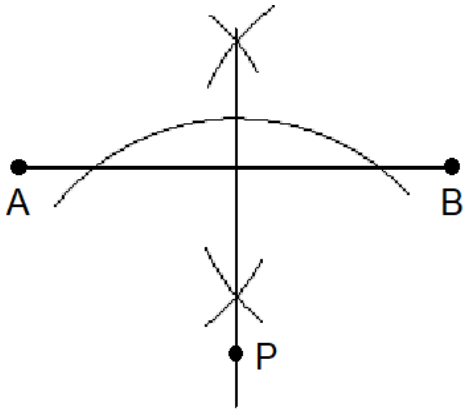

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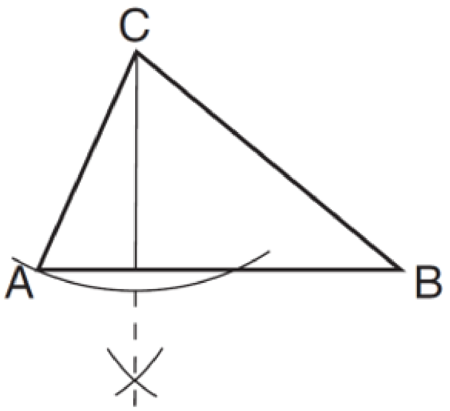

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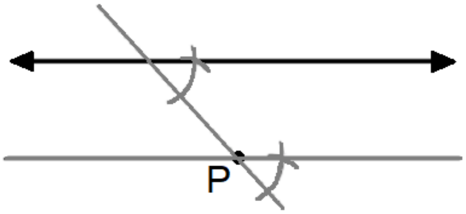
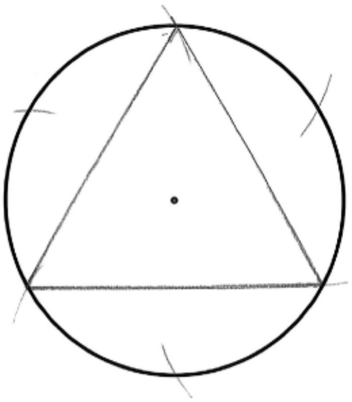


6.
a) 5 b) 2 c) 1 d) 4 e) 3 f) 6

17.6 Construct Inscribed Regular Polygons

1.



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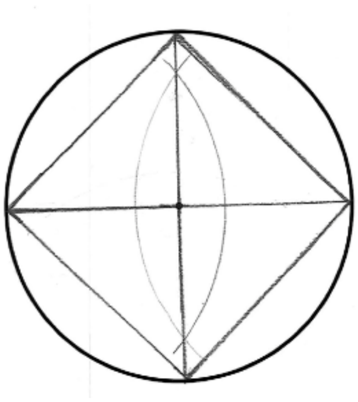
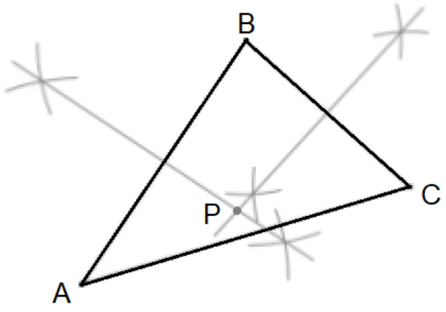


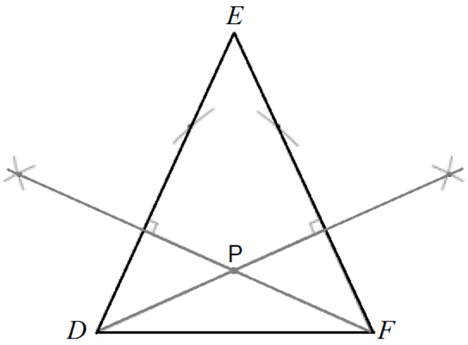
3.
a) Construct two perpendicular diameters (as if constructing an inscribed square).
b) Construct angle bisectors of the 90° central angles, forming eight 45° central angles.
c) The endpoints of the four diameters are the vertices of the regular octagon. Draw the chords between them.

17.7 Construct Points of Concurrency

1.



2.



17.8 Construct Circles of Triangles

1. (1)

2. (4)

3. a) 2 b) 1 c) 5 d) 4 e) 3

REGENTS QUESTIONS

CHAPTER 1. BASIC GEOMETRY

1.1 Lines, Angles and Shapes

1. CC JAN '16 [6] Ans: 1

1.2 Pythagorean Theorem

1. CC JAN '16 [32]
   $\frac{16}{9} = \frac{x}{20.6}$; $x \approx 36.6$
   $36.6^2 + 20.6^2 = c^2$
   $1763.92 = c^2$
   $42 \approx c$

1.3 Perimeter and Circumference

There are no Regents exam questions on this topic.

1.4 Area

1. CC JAN '17 [8] Ans: 1
2. CC AUG '17 [20] Ans: 1
3. CC JAN '19 [18] Ans: 1
4. CC JUN '19 [2] Ans: 3
5. CC AUG '19 [17] Ans: 4
6. CC JAN '24 [4] Ans: 3
7. CC JAN '16 [30]
   Dish A
   $d_A = \frac{40{,}000}{\pi(25.5)^2} \approx 19.6$
   $d_B = \frac{72{,}000}{\pi(37.5)^2} \approx 16.3$
8. CC AUG '17 [34]
   $x^2 + x^2 = 58^2$
   $2x^2 = 3364$
   $x = \sqrt{1682}$
   $A = \left(\sqrt{1682} + 8\right)^2 \approx 2402.2$
9. CC JAN '19 [31]
   Area of outer circle − Area of inner circle = $\pi 30^2 - \pi 20^2 = 500\pi$
   Area of each □ = $90 \times 10 = 900$
   Total area = $500\pi + 2(900)$
   ≈ 3,371 sq. ft
10. CC AUG '23 [28]
    $A = 5\pi(2)^2 + 5(6)(4) \approx 182.83$
    $\frac{182.83}{25} \approx 7.3$; 8 cans

CHAPTER 2. COORDINATE GEOMETRY

2.1 Forms of Linear Equations

There are no Regents exam questions on this topic.

2.2 Parallel and Perpendicular Lines

1. CC JUN ‘15 [9] Ans: 1
2. CC AUG ‘15 [10] Ans: 1
3. CC JAN ‘16 [2] Ans: 4
4. CC JAN ‘17 [1] Ans: 3
5. CC JUN ‘17 [19] Ans: 2
6. CC JAN ‘18 [20] Ans: 1
7. CC JUN ‘18 [12] Ans: 2
8. CC AUG ‘18 [11] Ans: 1
9. CC JUN ‘19 [16] Ans: 2
10. CC AUG ‘19 [8] Ans: 1
11. CC JAN ‘23 [13] Ans: 4
12. CC AUG ‘23 [8] Ans: 2
13. CC JAN ’24 [5] Ans: 1
14. CC JAN ‘19 [25]
    $y = \frac{2}{3}x - \frac{7}{3}, m = \frac{2}{3}$
    $y - 6 = \frac{2}{3}(x - 2)$
15. CC JAN ‘20 [31]
    $y = \frac{5}{4}x - \frac{5}{2}, m = \frac{5}{4}$
    $y - 12 = -\frac{4}{5}(x - 5)$

2.3 Distance Formula

1. CC JUN ‘15 [3] Ans: 3
2. CC JAN ‘16 [15] Ans: 2

2.4 Midpoint Formula

There are no Regents exam questions on this topic.

2.5 Perpendicular Bisectors

1. CC JUN ‘16 [12] Ans: 1
2. CC AUG ‘17 [24] Ans: 4
3. CC JUN ‘22 [20] Ans: 4
4. CC JUN ‘23 [24] Ans: 4

2.6 Directed Line Segments

1. CC FALL ‘14 [14] Ans: 4
2. CC AUG ‘16 [18] Ans: 4
3. CC JAN ‘17 [20] Ans: 1
4. CC JUN ‘17 [15] Ans: 2
5. CC AUG ‘17 [17] Ans: 1
6. CC JAN ‘18 [6] Ans: 1
7. CC JUN ‘18 [14] Ans: 2
8. CC AUG ‘18 [15] Ans: 1
9. CC JAN ‘19 [15] Ans: 1
10. CC JUN ‘19 [19] Ans: 4
11. CC AUG ‘19 [3] Ans: 3
12. CC JAN ‘20 [5] Ans: 4
13. CC JUN ‘22 [22] Ans: 2
14. CC AUG ‘22 [13] Ans: 1
15. CC AUG ‘23 [2] Ans: 4
16. CC JAN ’24 [10] Ans: 4

17. CC JUN ‘15 [27]

$P_x = -6 + \frac{2}{5}(4 + 6) = -2;$

$P_y = -5 + \frac{2}{5}(0 + 5) = -3$

$P(-2, -3)$

18. CC AUG ‘15 [31]

$E_x = 1 + \frac{2}{5}(16 - 1) = 7;$

$E_y = 4 + \frac{2}{5}(14 - 4) = 8$

$E(7,8)$

19. CC JAN ‘16 [27]

$J_x = -2 + \frac{2}{3}(4 + 2) = 2;$

$J_y = 1 + \frac{2}{3}(7 - 1) = 5$

$J(2,5)$

20. CC JUN ‘16 [26]

$P_x = 4 + \frac{4}{9}(22 - 4) = 12;$

$P_y = 2 + \frac{4}{9}(2 - 2) = 2$

$P(12,2)$

21. CC JAN ‘23 [28]

$P_x = -2 + \frac{3}{5}(8 + 2) = 4;$

$P_y = 5 + \frac{3}{5}(-1 - 5) = \frac{7}{5}$

$P\left(4, \frac{7}{5}\right)$

22. CC JUN ‘23 [27]

$R_x = -5 + \frac{2}{5}(5 + 5) = -1;$

$R_y = 1 + \frac{2}{5}(6 - 1) = 3$

$R(-1,3)$

CHAPTER 3. POLYGONS IN THE COORDINATE PLANE

3.1 Triangles in the Coordinate Plane

1. CC JAN ‘16 [18] Ans: 1
2. CC JUN ‘16 [14] Ans: 4
3. CC AUG ‘16 [15] Ans: 3
4. CC AUG ‘15 [33]

   $m_{\overline{BC}} = -\frac{3}{2}$ $m_{\perp} = \frac{2}{3}$

   The right ∠ may be at B or C.

   Right ∠ at B means:

   $-1 = \frac{2}{3}(-3) + b$ $3 = \frac{2}{3}x + 1$

   $b = 1$ $x = 3$

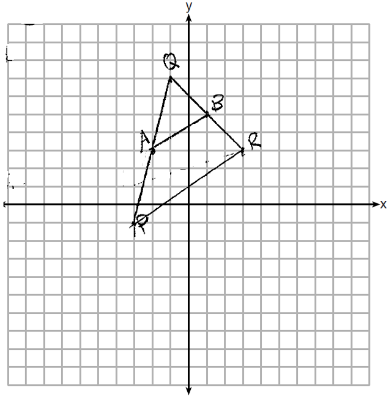
   Right ∠ at C means:

   $-4 = \frac{2}{3}(-1) + b$ $3 = \frac{2}{3}x - \frac{10}{3}$

   $b = -\frac{10}{3}$ $x = \frac{19}{2} = 9.5$

   Either answer, $x = 3$ or $x = 9.5$, is correct.

5. CC AUG ‘17 [32]



   $A(-2,3)$ and $B(1,5)$

   $m_{\overline{AB}} = \frac{5-3}{1+2} = \frac{2}{3}$

   $m_{\overline{PR}} = \frac{3+1}{3+3} = \frac{4}{6} = \frac{2}{3}$

   $\overline{AB} \parallel \overline{PR}$ because same slopes.

   $AB = \sqrt{3^2 + 2^2} = \sqrt{13}$

   $PR = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$

   Therefore, $AB = \frac{1}{2}PR$.

6. CC JUN ‘18 [32]

   $AB = \sqrt{5^2 + 1^2} = \sqrt{26}$

   $BC = \sqrt{4^2 + 4^2} = \sqrt{32}$

   $AC = \sqrt{1^2 + 5^2} = \sqrt{26}$

   $\triangle ABC$ has two ≅ sides, not three, so it is isosceles and not equilateral.

7. CC JAN ‘19 [30]

   No. (a) If $\overline{EG}$ is a median, then G is the midpoint of $\overline{DF}$.

   But midpoint of $\overline{DF}$ is $\left(\frac{1+4}{2}, \frac{-1+2}{2}\right) =$ (2.5,0.5), or

   (b) If $\overline{EG}$ is a median, then $DG = GF$.

   But, $DG = \sqrt{2^2 + 2^2} = \sqrt{8}$ and $GF = \sqrt{1^2 + 1^2} = \sqrt{2}$.

8. CC JAN ‘19 [32]

   $AC = \sqrt{3^2 + (-5)^2} = \sqrt{34}$

   $BC = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34}$

   Isosceles because $AC = BC$

   $m_{\overline{AC}} = \frac{-1-4}{1-(-2)} = -\frac{5}{3}$ $m_{\overline{BC}} = \frac{-1-2}{1-6} = \frac{3}{5}$

   $\overline{AC} \perp \overline{BC}$ because their slopes are opp reciprocals, so $\angle C$ is a right ∠.

9. CC JUN ‘23 [31]

   $m = \frac{-2+4}{-3-4} = -\frac{2}{7}$; $y - 2 = -\frac{2}{7}(x - 3)$

3.2 Quadrilaterals in the Coordinate Plane

1. CC AUG ‘15 [22] Ans: 4
2. CC AUG ‘16 [14] Ans: 1
3. CC JAN ‘17 [19] Ans: 3
4. CC AUG ‘19 [2] Ans: 3
5. CC JUN ‘23 [15] Ans: 1
6. CC AUG ‘23 [21] Ans: 4

7. CC FALL '14 [11]

Midpoint of $\overline{MT}$ is $\left(\frac{0+4}{2},\frac{-1+6}{2}\right)=\left(2,\frac{5}{2}\right)$.

$m_{\overline{MT}}=\frac{6+1}{4-0}=\frac{7}{4}$. In a rhombus, diagonals are $\perp$, so $m_{\overline{AH}}=-\frac{4}{7}$.

$y-\frac{5}{2}=-\frac{4}{7}(x-2)$

The diagonals, $\overline{MT}$ and $\overline{AH}$, of rhombus $MATH$ are $\perp$ bisectors of each other.

8. CC JUN '15 [36]

$m_{\overline{TS}}=-\frac{5}{3}$ $\quad$ $m_{\overline{SR}}=\frac{3}{5}$

Since the slopes of $\overline{TS}$ and $\overline{SR}$ are opp reciprocals, they are $\perp$ and form right $\angle$. $\triangle RST$ is a right $\triangle$ ($\angle S$ is a right $\angle$)

$P(0,9)$

$m_{\overline{RP}}=-\frac{5}{3}$ $\quad$ $m_{\overline{PT}}=\frac{3}{5}$

Since the slopes of all four adjacent sides are opp reciprocals, they are $\perp$ and form right $\angle$'s. Quad $RSTP$ is a □ because it has four right $\angle$'s.

9. CC JAN '17 [31]

$(1,3)$ and $(-3,-3)$

10. CC JUN '17 [35]

$PQ=\sqrt{5^2+5^2}=\sqrt{50}$

$QR=\sqrt{(-7)^2+1^2}=\sqrt{50}$

$RS=\sqrt{(-5)^2+(-5)^2}=\sqrt{50}$

$PS=\sqrt{(-7)^2+1^2}=\sqrt{50}$

$PQRS$ is a rhombus b/c all sides are $\cong$.

The slope of $\overline{PQ}$ is 1 and the slope of $\overline{QR}$ is -7. Because the slopes of adjacent sides are not opp reciprocals, they are not $\perp$ and do not form a right $\angle$. Therefore, $PQRS$ is not a square.

11. CC JAN '18 [35]

$PA=\sqrt{(-4+1)^2+(5+6)^2}=\sqrt{130}$

$AT=\sqrt{(5+4)^2+(-2-5)^2}=\sqrt{130}$

$PA=AT$, so $\triangle PAT$ is isosceles.

$R(2,9)$

$m_{\overline{PA}}=-\frac{11}{3}$ and $m_{\overline{RT}}=-\frac{11}{3}$, so $\overline{PA}\parallel\overline{RT}$.

$RT=\sqrt{(5-2)^2+(-2-9)^2}=\sqrt{130}$, so $\overline{PA}\cong\overline{RT}$.

$PART$ is a ▱ because it has a pair of opp sides that are both $\cong$ and $\parallel$.

12. CC AUG '18 [35]

$m_{\overline{MH}}=\frac{6}{10}=\frac{3}{5}$ and $m_{\overline{AT}}=\frac{6}{10}=\frac{3}{5}$

$m_{\overline{MA}}=-\frac{5}{3}$ and $m_{\overline{HT}}=-\frac{5}{3}$

$\overline{MH}\parallel\overline{AT}$ and $\overline{MA}\parallel\overline{HT}$, so $MATH$ is a ▱ since both sides of opp sides are $\parallel$.

Since their slopes are negative reciprocals, $\overline{MA}\perp\overline{AT}$ and $\angle A$ is a right $\angle$. $MATH$ is a □ because it is a ▱ with a right $\angle$.

13. CC JUN '19 [32]

$m_{\overline{AD}}=\frac{0-6}{1-(-1)}=-3$

$m_{\overline{BC}}=\frac{-1-8}{6-3}=-3$

$\overline{AD}\parallel\overline{BC}$ because their slopes are equal, so ABCD is a trapezoid.

$AC=\sqrt{(-1-6)^2+(6+1)^2}=\sqrt{98}$

$BD=\sqrt{(8-0)^2+(3-1)^2}=\sqrt{68}$.

$ABCD$ is not an isosceles trapezoid because its diagonals are not $\cong$.

14. CC AUG '19 [35]

$AB=\sqrt{(-5-1)^2+(3-2)^2}=\sqrt{37}$

$BC=\sqrt{(-5+6)^2+(3+3)^2}=\sqrt{37}$

$\triangle ABC$ is isosceles because $AB=BC$.

$D(0,-4)$

$AD=\sqrt{(1-0)^2+(2+4)^2}=\sqrt{37}$

$CD=\sqrt{(-6-0)^2+(3+4)^2}=\sqrt{37}$

$m_{\overline{AB}}=\frac{3-2}{-5-1}=-\frac{1}{6}$ $\quad$ $m_{\overline{BC}}=\frac{3+3}{-5+6}=6$;

$\overline{AB}\perp\overline{BC}$ (slopes are opp reciprocals), so $\angle B$ is a right $\angle$;

$ABCD$ is a square because all four sides are $\cong$ and it has a right $\angle$.

15. CC JAN '20 [32]

$NA=\sqrt{(1+4)^2+(2+3)^2}=\sqrt{50}$

$AT=\sqrt{(8-1)^2+(1-2)^2}=\sqrt{50}$

$TS=\sqrt{(3-8)^2+(-4-1)^2}=\sqrt{50}$

$SN=\sqrt{(-4-3)^2+(-3+4)^2}=\sqrt{50}$

All four sides are $\cong$, so $NATS$ is a rhombus.

16. CC AUG ‘22 [33]

$m_{\overline{HY}} = \frac{9-6}{2+3} = \frac{3}{5}$; $m_{\overline{PE}} = \frac{-4+1}{3-8} = \frac{3}{5}$;

$m_{\overline{HE}} = \frac{6+4}{-3-3} = -\frac{5}{3}$; $m_{\overline{YP}} = \frac{9+1}{2-8} = -\frac{5}{3}$;

$HYPE$ is a ▱ since both pairs of opp sides are ∥ (slopes are equal);

$\overline{HY} \perp \overline{HE}$ (slopes are opp reciprocals)

$HYPE$ is a ▭ (▱ with a right ∠)

17. CC AUG ‘23 [34]

$m_{\overline{AD}} = \frac{-1-4}{5-0} = -1$; $m_{\overline{BC}} = \frac{3-8}{8-3} = -1$;

$m_{\overline{AB}} = \frac{8-4}{3-0} = \frac{4}{3}$; $m_{\overline{CD}} = \frac{-1-3}{5-8} = \frac{4}{3}$;

$ABCD$ is a ▱ since both pairs of opp sides are ∥ (slopes are equal);

The slopes of $\overline{AB}$ and $\overline{BC}$ are not opp reciprocals, so $\overline{AB}$ is not perp to $\overline{BC}$ and $\angle B$ is not a right angle. Therefore, $ABCD$ is not a rectangle.

18. CC JAN ’24 [35]

$m_{\overline{MA}} = \frac{5-7}{3+1} = -\frac{1}{2}$; $m_{\overline{TH}} = \frac{-3+7}{-6-2} = -\frac{1}{2}$;

$\overline{MA} \parallel \overline{TH}$ because their slopes are equal; MATH is a trapezoid because it has a pair of parallel sides;

$Y(7,3)$;

$m_{\overline{MY}} = \frac{3-7}{7+1} = -\frac{1}{2}$; $m_{\overline{TH}} = -\frac{1}{2}$;

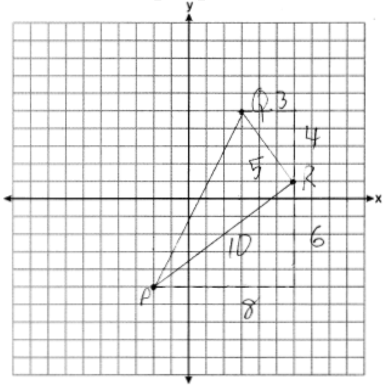
$m_{\overline{HM}} = \frac{7+3}{-1+6} = 2$; $m_{\overline{TY}} = \frac{3+7}{7-2} = 2$;

Slopes of all pairs of adjacent sides are opposite reciprocals, so they are ⊥.

⊥ side form right ∠’s, so MYTH has four right ∠’s and is a rectangle.

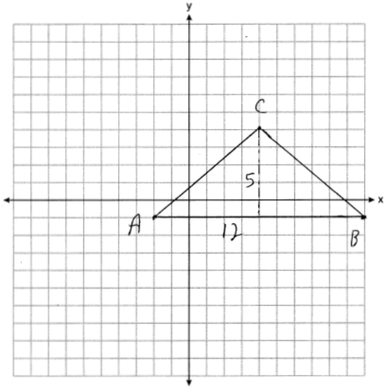
3.3 Perimeter and Area using Coordinates

1. CC JUN ‘16 [22] Ans: 3
2. CC JUN ‘17 [2] Ans: 3
3. CC AUG ‘17 [3] Ans: 3
4. CC JUN ‘18 [15] Ans: 1
5. CC AUG ‘18 [8] Ans: 4
6. CC JAN ‘19 [21] Ans: 4
7. CC JAN ‘20 [18] Ans: 2
8. CC AUG ‘22 [14] Ans: 4
9. CC JAN ’24 [7] Ans: 2
10. CC JUN ‘19 [26]



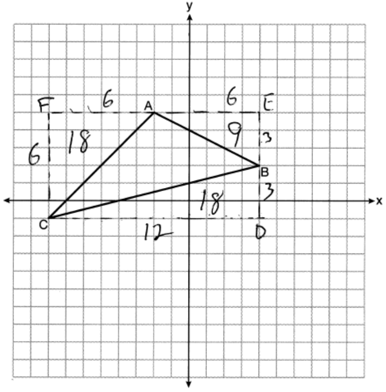
$A = \frac{1}{2}(5)(10) = 25$

11. CC AUG ‘19 [28]



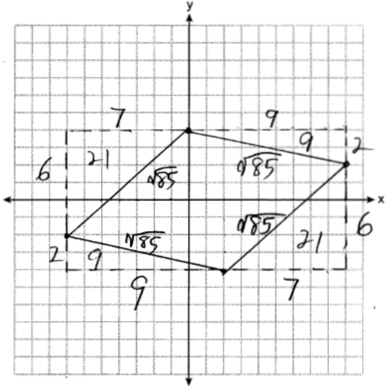
$A = \frac{1}{2}(5)(12) = 30$

12. CC JAN ‘23 [31]



$A = (6)(12) - \frac{1}{2}(6)(6) - \frac{1}{2}(6)(3) - \frac{1}{2}(12)(3) = 72 - 18 - 9 - 18 = 27$

13. CC JUN ‘23 [34]



All four sides measure $\sqrt{85}$, so $MATH$ is a rhombus.
$A = (16)(8) - 21 - 9 - 9 - 21 = 68$.

CHAPTER 4. RIGID MOTIONS

4.1 Translations

1. CC JUN '22 [35]

   $AB = \sqrt{(-2+7)^2 + (4+1)^2} = \sqrt{50} = 5\sqrt{2}$

   $AC = \sqrt{(-2+3)^2 + (4+3)^2} = \sqrt{50} = 5\sqrt{2}$

   $AB = AC$, so $\triangle ABC$ is isosceles

   $A(-2,4) \rightarrow A'(3,-1)$

   $B(-7,-1) \rightarrow B'(-2,-6)$

   $C(-3,-3) \rightarrow C'(2,-8)$

   $A'C' = AC = 5\sqrt{2}$ because translations preserve distance

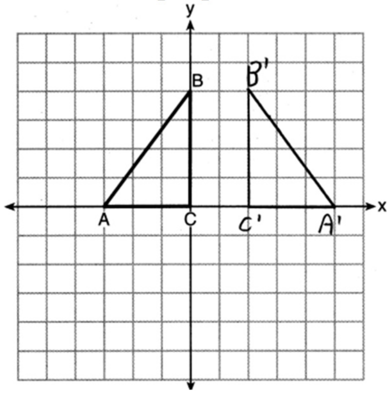
   $AA' = \sqrt{(3+2)^2 + (-1-4)^2} = \sqrt{50} = 5\sqrt{2}$

   $CC' = \sqrt{(2+3)^2 + (-8+3)^2} = \sqrt{50} = 5\sqrt{2}$

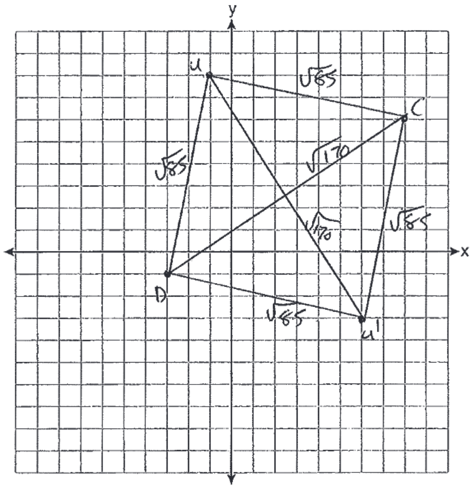
   $AA'C'C$ is a quad with 4 ≅ sides, so it is a rhombus

4.2 Line Reflections

1. CC AUG '22 [1] Ans: 2
2. CC AUG '23 [17] Ans: 3
3. CC JAN '16 [25]



4. CC JAN '23 [35]



$m_{\overline{DU}} = \frac{9}{2}$ and $m_{\overline{UC}} = -\frac{2}{9}$;

$\overline{DU} \perp \overline{UC}$ (opposite reciprocal slopes);

$\triangle DUC$ is a right triangle ($\angle DUC$ is a right $\angle$);

Each side of $DUCU'$ is $\sqrt{9^2 + 2^2} = \sqrt{85}$;

Quad $DUCU'$ is a square (4 ≅ sides and a right $\angle$);

4.3 Rotations

1. CC FALL '14 [2] Ans: 4
2. CC JAN '16 [11] Ans: 4
3. CC AUG '16 [5] Ans: 1
4. CC JUN '22 [5] Ans: 3

5. CC AUG ‘22 [9] Ans: 1
6. CC JUN ‘23 [2] Ans: 3
7. CC AUG ‘16 [29]
   $\text{m}\angle P = \text{m}\angle L = 47^\circ$ because rotations preserve $\angle$’s.
   $\text{m}\angle M = 180 - (47 + 57) = 76$ because the $\angle$’s of a $\triangle$ add to 180°.

4.4 Point Reflections

There are no Regents exam questions on this topic.

4.5 Carry a Polygon onto Itself

1. CC FALL ‘14 [15] Ans: 2
2. CC JUN ‘15 [10] Ans: 1
3. CC AUG ‘15 [5] Ans: 1
4. CC JAN ‘17 [17] Ans: 4
5. CC JUN ‘17 [7] Ans: 1
6. CC AUG ‘17 [6] Ans: 3
7. CC AUG ‘17 [22] Ans: 4
8. CC JAN ‘18 [15] Ans: 3
9. CC JUN ‘18 [19] Ans: 3
10. CC AUG ‘18 [17] Ans: 3
11. CC JAN ‘19 [4] Ans: 3
12. CC JUN ‘19 [4] Ans: 4
13. CC AUG ‘19 [23] Ans: 4
14. CC JAN ‘20 [11] Ans: 3
15. CC JUN ‘22 [4] Ans: 1
16. CC AUG ‘22 [5] Ans: 4
17. CC JAN ‘23 [11] Ans: 1
18. CC JUN ‘23 [20] Ans: 3
19. CC AUG ‘23 [6] Ans: 4
20. CC JAN ’24 [3] Ans: 1
21. CC AUG ‘16 [27]
    $\frac{360}{6} = 60$

CHAPTER 5. DILATIONS

5.1 Dilations of Line Segments

1.	CC JUN ‘15 [18]	Ans: 1
2.	CC JAN ‘16 [10]	Ans: 2
3.	CC JUN ‘16 [2]	Ans: 4
4.	CC AUG ‘16 [21]	Ans: 4
5.	CC JAN ‘17 [13]	Ans: 1
6.	CC JUN ‘17 [6]	Ans: 3
7.	CC AUG ‘17 [10]	Ans: 1
8.	CC JAN ‘19 [1]	Ans: 4
9.	CC JUN ‘19 [5]	Ans: 1
10.	CC AUG ‘19 [1]	Ans: 2
11.	CC JAN ‘23 [22]	Ans: 4

12. CC AUG ‘17 [29]

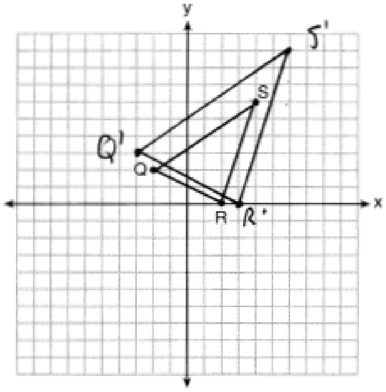
$A'B' = \frac{1}{2}AB =$

$\frac{1}{2}\sqrt{(5-2)^2+(-1-3)^2} = \frac{1}{2}\sqrt{25} = 2.5$

5.2 Dilations of Polygons

1.	CC JUN ‘15 [16]	Ans: 2
2.	CC AUG ‘15 [6]	Ans: 4
3.	CC AUG ‘15 [20]	Ans: 1
4.	CC AUG ‘15 [23]	Ans: 1
5.	CC JAN ‘18 [11]	Ans: 1
6.	CC JUN ‘18 [5]	Ans: 4
7.	CC AUG ‘18 [23]	Ans: 3
8.	CC JUN ‘22 [1]	Ans: 2
9.	CC JUN ‘22 [3]	Ans: 1
10.	CC AUG ‘22 [6]	Ans: 1
11.	CC JUN ‘23 [13]	Ans: 2
12.	CC AUG ‘23 [23]	Ans: 3
13.	CC JAN ’24 [9]	Ans: 2

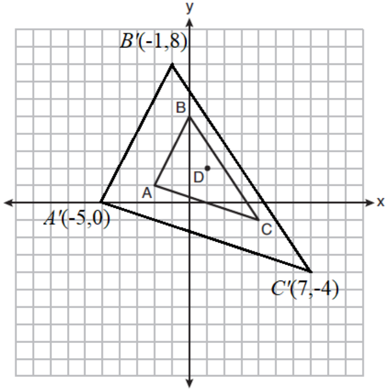
14. CC JAN ‘17 [32]



A dilation preserves slope, so the slopes of $\overline{QR}$ and $\overline{Q'R'}$ are equal.

Therefore, $\overline{Q'R'} \parallel \overline{QR}$.

15. CC JUN ‘18 [26]



5.3 Dilations of Lines

1.	CC FALL ‘14 [3]	Ans: 2
2.	CC FALL ‘14 [16]	Ans: 2
3.	CC JUN ‘15 [22]	Ans: 1
4.	CC AUG ‘15 [24]	Ans: 4
5.	CC JAN ‘18 [14]	Ans: 1
6.	CC JUN ‘18 [24]	Ans: 2

7. CC JAN ‘19 [24] Ans: 4

8. CC JUN ‘19 [7] Ans: 2

9. CC AUG ‘19 [10] Ans: 1

10. CC JAN ‘20 [8] Ans: 1

11. CC JUN ‘22 [23] Ans: 4

12. CC AUG ‘22 [12] Ans: 3

13. CC JAN ‘23 [19] Ans: 4

14. CC JUN ‘23 [19] Ans: 2

15. CC JAN ’24 [16] Ans: 2

16. CC JAN ‘16 [31]
$\ell : y = 3x - 4; m : y = 3x - 8$

17. CC JUN ‘17 [31]
The center of dilation is on the original line, so the line does not change. Line p is $3x + 4y = 20$.

18. CC AUG ‘18 [30]
No, the line passes through the center of dilation, so the dilated line is not distinct. To show the given lines are distinct:
$4x + 3y = 24$
$3y = -4x + 24$
$y = -\frac{4}{3}x + 8$
This has a different y-intercept than
$y = -\frac{4}{3}x + 16$.

19. CC AUG ‘23 [31]
Nathan; A line dilated through a point on the line results in the same line.

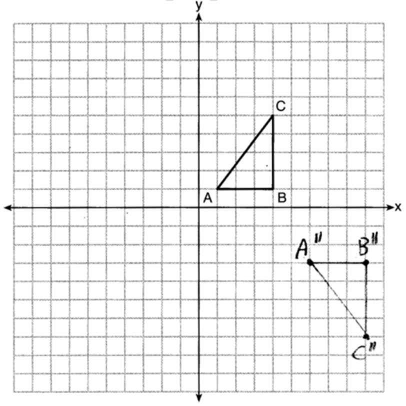
CHAPTER 6. TRANSFORMATION PROOFS

6.1 Properties of Transformations

1. CC AUG ‘15 [13] Ans: 2
2. CC JAN ‘16 [5] Ans: 3
3. CC JUN ‘16 [4] Ans: 1
4. CC AUG ‘16 [2] Ans: 2
5. CC JAN ‘18 [4] Ans: 4
6. CC JAN ‘18 [8] Ans: 4
7. CC JUN ‘18 [1] Ans: 1
8. CC JAN ‘19 [2] Ans: 4
9. CC JUN ‘22 [10] Ans: 3

6.2 Sequences of Transformations

1. CC JUN ‘15 [4] Ans: 4
2. CC AUG ‘15 [7] Ans: 1
3. CC JAN ‘16 [8] Ans: 1
4. CC JUN ‘16 [8] Ans: 4
5. CC JAN ‘17 [10] Ans: 3
6. CC JUN ‘17 [1] Ans: 2
7. CC JUN ‘18 [3] Ans: 4
8. CC AUG ‘18 [4] Ans: 1
9. CC JAN ‘19 [3] Ans: 3
10. CC JUN ‘19 [1] Ans: 4
11. CC AUG ‘19 [9] Ans: 2
12. CC JAN ‘20 [17] Ans: 2
13. CC JAN ‘20 [22] Ans: 1
14. CC JUN ‘22 [18] Ans: 3
15. CC AUG ‘22 [20] Ans: 2
16. CC JUN ‘23 [8] Ans: 1
17. CC AUG ‘23 [22] Ans: 2
18. CC JUN ‘16 [25]
    translation 6 units right and reflection over x-axis
19. CC AUG ‘16 [26]

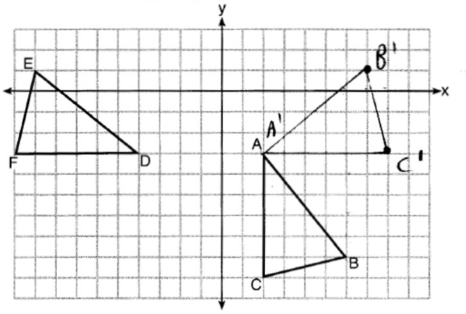


20. CC JAN ‘17 [26]
    $T_{0,-2}$ and r_{y-axis}
21. CC JUN ‘17 [30]
    Rotate $\triangle ABC$ about point C until $\overline{DF} \parallel \overline{AC}$.
    Translate $\triangle ABC$ along $\overline{CF}$ so that C maps onto F.
22. CC AUG ‘17 [27]
    rotation 180° about $\left(-\frac{1}{2},\frac{1}{2}\right)$;
    or rotation 180° about the origin followed by a translation of $-1, 1$.
23. CC JUN ‘18 [27]
    reflection over the y-axis followed by a translation up 5.
24. CC AUG ‘18 [28]
    rotation 180° about $(0,-1)$;
    or rotation 180° about the origin followed by a translation 2 units down;
    or reflection over x-axis, translation 2 units down, reflection over y-axis.
25. CC JAN ‘19 [28]
    reflection over line: $r_{y=x+4}$;
    or rotation $R_{(-5,-1),90°}$ followed by reflection $r_{x=-5}$;
    or r_{x-axis} followed by $T_{3,-1}$ and rotation $R_{Q,90°}$.
26. CC JUN ‘19 [29]
    $R_{(0,0),90°}$; or $T_{2,-6}$ followed by $R_{(-2,-4),90°}$.
27. CC AUG ‘19 [27]
    reflection $r_{y=2}$ and reflection r_{y-axis}
28. CC AUG ‘22 [25]
    r_{y-axis} and $T_{0,5}$

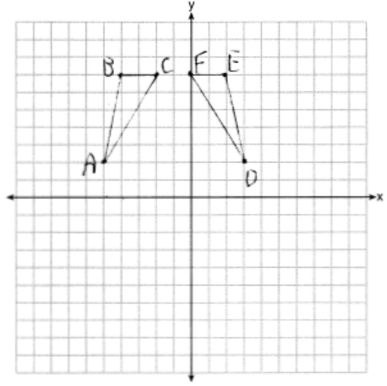
29. CC JAN '23 [26]
rotation 270° (or 90° clockwise) about B and translate 4 down and 3 right.
30. CC JUN '23 [26]
$T_{4,-4}$ followed by a rotation 270° (or 90° clockwise) about D.
31. CC AUG '23 [25]
rotation 180° about $\left(-1,\frac{1}{2}\right)$; or translation 3 up and 4 left followed by a rotation 180° about K'.
32. CC JAN '24 [28]
rotation 90° counterclockwise about the origin; or translation 3 down and 7 left followed by a 90° rotation about D.

6.3 Transformations and Congruence

1. CC JUN '15 [2] Ans: 4
2. CC AUG '15 [2] Ans: 3
3. CC JUN '16 [16] Ans: 3
4. CC JAN '17 [6] Ans: 4
5. CC AUG '17 [2] Ans: 4
6. CC AUG '22 [3] Ans: 3
7. CC JAN '23 [1] Ans: 1
8. CC JAN '24 [8] Ans: 3
9. CC FALL '14 [4]
Translate $\triangle ABC$ such that point C maps onto point F, then reflect over $\overline{DF}$; or reflect $\triangle ABC$ over the ⊥ bisector of $\overline{EB}$.
10. CC JUN '15 [30]
Reflections are rigid motions that preserve congruency.
11. CC AUG '15 [30]
$\triangle XYZ$ is the image of $\triangle ABC$ after a rotation of 180° around the origin. Rotations are rigid motions that preserve congruency.
12. CC AUG '15 [34]
Translations preserve distance, so if point D is mapped onto point A, then point F would map onto point C. $\triangle DEF \cong \triangle ABC$ since $\triangle DEF$ can be mapped onto $\triangle ABC$ by a sequence of rigid motions.
13. CC JAN '16 [28]
Yes. The sequence of transformations consists of a reflection r_{y-axis} and a translation $T_{0,-3}$, which are rigid motions which preserve congruency.
14. CC AUG '16 [33]



Mapping $C(2,-9)$ to $C'(8,-3)$ is a 90° rotation about A. This would map $B(6,-8)$ to $B'(7,1)$. $\triangle DEF \cong \triangle A'B'C'$ because $\triangle DEF$ is a reflection of $\triangle A'B'C'$ over $x=-1$ and reflections preserve congruency.
15. CC JUN '17 [32]



Reflection $r_{x=-1}$. The $\triangle$s are $\cong$ because reflections are rigid motions that preserve distance.
16. CC JAN '18 [30]
$AB=\sqrt{3^2+8^2}=\sqrt{73}$ and $RS=\sqrt{3^2+7^2}=\sqrt{58}$, so $\overline{AB}$ is not $\cong$ to $\overline{RS}$. Therefore, $\triangle ABC$ is not $\cong$ to $\triangle RST$. Since they are not $\cong$, there is no sequence of rigid motions that would map $\triangle ABC$ onto $\triangle RST$.

17. CC JUN ‘18 [25]
    Yes, translations are rigid motions that preserve distance and angles.
18. CC JUN ‘19 [25]
    No, dilations do not preserve distance, and therefore do not preserve congruence.
19. CC JUN ‘22 [28]
    Reflections preserve distance and angles, and therefore congruence.

6.4 Transformations and Similarity

1. CC AUG ‘16 [9] Ans: 4
2. CC JAN ‘17 [2] Ans: 2
3. CC JUN ‘17 [14] Ans: 1
4. CC AUG ‘18 [2] Ans: 1
5. CC FALL ‘14 [17]
   Circle A can be mapped onto circle B by first translating circle A such that A maps onto B, and then dilating circle A, centered at A, by a scale factor of $\frac{5}{3}$.
   Since there exists a sequence of transformations that maps circle A onto circle B, circle A is similar to circle B.
6. CC FALL ‘14 [19]
   Let $\triangle X'Y'Z'$ be the image of $\triangle XYZ$ after a rotation about point Z such that $\overline{ZX'}$ coincides with $\overline{ZU}$. Since rotations preserve angle measure, $\overline{ZY'}$ coincides with $\overline{ZV}$. Then, dilate $\triangle X'Y'Z'$ by a scale factor of $\frac{ZU}{ZX'}$ with its center at point Z. Since dilations preserve angles, $\overline{X'Y'}$ maps onto $\overline{UV}$.
   Therefore, $\triangle XYZ \sim \triangle UVZ$.
7. CC JUN ‘16 [34]
   A dilation of $\frac{5}{2}$ about the origin.
   Dilations preserve similarity.
8. CC JAN ‘18 [32]
   Dilation by a scale factor of 3 with its center at point A. Dilations preserves similarity.

Chapter 7. Circles in the Coordinate Plane

7.1 Equation of a Circle

1. CC JUN ‘15 [14] Ans: 2
2. CC AUG ‘15 [9] Ans: 3
3. CC JAN ‘16 [17] Ans: 4
4. CC JUN ‘16 [3] Ans: 2
5. CC JUN ‘16 [23] Ans: 1
6. CC AUG ‘16 [16] Ans: 1
7. CC JAN ‘17 [18] Ans: 1
8. CC JAN ‘17 [22] Ans: 3
9. CC JUN ‘17 [12] Ans: 1
10. CC JAN ‘18 [12] Ans: 2
11. CC JUN ‘18 [20] Ans: 2
12. CC AUG ‘18 [21] Ans: 4
13. CC JAN ‘19 [20] Ans: 1
14. CC JUN ‘19 [20] Ans: 4
15. CC AUG ‘19 [6] Ans: 4
16. CC JAN ‘20 [20] Ans: 2
17. CC AUG ‘22 [19] Ans: 1
18. CC JAN ‘23 [14] Ans: 2
19. CC JUN ‘23 [9] Ans: 4
20. CC AUG ‘23 [13] Ans: 3
21. CC AUG ‘16 [30]
    Yes.
    $(x-1)^2+(y+2)^2=4^2$
    $(3.4-1)^2+(1.2+2)^2=4^2$
    $5.76+10.24=16$ ✓
22. CC AUG ‘17 [31]
    $x^2-6x+y^2+8y=56$
    $x^2-6x+9+y^2+8y+16=$
    $56+9+16$
    $(x-3)^2+(y+4)^2=81$
    $(3,-4), r=9$
23. CC JUN ‘22 [30]
    $x^2+6x+y^2-6y=63$
    $(x^2+6x+9)+(y^2-6y+9)=$
    $63+9+9$
    $(x+3)^2+(y-3)^2=81$
    $(-3,1), r=9$
24. CC JAN’24 [30]
    $x^2+16x+y^2+12y=44$
    $(x^2+16x+64)+(y^2+12y+36)=$
    $44+64+36$
    $(x+8)^2+(y+6)^2=144$
    $(-8,-6), r=12$

7.2 Graph Circles

There are no Regents exam questions on this topic.

CHAPTER 8. FOUNDATIONS OF EUCLIDEAN GEOMETRY

8.1 Postulates, Theorems and Proofs

There are no Regents exam questions on this topic.

8.2 Parallel Lines and Transversals

1.	CC JUN ‘15 [17]	Ans: 1
2.	CC AUG ‘16 [1]	Ans: 2

CHAPTER 9. TRIANGLES AND CONGRUENCE

9.1 Angles of Triangles

1. CC AUG ‘16 [4] Ans: 2
2. CC JUN ‘17 [17] Ans: 4
3. CC JAN ‘18 [9] Ans: 3
4. CC JAN ‘18 [18] Ans: 2
5. CC JUN ‘18 [2] Ans: 3
6. CC AUG ‘18 [1] Ans: 4
7. CC JAN ‘19 [16] Ans: 4
8. CC JAN ‘20 [1] Ans: 3
9. CC JUN ‘22 [15] Ans: 3
10. CC FALL ‘14 [10]
    The sum of the measures of the ∠’s of a △ is 180°, so
    $\mathrm{m}\angle ABC + \mathrm{m}\angle BCA + \mathrm{m}\angle CAB = 180°$.
    Each interior ∠ of the △ and its exterior ∠ form a linear pair.
    Linear pairs are supplementary, so
    $\mathrm{m}\angle ABC + \mathrm{m}\angle FBC = 180°$,
    $\mathrm{m}\angle BCA + \mathrm{m}\angle DCA = 180°$, and
    $\mathrm{m}\angle CAB + \mathrm{m}\angle EAB = 180°$.
    By addition, the sum of these linear pairs is 540°. When the ∠ measures of the △ are subtracted from this sum, the result is 360°, the sum of the ext ∠’s of the △.
11. CC JAN ‘16 [33]
    (2) Parallel Postulate
    (3) Alt Int ∠’s Thm
    (4) Consecutive adjacent ∠’s on a straight line add to 180°
    (5) Substitution

9.2 Triangle Inequality Theorem

1. CC JAN ‘19 [19] Ans: 3
2. CC AUG ‘23 [10] Ans: 1

9.3 Segments in Triangles

1. CC JAN ‘23 [5] Ans: 4

9.4 Isosceles and Equilateral Triangles

1. CC AUG ‘16 [8] Ans: 3
2. CC AUG ‘17 [11] Ans: 4
3. CC AUG ‘19 [5] Ans: 3
4. CC JAN ‘20 [12] Ans: 2
5. CC JUN ‘22 [7] Ans: 4
6. CC JUN ‘23 [17] Ans: 2
7. CC JUN ‘23 [18] Ans: 4
8. CC AUG ‘23 [15] Ans: 3
9. CC JAN ’24 [21] Ans: 4
10. CC FALL ‘14 [5]
    In an isosceles △, the bisector of the vertex ∠ is also a median. Therefore, $MO = \frac{1}{2}MP$, so $MO = 8$.

11. CC FALL ‘14 [24]
$\triangle XYZ, \overline{XY} \cong \overline{ZY}$ and $\overline{YW}$ bisects $\angle XYZ$ (Given)
$\triangle XYZ$ is isosceles (Def of isosceles $\triangle$)
$\overline{YW}$ is an altitude of $\triangle XYZ$
($\angle$ bisector of the vertex of an isosceles $\triangle$ is also an altitude)
$\overline{YW} \perp \overline{XZ}$ (Def of altitude)
$\angle YWZ$ is a right $\angle$ (Def of $\perp$)

12. CC JUN ‘15 [32]
Since linear pairs are supplementary, $\text{m}\angle GIH = 65°$.
Since $\overline{GH} \cong \overline{IH}$, $\text{m}\angle IGH = \text{m}\angle \text{GIH} = 65°$ and $\text{m}\angle GHI = 180 - (65 + 65) = 50°$.
Since $\angle EGB \cong \angle GHI$, the corresponding $\angle$’s formed by the transversal and lines are $\cong$ and $\overline{AB} \parallel \overline{CD}$.

13. CC JAN ‘17 [30]
$\text{m}\angle DAC = \text{m}\angle ECA = 25°$
$\text{m}\angle AXC = 180 - 2(25) = 130°$

14. CC AUG ‘23 [26]
$5x - 14 = 3x + 10$
$2x = 24$
$x = 12$

9.5 Triangle Congruence Methods

1. CC JUN ‘15 [24] Ans: 3
2. CC JAN ‘17 [3] Ans: 1
3. CC JUN ‘17 [9] Ans: 2
4. CC JAN ‘18 [1] Ans: 1
5. CC AUG ‘18 [10] Ans: 4
6. CC JUN ‘19 [8] Ans: 4
7. CC JUN ‘19 [14] Ans: 4
8. CC JUN ‘22 [16] Ans: 4
9. CC AUG ‘23 [16] Ans: 1
10. CC JAN ’24 [22] Ans: 3

11. CC AUG ‘17 [30]
Yes, a sequence of rigid motions preserves distance and angle measure, so $\triangle ABC \cong \triangle XYZ$ by ASA.
$\overline{BC} \cong \overline{YZ}$ by CPCTC.

12. CC JAN ‘20 [25]
Various answers, such as: $\angle Q \cong \angle M$, $\angle P \cong \angle N$, and $\overline{QP} \cong \overline{MN}$.

13. CC JAN ‘23 [29]
Yes, the triangles are both 5-12-13 triangles congruent by SSS. All congruent triangles are similar.

9.6 Prove Triangles Congruent

1. CC JUN ‘17 [33]
(Givens omitted)
$\overline{TX} \cong \overline{XV}$ and $\overline{RX} \cong \overline{XS}$ (def of bisector)
$\angle TXR \cong \angle VXS$ (vertical $\angle$’s)
$\triangle TXR \cong \triangle VXS$ (SAS)
$\angle T \cong \angle V$ (CPCTC)
$\overline{TR} \parallel \overline{SV}$ (alt int $\angle$’s converse)

2. CC JAN ‘23 [33]
(Givens omitted)
$\angle A \cong \angle D$ (alt int $\angle$’s are $\cong$)
$\angle EBA \cong \angle FCD$ (alt ext $\angle$’s are $\cong$)
$\overline{BC} \cong \overline{BC}$ (reflexive)
$\overline{AB} \cong \overline{CD}$ (segment subtraction)
$\triangle EAB \cong \triangle FDC$ (ASA)

3. CC JAN ’24 [34]
(Givens omitted)
$\angle ABF \cong \angle CDE$ (alt int $\angle$’s are $\cong$)
$\overline{EF} \cong \overline{EF}$ (reflexive)
$\overline{BF} \cong \overline{DE}$ (addition)
$\triangle AFB \cong \triangle CED$ (SAS)
$\overline{CE} \cong \overline{AF}$ (CPCTC)

9.7 Overlapping Triangles

1. CC AUG '16 [22] Ans: 3
2. CC AUG '19 [33]
   $\triangle ABE \cong \triangle CBD$ (given)
   $\angle A \cong \angle C$ (CPCTC)
   $\angle AFD \cong \angle CFE$ (vertical ∠'s are ≅)
   $\overline{AB} \cong \overline{CB}, \overline{DB} \cong \overline{EB}$ (CPCTC)
   $\overline{AD} \cong \overline{CE}$ (subtraction)
   $\triangle AFD \cong \triangle CFE$ (AAS)
3. CC JAN '20 [35]
   (Givens omitted)
   $\overline{BD} \cong \overline{BD}$ (reflexive prop)
   $\triangle ABD \cong \triangle CDB$ (SAS)
   $\angle CBD \cong \angle ADB$ (CPCTC)
   $\overline{BC} \cong \overline{DA}$ (CPCTC)
   $\overline{BC} - \overline{CE} = \overline{DA} - \overline{AF}$, so $\overline{BE} \cong \overline{DF}$
   (subtraction prop)
   $\angle BGE \cong \angle DGF$ (vertical ∠'s are ≅)
   $\triangle EBG \cong \triangle FDG$ (AAS)
   $\overline{FG} \cong \overline{EG}$ (CPCTC)

CHAPTER 10. TRIANGLES AND SIMILARITY

10.1 Properties of Similar Triangles

1. CC JUN ‘15 [21] Ans: 4
2. CC AUG ‘15 [14] Ans: 4
3. CC AUG ‘15 [19] Ans: 2
4. CC JAN ‘16 [20] Ans: 4
5. CC AUG ‘22 [16] Ans: 2
6. CC JUN ‘18 [30]
   Yes, both are 5-12-13 △s by the Pythagorean Thm, so they are ≅ by SSS. All ≅ △s are also similar.

10.2 Triangle Similarity Methods

1. CC JUN ‘15 [15] Ans: 3
2. CC JAN ‘16 [13] Ans: 1
3. CC JAN ‘16 [24] Ans: 3
4. CC JUN ‘16 [5] Ans: 3
5. CC JUN ‘16 [17] Ans: 1
6. CC AUG ‘16 [12] Ans: 3
7. CC AUG ‘17 [5] Ans: 4
8. CC AUG ‘17 [9] Ans: 4
9. CC JAN ‘18 [13] Ans: 3
10. CC JAN ‘18 [17] Ans: 4
11. CC JUN ‘18 [4] Ans: 3
12. CC JUN ‘18 [9] Ans: 4
13. CC JAN ‘19 [8] Ans: 1
14. CC JUN ‘19 [15] Ans: 2
15. CC AUG ‘19 [18] Ans: 3
16. CC JAN ‘20 [3] Ans: 2
17. CC JAN ‘20 [6] Ans: 3
18. CC JAN ‘20 [24] Ans: 4
19. CC JUN ‘22 [11] Ans: 4
20. CC JUN ‘22 [14] Ans: 2
21. CC AUG ‘22 [2] Ans: 2
22. CC JAN ‘23 [24] Ans: 4
23. CC JUN ‘23 [14] Ans: 2
24. CC AUG ‘23 [24] Ans: 4
25. CC JAN ’24 [23] Ans: 1
26. CC AUG ‘15 [29]
    $\frac{6}{14} = \frac{9}{21}$; Yes (SAS~)
27. CC AUG ‘18 [29]
    △s are similar by AA~.
    The △s share the same ∠ at the stake, so these ∠’s are ≅; the ∠’s at the bases of the two poles are corresponding ∠’s formed by ∥ lines, so they are ≅.
    (The ∠’s formed by the poles with the support wire are also ≅ since they are corresponding ∠’s formed by ∥ lines.)
28. CC JAN ’24 [29]
    △s are similar by SAS~. $\angle A \cong \angle A$ and $\frac{AB}{AD} = \frac{AE}{AC}$, as shown by the cross-products of $\frac{4.1}{9.02} = \frac{5.6}{12.32}$ giving us $50.512 = 50.512$.

10.3 Prove Triangles Similar

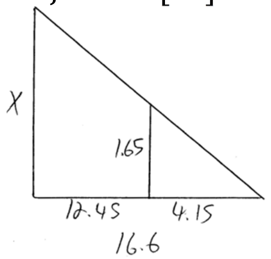
1. CC JAN ‘17 [29]
   (Givens omitted)
   $\angle I \cong \angle N$, $\angle G \cong \angle T$ (alt int ∠’s thm)
   $\triangle GIA \sim \triangle TNA$ (AA~)

10.4 Triangle Angle Bisector Theorem

There are no Regents exam questions on this topic.

10.5 Side Splitter Theorem

1. CC JUN ‘15 [11] Ans: 3
2. CC AUG ‘15 [17] Ans: 4
3. CC JUN ‘16 [21] Ans: 2
4. CC JUN ‘17 [5] Ans: 4
5. CC JUN ‘17 [10] Ans: 2
6. CC AUG ‘17 [7] Ans: 4
7. CC JUN ‘18 [11] Ans: 2
8. CC JUN ‘18 [21] Ans: 4
9. CC AUG ‘18 [12] Ans: 2
10. CC AUG ‘18 [16] Ans: 3
11. CC JAN ‘19 [6] Ans: 2
12. CC JUN ‘19 [11] Ans: 1
13. CC AUG ‘22 [22] Ans: 1
14. CC JAN ‘23 [3] Ans: 2
15. CC JAN ‘23 [8] Ans: 2
16. CC JUN ‘23 [7] Ans: 3
17. CC JUN ‘23 [21] Ans: 4
18. CC AUG ‘23 [14] Ans: 4
19. CC JAN ’24 [2] Ans: 2
20. CC JUN ‘15 [31]



$\frac{1.65}{4.15}=\frac{x}{16.6};\ x\approx 6.6$

21. CC AUG ‘15 [27]

$\frac{120}{230}=\frac{BC}{315};\ BC\approx 164$

22. CC JUN ‘16 [27]

$\frac{3.75}{5}=\frac{4.5}{6};\ 22.5=22.5$ ✓

$\overline{AB}\parallel\overline{CD}$ because $\overline{AB}$ divides the sides proportionately.

10.6 Triangle Midsegment Theorem

1. CC JAN ‘17 [4] Ans: 4
2. CC AUG ‘17 [16] Ans: 4
3. CC JUN ‘19 [23] Ans: 3
4. CC JAN ‘20 [9] Ans: 3
5. CC JAN ‘23 [21] Ans: 1
6. CC JUN ‘23 [22] Ans: 4

CHAPTER 11. POINTS OF CONCURRENCY

11.1 Incenter and Circumcenter

There are no Regents exam questions on this topic.

11.2 Orthocenter and Centroid

1. CC JUN '18 [18] Ans: 1
2. CC AUG '19 [4] Ans: 1
3. CC JAN '20 [30]
   $CX = 2(CE) = 10$; $CF = \frac{1}{3}(YF) = 7$;
   $XF = \frac{1}{2}(XZ) = 7.5$;
   $P = 10 + 7 + 7.5 = 24.5$

CHAPTER 12. RIGHT TRIANGLES AND TRIGONOMETRY

12.1 Congruent Right Triangles

1. CC JUN ‘16 [7] Ans: 3
2. CC FALL ‘14 [21]
   a) $\overline{LA} \cong \overline{DN}$, $\overline{CA} \cong \overline{CN}$, and $\overline{DAC} \perp \overline{LCN}$ (Given)
   $\angle LCA$ and $\angle DCN$ are right $\angle$’s (Def of $\perp$)
   $\triangle LAC$ and $\triangle DNC$ are right $\triangle$s (Def of a right $\triangle$)
   $\triangle LAC \cong \triangle DNC$ (HL)
   b) Rotate $\triangle LAC$ counterclockwise 90° about point C such that point L maps onto point D.

12.2 Equidistance Theorems

1. CC JUN ‘16 [19] Ans: 2
2. CC AUG ‘16 [11] Ans: 4
3. CC AUG ‘18 [22] Ans: 4
4. CC JAN ‘23 [16] Ans: 1
5. CC AUG ‘18 [32]
   (2) Reflexive; (4) $\angle BDA \cong \angle BDC$;
   (6) CPCTC;
   (7) If points B and D are equidistant from the endpoints of $\overline{AC}$, then B and D are on the $\perp$ bisector of $\overline{AC}$.

12.3 Geometric Mean Theorems

1. CC JAN ‘16 [22] Ans: 2
2. CC JUN ‘16 [13] Ans: 2
3. CC AUG ‘16 [10] Ans: 2
4. CC AUG ‘17 [18] Ans: 2
5. CC JAN ‘18 [23] Ans: 2
6. CC JUN ‘18 [23] Ans: 1
7. CC AUG ‘18 [7] Ans: 3
8. CC AUG ‘18 [20] Ans: 2
9. CC JAN ‘19 [10] Ans: 3
10. CC AUG ‘19 [16] Ans: 1
11. CC AUG ‘19 [20] Ans: 2
12. CC JAN ‘20 [16] Ans: 4
13. CC JUN ‘22 [13] Ans: 3
14. CC JAN ‘23 [15] Ans: 4
15. CC JAN ’24 [12] Ans: 1
16. CC JAN ’24 [18] Ans: 1
17. CC JUN ‘15 [34]
   $\sqrt{0.55^2 - 0.25^2} \approx 0.49$
   No, $0.49^2 = 0.25y$
   $0.9604 = y$
   $0.9604 + 0.25 < 1.5$
18. CC JUN ‘17 [29]
   If an altitude is drawn to the hypotenuse of a $\triangle$, it divides the $\triangle$ into two right $\triangle$s that are similar to each other and to the original $\triangle$.
19. CC JUN ‘19 [30]
   $\frac{x}{15} = \frac{15}{17};\ 17x = 225;\ x \approx 13.2$
20. CC AUG ‘22 [29]
   $\frac{x}{6} = \frac{6}{4x}; 4x^2 = 36; x^2 = 9; x = 3$
21. CC JUN ‘23 [30]
   $\frac{2}{6} = \frac{6}{x+2}; 2(x+2) = 36;$
   $2x + 4 = 36; 2x = 32; x = 16$
22. CC AUG ‘23 [30]
   Let $x = SQ$ and $4x = QR$;
   $(4x)(x) = 8^2; 4x^2 = 64; x = 4;$
   $SR = 4 + 4x = 20$

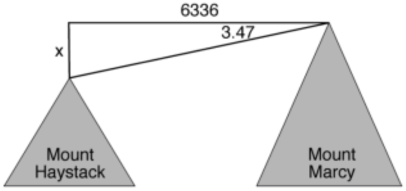
CHAPTER 13. TRIGONOMETRY

13.1 Trigonometric Ratios

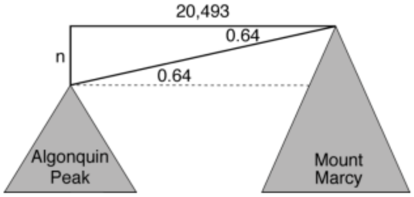
1. CC JUN ‘16 [15] Ans: 4
2. CC JAN ‘17 [14] Ans: 3
3. CC JAN ‘19 [17] Ans: 4
4. CC JAN ‘23 [7] Ans: 1

13.2 Use Trigonometry to Find a Side

1. CC JUN ‘15 [5] Ans: 3
2. CC JUN ‘16 [11] Ans: 4
3. CC JAN ‘17 [7] Ans: 2
4. CC JAN ‘17 [12] Ans: 3
5. CC JUN ‘17 [21] Ans: 4
6. CC AUG ‘17 [19] Ans: 1
7. CC JAN ‘18 [4] Ans: 1
8. CC AUG ‘18 [6] Ans: 4
9. CC JAN ‘19 [13] Ans: 2
10. CC AUG ‘19 [15] Ans: 2
11. CC AUG ‘19 [24] Ans: 1
12. CC JUN ‘22 [17] Ans: 1
13. CC JUN ‘23 [16] Ans: 4
14. CC JAN ’24 [6] Ans: 4
15. CC JAN ’24 [11] Ans: 4
16. CC FALL ‘14 [13]



$\tan 3.47 = \frac{M}{6336}$; $M \approx 384$;
$4960 + 384 = 5344$



$\tan 0.64 = \frac{A}{20493}$; $A \approx 229$;
$5344 - 229 = 5115$

17. CC FALL ‘14 [22]
x represents the distance between the lighthouse and the canoe at 5:00;
y represents the distance between the lighthouse and the canoe at 5:05.
$\tan 6 = \frac{112-1.5}{x}$; $x \approx 1051.3$
$\tan 55 = \frac{112-1.5}{y}$; $y \approx 77.4$
$\frac{1051.3-77.4}{5} \approx 195$

18. CC AUG ‘15 [32]
$\tan 7 = \frac{125}{AC}$; $AC = \frac{125}{\tan 7} \approx 1018.0$
$\tan 16 = \frac{125}{DC}$; $DC = \frac{125}{\tan 16} \approx 435.9$
$AD = AC - DC \approx$
$1018.0 - 435.9 \approx 582$

19. CC JAN ‘16 [29]
$\sin 70 = \frac{30}{x}$; $x \approx 32$

20. CC JAN ‘16 [36]
$\tan 52.8 = \frac{h}{x}$;
$h = x \tan 52.8 \approx 1.32x$;
$\tan 34.9 = \frac{h}{x+8}$;
$h = (x+8)\tan 34.9 \approx 0.70(x+8)$;
$1.32x = 0.7(x+8)$; $x \approx 9.0$
$\tan 52.8 \approx \frac{h}{9}$; $h \approx 9 \tan 52.8 \approx 11.86$
$11.86 + 1.7 \approx 13.6$

21. CC AUG ‘16 [31]
$\sin 75 = \frac{15}{x}$; $x \approx 15.5$

22. CC JUN ‘17 [36]
$\tan 15 = \frac{6250}{x}$; $x \approx 23{,}325.3$
$\tan 52 = \frac{6250}{y}$; $y \approx 4{,}883.0$
$23{,}325.3 - 4{,}883.0 = 18{,}442.3$
Plane traveled 18,442 ft. in 1 min.
$\frac{18442\,\cancel{ft}}{1\,\cancel{min}} \cdot \frac{1\,mile}{5280\,\cancel{ft}} \cdot \frac{60\,\cancel{min}}{1\,hr} \approx 210\,mph$

23. CC JAN ‘18 [34]
$\cos 54 = \frac{4.5}{HI}; HI \approx 7.7$ mi
$\tan 54 = \frac{IM}{4.5}; IM \approx 6.2$ mi

24. CC JUN ‘18 [33]
$\tan 72° = \frac{ST}{400}; ST \approx 1231.07$
$\sin 55° = \frac{ST}{CT} \approx \frac{1231.07}{CT}; CT \approx 1503$

25. CC AUG ‘18 [33]
$\text{m}\angle AGH = 36$
$\tan 36 = \frac{HA}{10};\ HA \approx 7.2654$
$HA = FG = DE \approx 7.3$
$AG = \sqrt{(HA)^2 + (HG)^2} =$
$\sqrt{7.2654^2 + 10^2} \approx 12.3607$
$AC = 3 \times 12.3607 \approx 37$

26. CC JAN ‘19 [34]
$\sin 4.76° = \frac{6 \cdot 3}{x};$
$x \approx 216.914$ in. ≈ 18.1 ft.
$\tan 4.76° = \frac{18}{y};\ y \approx 216.166$ in.
$d = 216.166 - 16 =$
200.166 in. ≈ 16.7 ft.

27. CC JUN ‘19 [27]
$\cos 68 = \frac{10}{x};\ x \approx 27$

28. CC JUN ‘19 [34]
$\tan 30 = \frac{y}{440};\ y \approx 254$
$\tan 38.8 = \frac{h}{440};\ h \approx 353.8$
$353.8 - 254 \approx 100$

29. CC JAN ‘20 [26]
$\sin 38° = \frac{24.5}{x};\ x \approx 40$ inches

30. CC JAN ‘20 [33]
$\tan 56° = \frac{x}{1.3};\ x \approx 1.927;$
$x + 1.5 \approx 3.427;$
$(3.427)^2 + (1.3)^2 = c^2;\ c \approx 3.7$ m

31. CC JUN ‘22 [25]
$\sin 86.03 = \frac{183.27}{x}$
$x \approx 183.71$

32. CC AUG ‘22 [28]
$5 - 1.2 = 3.8$
$\cos 14 = \frac{3.8}{x}$
$x \approx 3.92$

33. CC AUG ‘22 [32]
$\tan 22.2 = \frac{50}{x}; x \approx 122.52$
$\tan 13.3 = \frac{y}{122.52}; y \approx 28.96$
$50 - 28.96 \approx 21$ meters

34. CC JAN ‘23 [34]
$\text{m}\angle ABH = 180 - 80 = 100°$
$\text{m}\angle AHB = 180 - 100 - 40 = 40°$
$\triangle ABH$ is isosceles because it has congruent base angles.
$BH = AB = 85$ ft.;
$\cos 80 = \frac{BC}{85}; BC \approx 14.8;$
$(CH)^2 + 14.8^2 = 85^2; CH \approx 84$

35. CC JUN ‘23 [32]
$\tan 15 = \frac{BC}{3280}; BC \approx 878.9$
$\tan 31 = \frac{BD}{3280}; BD \approx 1970.8$
$1970.8 - 878.9 \approx 1092$ ft.

36. CC AUG ‘23 [27]
$\tan 53 = \frac{x}{91}; x \approx 120.8$

37. CC AUG ‘23 [33]
$\sin 65 = \frac{7.7}{x}; x \approx 8.5$ m
$\tan 65 = \frac{7.7}{y}; y \approx 3.6$ m

38. CC JAN ’24 [32]
$\tan 35° = \frac{BD}{85}; BD \approx 59.5$
$\tan 75° = \frac{AD}{85}; AD \approx 317.2$
$AB = AD + BD \approx 377$ m

13.3 Use Trigonometry to Find an Angle

1. CC FALL ‘14 [1] Ans: 1
2. CC JAN ‘16 [16] Ans: 3
3. CC JUN ‘17 [13] Ans: 1
4. CC AUG ‘17 [15] Ans: 1
5. CC JUN ‘18 [6] Ans: 2
6. CC AUG ‘18 [9] Ans: 1
7. CC JUN ‘19 [22] Ans: 4
8. CC JAN ‘20 [7] Ans: 1
9. CC AUG ‘22 [7] Ans: 4
10. CC JUN ‘23 [4] Ans: 1
11. CC AUG ‘23 [3] Ans: 3
12. CC JUN ‘15 [28]
$\sin x = \frac{4.5}{11.75}; x \approx 23°$
13. CC JUN ‘16 [30]
$\tan x = \frac{10}{4}; x \approx 68°$

14. CC AUG ‘16 [34]
$\tan x = \frac{12}{75}; x \approx 9.09$
$\tan y = \frac{72}{75}; y \approx 43.83; y - x \approx 34.7°$
15. CC JAN ‘18 [31]
$\cos x = \frac{6}{18}; x \approx 71°$
16. CC AUG ‘19 [26]
$\sin x = \frac{5}{25}; x \approx 11.5°$
17. CC JUN ‘22 [32]
$\tan x = \frac{0.41}{3.74}; x \approx 6.26$
$\tan y = \frac{1.58}{3.74}; y \approx 22.90; y - x \approx 16.6°$
18. CC JAN ‘23 [27]
$\tan x = \frac{4}{12}; x \approx 18°$
19. CC JAN ’24 [31]
$\cos J = \frac{3}{5}; \text{m}\angle J \approx 53°;$
$\text{m}\angle JCM \approx 90 - 53 \approx 37°;$
$\angle S \cong \angle JCM$, so $\text{m}\angle S \approx 37°$

13.4 Special Triangles

1. CC JAN ‘17 [9] Ans: 2

13.5 Cofunctions

1. CC JUN ‘15 [12] Ans: 4
2. CC AUG ‘15 [4] Ans: 1
3. CC JAN ‘16 [9] Ans: 4
4. CC AUG ‘16 [6] Ans: 1
5. CC JUN ‘17 [3] Ans: 3
6. CC AUG ‘17 [21] Ans: 4
7. CC JUN ‘18 [8] Ans: 1
8. CC AUG ‘18 [24] Ans: 2
9. CC JAN ‘19 [22] Ans: 1
10. CC JUN ‘19 [9] Ans: 2
11. CC AUG ‘19 [19] Ans: 1
12. CC JAN ‘20 [21] Ans: 3
13. CC JAN ‘20 [23] Ans: 2
14. CC JUN ‘22 [6] Ans: 3
15. CC AUG ‘22 [10] Ans: 4
16. CC JAN ‘23 [4] Ans: 1
17. CC JUN ‘23 [12] Ans: 1
18. CC AUG ‘23 [11] Ans: 2
19. CC JAN ’24 [1] Ans: 3
20. CC FALL ‘14 [7]
$2x + 0.1 = 4x - 0.7;\ x = 0.4$
A and B are complementary ∠’s, and cofunctions of complementary ∠’s are equal.
21. CC FALL ‘14 [20]
The acute ∠’s in a right △ are always complementary. The sine of any acute ∠ is equal to the cosine of its complement.
22. CC JUN ‘16 [28]
$R = 90 - 73 = 17°$
Cofunctions of complementary ∠’s are equal.
23. CC JAN ‘17 [27]
Yes, because 28° and 62° ∠’s are complementary. The sine of an ∠ equals the cosine of its complement.
24. CC JAN ‘18 [27]
Since ∠’s A and B are complementary and sine and cosine are cofunctions, $\sin A = \cos B$. Therefore, when $\sin A$ increases, $\cos B$ increases.

13.6 SAS Sine Formula for Area of a Triangle

There are no Regents exam questions on this topic.

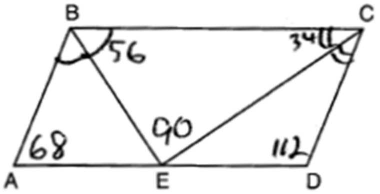
CHAPTER 14. QUADRILATERALS

14.1 Angles of Polygons

There are no Regents exam questions on this topic.

14.2 Properties of Quadrilaterals

1. CC AUG ‘15 [8] Ans: 3
2. CC JAN ‘16 [3] Ans: 3
3. CC AUG ‘16 [24] Ans: 1
4. CC AUG ‘17 [8] Ans: 4
5. CC JAN ‘18 [2] Ans: 2
6. CC AUG ‘18 [13] Ans: 4
7. CC JAN ‘19 [7] Ans: 2
8. CC JAN ‘19 [12] Ans: 2
9. CC JUN ‘19 [17] Ans: 2
10. CC JUN ‘19 [21] Ans: 2
11. CC AUG ‘19 [7] Ans: 2
12. CC JAN ‘20 [15] Ans: 4
13. CC JUN ‘22 [21] Ans: 1
14. CC AUG ‘22 [15] Ans: 3
15. CC JAN ‘23 [9] Ans: 3
16. CC JUN ‘23 [6] Ans: 3
17. CC AUG ‘23 [5] Ans: 2
18. CC JAN ’24 [13] Ans: 3
19. CC JAN ’24 [17] Ans: 3
20. CC JUN ‘15 [26]
    Opp ∠’s in a ▱ are ≅, so $m\angle O = 118°$.
    The int ∠’s of a △ equal 180°.
    $180 - (118 + 22) = 40$.
21. CC AUG ‘17 [26]
    The four small △s are 8-15-17 △s.
    $4 \times 17 = 68$
22. CC AUG ‘18 [26]
    Ans: 90°

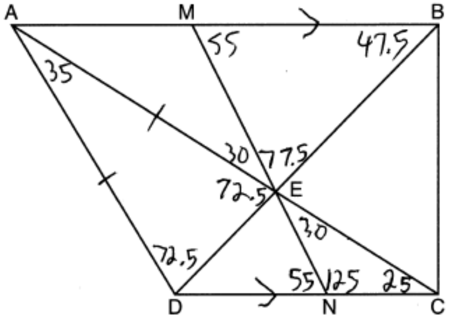


23. CC JAN ‘19 [26]
    $m\angle ADC = m\angle B = 118°$
    (▱ → opp ∠’s ≅)
    $m\angle DGF = m\angle ADC = 118°$
    (alt int ∠’s of ∥ lines $\overline{FG}$ and $\overline{EDC}$)
    $m\angle GFH = m\angle AHC - m\angle DGF =$
    $138° - 118° = 20°$
    ($\angle AHC$ is an ext ∠ of $\triangle FGH$)
24. CC AUG ‘19 [25]
    $m\angle D = 46°$ (∠’s of a △ add to 180°)
    $m\angle B = 46°$ (▱ → opp ∠’s ≅)

14.3 Trapezoids

1. CC JUN ‘23 [23] Ans: 3
2. CC AUG ‘17 [35]
    (Givens omitted)
    $\overline{AD} \cong \overline{BC}$ (isos trap → ≅ legs)
    $\angle DEA$ and $\angle CEB$ are right ∠’s (def of ⊥)
    $\angle DEA \cong \angle CEB$ (right ∠’s are ≅)
    $\angle CDA \cong \angle DCB$ (isos trap → ≅ base ∠’s)
    $\angle CDA - \angle CDE \cong \angle DCB - \angle DCE$
    (subtraction)
    $\angle ADE \cong \angle BCE$ (substitution)
    $\triangle ADE \cong \triangle BCE$ (AAS)
    $\overline{EA} \cong \overline{EB}$ (CPCTC)
    $\triangle AEB$ is isosceles (def of isosceles)

3. CC AUG ‘22 [30]



m$\angle ABD = 47.5°$

14.4 Use Quadrilateral Properties in Proofs

1. CC AUG ‘22 [17] Ans: 3
2. CC JUN ‘15 [33]
   Quad $ABCD$ is a ▱ with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E (Given)
   $\overline{AD} \cong \overline{BC}$ (▱ → opp sides ≅)
   $\angle AED \cong \angle CEB$ (Vertical ∠’s are ≅)
   $\overline{BC} \parallel \overline{DA}$ (▱ → opp sides ∥)
   $\angle DBC \cong \angle BDA$ (alt int ∠’s thm)
   $\triangle AED \cong \triangle CEB$ (AAS)
   180° rotation of $\triangle AED$ around point E.
3. CC AUG ‘15 [28]
   (Givens omitted)
   $\overline{DC} \parallel \overline{AB}$; $\overline{DA} \parallel \overline{CB}$ (▱ → opp sides ∥)
   $\angle ACD \cong \angle CAB$ (alt int ∠’s thm)
4. CC JUN ‘16 [33]
   (Givens omitted)
   $\angle DFE \cong \angle BFG$ (vertical ∠’s are ≅)
   $\overline{AD} \parallel \overline{CB}$ (▱ → opp sides ∥)
   $\angle EDF \cong \angle GBF$ (alt int ∠’s thm)
   $\triangle DEF \sim \triangle BGF$ (AA~)
5. CC JAN ‘18 [25]
   (Givens omitted)
   $\overline{AB} \cong \overline{CD}, \overline{AD} \cong \overline{BC}$ (▱ → opp sides ≅)
   $\overline{AC} \cong \overline{AC}$ (Reflexive prop)
   $\triangle ABC \cong \triangle CDA$ (SSS)
6. CC JUN ‘22 [33]
   (Givens omitted)
   $\overline{RS} \cong \overline{PQ}$ (▱ → opp sides ≅)
   $\angle P \cong \angle R$ (▱ → opp ∠’s ≅)
   $\angle RUS$ and $\angle PTQ$ are right ∠’s (def of ⊥)
   $\angle RUS \cong \angle PTQ$ (all right ∠’s are ≅)
   $\triangle RUS \cong \triangle PTQ$ (AAS)
   $\overline{PT} \cong \overline{RU}$ (CPCTC)

14.5 Prove Types of Quadrilaterals

1. CC JUN ‘15 [13] Ans: 4
2. CC AUG ‘15 [1] Ans: 2
3. CC JUN ‘16 [9] Ans: 1
4. CC AUG ‘16 [7] Ans: 3
5. CC JAN ‘17 [5] Ans: 4
6. CC JAN ‘17 [16] Ans: 1
7. CC JUN ‘17 [11] Ans: 4
8. CC JUN ‘17 [20] Ans: 2
9. CC AUG ‘17 [14] Ans: 3
10. CC JAN ‘18 [19] Ans: 4
11. CC JUN ‘18 [13] Ans: 4
12. CC JUN ‘19 [12] Ans: 3
13. CC JUN ‘19 [24] Ans: 3
14. CC AUG ‘19 [13] Ans: 3
15. CC JAN ‘20 [4] Ans: 1
16. CC JUN ‘22 [9] Ans: 3
17. CC AUG ‘22 [4] Ans: 2
18. CC JAN ‘23 [18] Ans: 3
19. CC JUN ‘23 [10] Ans: 3
20. CC AUG ‘23 [19] Ans: 4
21. CC JAN ’24 [20] Ans: 1

22. CC AUG ‘15 [35]
(Givens omitted)
$\angle BEC$ and $\angle DFC$ are right ∠’s (def of ⊥)
$\angle BEC \cong \angle DFC$ (right ∠’s are ≅)
$\angle FCD \cong \angle BCE$ (reflexive prop)
$\triangle BEC \cong \triangle DFC$ (ASA)
$\overline{BC} \cong \overline{CD}$ (CPCTC)
$ABCD$ is a rhombus (▱ with consecutive ≅ sides → rhombus)

23. CC JAN ‘16 [35]
(Givens omitted)
$\overline{AN} \cong \overline{RD}, \overline{AR} \cong \overline{DN}$ (▱ → opp sides ≅)
$\frac{1}{2}AR = \frac{1}{2}DN$ (division prop)
$RE = AE = \frac{1}{2}AR, NW = WD = \frac{1}{2}DN$ (def of bisector)
$\overline{RE} \cong \overline{NW}$ and $\overline{AE} \cong \overline{WD}$ (substitution)
$\angle R \cong \angle N$ (▱ → opp ∠’s ≅)
$\triangle ANW \cong \triangle DRE$ (SAS)
$\overline{AER} \parallel \overline{NWD}$ (▱ →opp sides ∥)
$AWDE$ is a ▱ (quad with pair of opp sides ≅ and ∥ → ▱)

24. CC JUN ‘16 [35]
(Givens omitted)
quad $ABCD$ is a ▱ (diagonals of a quad bisect each other → ▱)
quad $ABCD$ is a rhombus (diagonal of a ▱ bisects its ∠ → rhombus)
$\overline{AD} \cong \overline{DC}$ (rhombus → ≅ sides)
$\triangle ACD$ is isosceles △ (def of isosceles)
$\overline{AE} \perp \overline{BE}$ (diagonals of a rhombus are ⊥)
$\angle BEA$ is a right ∠ (def of ⊥)
$\triangle AEB$ is a right △ (def of right △)

25. CC JAN ‘17 [35]
(Givens omitted)
$\angle AED$ and $\angle CFB$ are right ∠’s (def of ⊥)
$\angle AED \cong \angle CFB$ (right ∠’s are ≅)
$ABCD$ is a ▱ (quad with pair of opp sides ≅ and ∥ → ▱)
$\overline{AD} \parallel \overline{BC}$ (▱ →opp sides ∥)
$\angle DAE \cong \angle BCF$ (alt int ∠’s thm)
$\overline{DA} \cong \overline{BC}$ (▱ →opp sides ≅)
$\triangle ADE \cong \triangle CBF$ (AAS)
$\overline{AE} \cong \overline{CF}$ (CPCTC)

26. CC JUN ‘18 [35]
(Givens omitted)
$\overline{BC} \parallel \overline{AD}$ (▱ →opp sides ∥)
$\overline{BE} \parallel \overline{FD}$ (parts of ∥ lines are ∥)
$\overline{BF} \parallel \overline{DE}$ (two lines ⊥ to same line are ∥)
$BEDF$ is a ▱ (quad with both pairs of opp sides ∥ → ▱)
$\angle DEB$ is a right ∠ (def of ⊥)
$BEDF$ is a □ (▱ with a right ∠ → □)

27. CC JAN ‘19 [35]
(Givens omitted)
$\overline{HF} \cong \overline{HF}$ (Reflexive prop)
$\overline{HF} + \overline{CF} \cong \overline{HF} + \overline{AH}$, so $\overline{AF} \cong \overline{CH}$ (Addition prop)
$\overline{AE} + \overline{BE} \cong \overline{CG} + \overline{DG}$, so $\overline{AB} \cong \overline{CD}$ (Addition prop)
$ABCD$ is a ▱ (both pairs of opp sides ≅)
$\overline{AB} \parallel \overline{CD}$ (▱ →opp sides ∥)
$\angle BAC \cong \angle DCA$ (alt int ∠’s thm)
$\triangle EAF \cong \triangle GCH$ (SAS)
$\overline{EF} \cong \overline{GH}$ (CPCTC)

28. CC JUN ‘19 [35]
(Givens omitted)
$\angle HEA$ and $\angle TAH$ are right ∠’s (def of ⊥)
$\angle HEA \cong \angle TAH$ (right ∠’s are ≅)
$MATH$ is a ▱ (quad with two pairs of opp sides ≅ → ▱)
$\overline{MA} \parallel \overline{TH}$ (opp sides of ▱ are ∥)
$\angle THA \cong \angle EAH$ (alt int ∠’s thm)
$\triangle HEA \sim \triangle TAH$ (AA~)
$\frac{HA}{TH} = \frac{HE}{TA}$ (CSSTP)
$TA \cdot HA = HE \cdot TH$ (cross prods =)

29. CC AUG ‘22 [35]
(Givens omitted)
$\overline{AD} \parallel \overline{BC}$ (transitive prop)
$ABCD$ is a ▱ (pair of opp sides ∥ and ≅)
$\angle AHE \cong \angle CHF$ (vertical ∠’s are ≅)
$\overline{AB} \parallel \overline{CD}$ (opp sides of a ▱ are ∥)
$\angle BAC \cong \angle DCA$ (alt int ∠’s thm)
$\triangle AHE \sim \triangle CHF$ (AA~)
$\frac{EH}{FH} = \frac{AH}{CH}$ (CSSTP)
$(EH)(CH) = (FH)(AH)$ (cross prods =)

30. CC JUN ‘23 [35]
(Givens omitted)
$ABCD$ is a ▱ (pair of opp sides both ∥ and ≅)
$\overline{AD} \cong \overline{CB}$ (opp sides of a ▱ are ≅)
$\overline{AE} \cong \overline{CF}$ (subtraction)
$\overline{AD} \parallel \overline{CB}$ (opp sides of a ▱ are ∥)
$\angle EAG \cong \angle FCG$ (alt int ∠’s thm)
$\angle AGE \cong \angle CGF$ (vertical angles)
$\triangle AEG \cong \triangle CFG$ (AAS)
$\overline{EG} \cong \overline{FG}$ (CPCTC)
G is the midpoint of $\overline{EF}$ (def of midpoint)

31. CC AUG ‘23 [35]
(Givens omitted)
$FACT$ is a ▱ (pair of opp sides both ∥ and ≅)
$\overline{AC} \parallel \overline{FT}$ (opp sides of a ▱ are ∥)
$\angle BAE \cong \angle RTE$ and $\angle ABE \cong \angle TRE$ (alt int ∠’s thm)
$\triangle ABE \sim \triangle TRE$ (AA~)
$\frac{AB}{AE} = \frac{TR}{TE}$ (CSSTP)
$(AB)(TE) = (AE)(TR)$ (cross prods =)

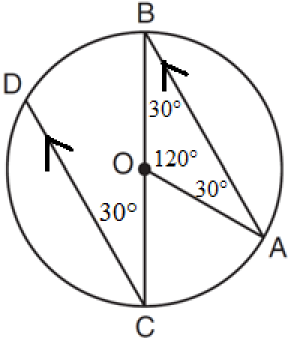
CHAPTER 15. CIRCLES

15.1 Circumference and Rotation

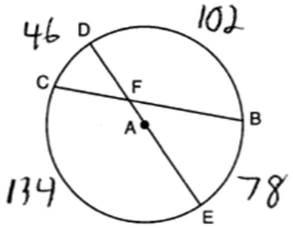
1. CC JAN ‘16 [23] Ans: 1

15.2 Arcs and Chords

1. CC JUN ‘15 [8] Ans: 1
2. CC AUG ‘15 [11] Ans: 2
3. CC AUG ‘15 [15] Ans: 3
4. CC JUN ‘16 [10] Ans: 2
5. CC AUG ‘16 [23] Ans: 1
6. CC JUN ‘17 [4] Ans: 4
7. CC JUN ‘17 [8] Ans: 2
8. CC AUG ‘17 [4] Ans: 1
9. CC JAN ‘18 [16] Ans: 4
10. CC JAN ‘18 [21] Ans: 4
11. CC JUN ‘18 [17] Ans: 3
12. CC JAN ‘19 [5] Ans: 4
13. CC JUN ‘19 [13] Ans: 3
14. CC AUG ‘19 [22] Ans: 4
15. CC AUG ‘22 [18] Ans: 4
16. CC AUG ‘22 [24] Ans: 4
17. CC JUN ‘23 [5] Ans: 2
18. CC JAN ‘16 [26]
    120°
    $\angle B \cong \angle C$ (alt int ∠’s)
    $\angle A \cong \angle B$ (base ∠’s of isosceles △ formed by two radii)



19. CC AUG ‘18 [27]
    118°



$\frac{134+102}{2} = 118$

20. CC JUN ‘22 [26]
    $2x + 3x + 5x + 5x = 360$, so $15x = 360$, or $x = 24$.
    $\text{m}\,\widehat{CD} + \text{m}\,\widehat{DA} = 2x + 3x = 5(24) = 120$
    $m\angle B = \frac{1}{2}(120) = 60$

15.3 Tangents

1. CC JUN ‘15 [20] Ans: 1
2. CC AUG ‘15 [12] Ans: 3
3. CC JAN ‘16 [21] Ans: 3
4. CC AUG ‘18 [14] Ans: 2
5. CC AUG ‘22 [23] Ans: 2
6. CC JAN ‘23 [23] Ans: 2
7. CC AUG ‘16 [25]
    $\frac{3}{8} \cdot 56 = 21$

15.4 Secants

1. CC JAN ‘17 [15] Ans: 2
2. CC AUG ‘17 [12] Ans: 2
3. CC JUN ‘19 [18] Ans: 1
4. CC JUN ‘22 [19] Ans: 1
5. CC JAN ‘23 [12] Ans: 2
6. CC AUG ‘23 [20] Ans: 1
7. CC JAN ’24 [14] Ans: 2
8. CC JAN ‘17 [28]
   $\frac{152-56}{2} = 48°$
9. CC JUN ‘18 [28]
   $10 \cdot 6 = 15x$
   $x = 4$
10. CC JAN ‘19 [27]
    $m\angle RPS = \frac{m\widehat{RS} - m\widehat{WT}}{2}$
    $35 = \frac{121-x}{2}$
    $-x = 70 - 121$
    $x = 51°$
11. CC AUG ‘19 [30]
    $\frac{124-56}{2} = 34$
12. CC JAN ‘20 [28]
    $(DA)^2 = (AB)(AC);\ (DA)^2 = (8)(12.5);$
    $(DA)^2 = 100;\ DA = 10$

15.5 Circle Proofs

1. CC FALL ‘14 [26]
   (Givens omitted)
   Chords $\overline{BC}$ and $\overline{BD}$ are drawn (auxiliary lines)
   $\angle A \cong \angle A$, $\widehat{BC} \cong \widehat{BC}$ (Reflexive prop)
   $m\angle BDC = \frac{1}{2} m\,\widehat{BC}$ (an inscribed ∠ is half the measure of its intercepted arc)
   $m\angle CBA = \frac{1}{2} m\,\widehat{BC}$ (an ∠ formed by a tangent and chord measures half the intercepted arc)
   $m\angle BDC = m\angle CBA$ (substitution)
   $\angle BDC \cong \angle CBA$ (def of ≅)
   $\triangle ABC \sim \triangle ADB$ (AA~)
   $\frac{AB}{AC} = \frac{AD}{AB}$ (Side Proportionality)
   $AC \cdot AD = AB^2$ (cross products =)
2. CC AUG ‘16 [35]
   (Givens omitted)
   Chords $\overline{CB}$ and $\overline{AD}$ are drawn (auxiliary lines)
   $\angle CEB \cong \angle AED$ (vertical ∠’s)
   $\angle C \cong \angle A$ (inscribed ∠’s that intercept the same arc are ≅)
   $\triangle BCE \sim \triangle DAE$ (AA~)
   $\frac{AE}{CE} = \frac{ED}{EB}$ (Side Proportionality)
   $AE \cdot EB = CE \cdot ED$ (cross products =)
3. CC AUG ‘17 [33]
   (Givens omitted)
   $\angle B$ is a right ∠
   (∠ inscribed in semi-circle is a right ∠)
   $\overleftrightarrow{EC} \perp \overleftrightarrow{OC}$ (radius drawn to a point of tangency is ⊥ to the tangent)
   $\angle ECA$ is a right ∠ (def of ⊥)
   $\angle B \cong \angle ECA$ (right ∠’s are ≅)
   $\angle BCA \cong \angle CAE$ (alt int ∠’s thm)
   $\triangle ABC \sim \triangle ECA$ (AA~)
   $\frac{BC}{CA} = \frac{AB}{EC}$ (CSSTP)

15.6 Arc Lengths and Sectors

1. CC JUN ‘15 [23] Ans: 2
2. CC AUG ‘15 [18] Ans: 3
3. CC JAN ‘16 [12] Ans: 3
4. CC JUN ‘16 [24] Ans: 3
5. CC AUG ‘16 [19] Ans: 2
6. CC JAN ‘17 [21] Ans: 4
7. CC JAN ‘18 [24] Ans: 3
8. CC JUN ‘18 [22] Ans: 4
9. CC AUG ‘18 [18] Ans: 2
10. CC JAN ‘19 [14] Ans: 2
11. CC AUG ‘19 [12] Ans: 4
12. CC JAN ‘20 [13] Ans: 3

13. CC JUN ‘22 [24] Ans: 4

14. CC JAN ‘23 [17] Ans: 4

15. CC AUG ‘23 [9] Ans: 2

16. CC FALL ‘14 [23]
$\text{m}\angle BOD = \frac{180-20}{2} = 80°$
$\frac{\theta}{360°} = \frac{S}{\pi r^2};\ \frac{80}{360} = \frac{S}{36\pi};\ S = 8\pi$

17. CC JUN ‘15 [29]
$\frac{\theta}{360°} = \frac{S}{\pi r^2};\ \frac{\theta}{360} = \frac{12\pi}{36\pi};\ \theta = 120°$

18. CC JUN ‘17 [26]
$\frac{\theta}{360°} = \frac{S}{\pi r^2};\ \frac{40}{360} = \frac{S}{20.25\pi};\ S = 2.25\pi$

19. CC JAN ‘18 [28]
$S = 625\pi - 500\pi = 125\pi$
$\frac{\theta}{360°} = \frac{S}{\pi r^2};\ \frac{\theta}{360} = \frac{125\pi}{625\pi};\ \theta = \frac{1}{5} \cdot 360 = 72°$

20. CC JUN ‘19 [28]
$\frac{\theta}{360°} = \frac{S}{\pi r^2};\ \frac{72}{360} = \frac{S}{100\pi};\ S = 20\pi$

21. CC AUG ‘22 [31]
$\frac{\theta}{360°} = \frac{S}{\pi r^2};\ \frac{72}{360} = \frac{S}{16\pi};\ S \approx 10.1$

22. CC JUN ‘23 [28]
$\frac{\theta}{360°} = \frac{S}{\pi r^2};\ \frac{80}{360} = \frac{S}{40.96\pi};\ S \approx 29$

23. CC JAN ’24 [26]
$\frac{\theta}{360°} = \frac{S}{\pi r^2}; \frac{102}{360} = \frac{S}{1444\pi}; S \approx 1285$

CHAPTER 16. SOLIDS

16.1 Volume of a Sphere

1. CC JAN ‘16 [14] Ans: 3
2. CC JUN ‘19 [10] Ans: 1
3. CC JAN ‘23 [10] Ans: 2
4. CC JUN ‘17 [28]
   $V = \frac{4}{3}\pi r^3$, so $r = \sqrt[3]{\frac{3V}{4\pi}}$.
   $\sqrt[3]{\frac{3(294)}{4\pi}} - \sqrt[3]{\frac{3(180)}{4\pi}} \approx 0.6$
5. CC JUN ‘18 [31]
   $2\pi r = 29.5$
   $r = \frac{29.5}{2\pi}$
   $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{29.5}{2\pi}\right)^3 \approx 434 \text{ in}^3$
6. CC JUN ‘22 [29]
   $V = \frac{1}{2} \cdot \frac{4}{3}\pi(2.8)^3 \approx 45.976$
   $100 \times 45.976 \approx 4598$
7. CC JUN ‘23 [29]
   $V = \frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi(3)^3 = 48\pi$

16.2 Volume of a Prism or Cylinder

1. CC JAN ‘16 [4] Ans: 2
2. CC AUG ‘16 [20] Ans: 4
3. CC JAN ‘17 [11] Ans: 2
4. CC JUN ‘17 [23] Ans: 3
5. CC JUN ‘18 [7] Ans: 1
6. CC JUN ‘23 [11] Ans: 3
7. CC AUG ‘23 [4] Ans: 1
8. CC JAN ’24 [19] Ans: 1
9. CC JUN ‘16 [32]
   $\frac{\pi(11.25)^2(33.5)}{231} \approx 57.7$
10. CC JUN ‘17 [34]
    $V = 20{,}000\,\cancel{g} \cdot \frac{1\,ft^3}{7.48\,\cancel{g}} \approx 2673.8\,ft^3$
    $V = \pi r^2 h$
    $2673.8 = \pi r^2(34.5)$
    $r \approx 4.967$
    $p \approx 2(4.967) + 1 \approx 10.9$
11. CC AUG ‘17 [36]
    $\tan 16.5 = \frac{x}{13.5}$
    $x \approx 4$; $4.5 + 4 = 8.5$ ft.
    $V_1 = 9 \cdot 16 \cdot 4.5 = 648$
    $V_2 = 13.5 \cdot 16 \cdot 4.5 = 972$
    $V_3 = \frac{1}{2} \cdot 13.5 \cdot 16 \cdot 4 = 432$
    $V_4 = 12.5 \cdot 16 \cdot 8.5 = 1700$
    $V_1 + V_2 + V_3 + V_4 = 3752 \text{ ft}^3$
    $3752 - (35 \cdot 16 \cdot 0.5) = 3472$
    $3472 \cdot 7.48 \approx 25{,}971$
    $\frac{25{,}971}{10.5} \approx 2473.4$
    $\frac{2473.4}{60} \approx 41$ hrs
12. CC JAN ‘18 [33]
    $V_{hemisphere} = \frac{1}{2} \cdot \frac{4}{3}\pi r^3 =$
    $\frac{2}{3}\pi \cdot 4^3 \approx 134.04$
    $V_{cylinder} = \pi r^2 h =$
    $\pi \cdot 4^2 \cdot (13 - 4) \approx 452.39$
    $V \approx 134.04 + 452.39 \approx 586 \text{ m}^3$
13. CC AUG ‘18 [31]
    $2\left(\frac{36}{12} \times \frac{36}{12} \times \frac{4}{12}\right) \times \$3.25 = \$19.50$

14. CC JAN ‘19 [33]
$V_T = 30 \cdot 15 \cdot 3.5 = 1{,}575$ cu. ft.
$1575 \cdot 7.48 = 11{,}781$ gallons
$\$3.95 \cdot 117.81 \approx \$465.35$
$V_N = \pi \cdot 12^2 \cdot 3.5 = 504\pi$ cu. ft.
$504\pi \cdot 7.48 \approx 11{,}843.553$ gallons
$\$200$ per $6000 = \$1$ per 30 gallons
$11{,}843.553 \div 30 \approx \$394.79$
Theresa paid more.

15. CC JUN ‘19 [33]
$r = \frac{6.5}{2} = 3.25;$
$V = \frac{2}{3}\pi(3.25)^2(1) \approx 22$
$22 \times 7.48 \approx 165$

16. CC AUG ‘19 [31]
$r = \frac{8.25}{2} = 4.125;$
$V = \pi(4.125)^2(2.5) \approx 134$

17. CC AUG ‘19 [34]
Altitude of △ base = $\sqrt{4^2 - 3^2} = \sqrt{7}$
Area of △ base = $\frac{1}{2}(6)\left(\sqrt{7}\right) = 3\sqrt{7}$
Area of ▭ base = $(10)(6) = 60$
Volume = $\left(60 + 3\sqrt{7}\right)(6.5) \approx 442$

18. CC JAN ‘20 [34]
$V_{cylinder} = \pi(7)^2(18) \approx 2770.8847$
$V_{prism} = 16x^2$
$16x^2 = 2770.8847;\ x^2 \approx 173.1803;$
$x \approx 13.2$
$\frac{80}{13.2} \approx 6.1$ and $\frac{60}{13.2} \approx 4.5$, so $6 \times 4 = 24$ containers fit

19. CC JUN ‘22 [34]
$V = \pi(0.5)^2(4) = \pi$
$10\pi \div \frac{2}{3} \approx 47.12$, so 48 bags are needed

20. CC JAN ‘23 [30]
$V = Bh$
$70 = \left(\frac{1}{2} \cdot 5w\right)(4)$
$70 = 10w$
$w = 7$

21. CC JUN ‘23 [33]
$V_S = \pi(3.5)^2(9) \approx 346;$
$V_L = \pi(4.5)^2(13) \approx 827.$
$\frac{827}{346} \approx 2.4$; 3 cans

22. CC AUG ‘23 [32]
$V = (3.5)^2(1.5) - (2)^2(1.5) = 12.375$
$\frac{12.375}{0.6} = 20.625$; 21 bags

16.3 Volume of a Pyramid or Cone

1.	CC AUG ‘15 [21]	Ans: 4
2.	CC JAN ‘16 [7]	Ans: 2
3.	CC JUN ‘16 [6]	Ans: 4
4.	CC JAN ‘17 [24]	Ans: 1
5.	CC JUN ‘17 [16]	Ans: 1
6.	CC JAN ‘18 [7]	Ans: 3
7.	CC JAN ‘18 [22]	Ans: 2
8.	CC JUN ‘18 [10]	Ans: 1
9.	CC AUG ‘18 [19]	Ans: 2
10.	CC JAN ‘19 [9]	Ans: 2
11.	CC JAN ‘19 [23]	Ans: 1
12.	CC JUN ‘19 [6]	Ans: 2
13.	CC AUG ‘19 [21]	Ans: 3
14.	CC JAN ‘20 [2]	Ans: 2
15.	CC JAN ‘20 [10]	Ans: 1
16.	CC AUG ‘22 [8]	Ans: 2
17.	CC JUN ‘23 [3]	Ans: 2
18.	CC AUG ‘23 [18]	Ans: 1

19. CC JUN ‘16 [36]
Similar △s are required to model and solve a proportion.
Let x = height of the rest of the cone
$\frac{height\ of\ cone}{radius\ of\ top} = \frac{height\ below\ glass}{radius\ of\ bottom}$
$\frac{x+5}{1.5} = \frac{x}{1}$
$x = 10$
$h = 10 + 5 = 15$
$V_{glass} = V_{cone} - V_{below\ glass}$
$V_{glass} = \frac{1}{3}\pi(1.5)^2(15) - \frac{1}{3}\pi(1)^2(10) \approx 24.9$

20. CC JAN ‘17 [34]
$C = 2\pi r$
$31.416 = 2\pi r$
$r = \frac{31.416}{2\pi} \approx 5$
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}(25)(13)\pi \approx 340$

21. CC JUN '22 [27]
$r = \frac{1}{2}(10) = 5$
$h^2 + 5^2 = 13^2$, so $h = 12$
$V = \frac{1}{3}\pi(5^2)(12) = 100\pi$

22. CC JAN '23 [32]
$V_S = \pi(2^2)(8) \approx 100.5$ cm^3
$V_M = \frac{1}{3}\pi(3.5^2)(12.5) \approx 160.4$ cm^3;
$160.4 - 100.5 \approx 60$ cm^3

16.4 Density

1. CC JUN '15 [7] Ans: 3
2. CC AUG '15 [16] Ans: 1
3. CC JAN '16 [19] Ans: 2
4. CC JUN '16 [18] Ans: 2
5. CC JUN '16 [20] Ans: 1
6. CC AUG '16 [17] Ans: 2
7. CC AUG '19 [14] Ans: 2
8. CC JAN '20 [14] Ans: 1
9. CC JUN '22 [12] Ans: 1
10. CC AUG '22 [21] Ans: 1
11. CC JAN '23 [6] Ans: 2
12. CC JAN '23 [20] Ans: 3
13. CC AUG '23 [12] Ans: 2
14. CC JAN '24 [24] Ans: 2
15. CC FALL '14 [6]
$5.1 \cdot 10.2 \cdot 20.3 = 1{,}056.006$ cm^3
$500 \cdot 1{,}056.006 = 528{,}003$ cm^3
$528{,}003\ \cancel{cm^3} \cdot \frac{1\ m^3}{100\ \cancel{cm^3}} = 0.528003\ m^3$
$\frac{1920\ kg}{\cancel{m^3}} \cdot 0.528003\ \cancel{m^3} \approx 1013\ kg$
No, the weight of the bricks is greater than 900 kg
16. CC FALL '14 [25]
$r = 25\ \cancel{cm} \cdot \frac{1\ m}{100\ \cancel{cm}} = 0.25\ m$
$V = \pi(0.25)^2(10) = 0.625\pi\ m^3$
$W = 0.625\pi\ \cancel{m^3} \cdot \frac{380\ kg}{\cancel{m^3}} \approx 746.1\ kg$
$\$4.75 \times 746.1 \approx \$3{,}544$ per tree
$\frac{\$50{,}000}{\$3{,}544} \approx 14.1$
Need to sell 15 trees
17. CC JUN '15 [35]
$\tan 47° = \frac{x}{8.5}$
$x \approx 9.115$
$V_{cone} = \frac{1}{3}\pi(8.5)^2(9.115) \approx 689.6$
$V_{cylinder} = \pi(8.5)^2(25) \approx 5674.5$
$V_{hemisphere} = \frac{1}{2} \cdot \frac{4}{3}\pi(8.5)^3 \approx 1286.3$
$V_{tower} \approx 689.6 + 5674.5 + 1286.3 \approx$ 7650 ft^3
$85\% \cdot 7650\ \cancel{ft^3} \cdot \frac{62.4\ lbs}{\cancel{ft^3}} = 405{,}756\ lbs$
No, the weight exceeds 400,000 lbs
18. CC AUG '15 [25]
$\frac{137.8}{6^3} \approx 0.638\ g/cm^3$; Ash
19. CC AUG '15 [36]
$V_{cone} = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2(8) \approx 18.85\ in^3$
Total $\approx 100 \times 18.85 \approx 1885\ in^3$
$(0.52 \times 1885) \times \$0.10 = \$98.02$
$(100 \times \$1.95) - (\$98.02 + \$37.83) = \$59.15$
20. CC AUG '16 [36]
$V = \frac{1}{3}\pi\left(\frac{8.3}{2}\right)^2(10.2) + \frac{1}{2} \cdot \frac{4}{3}\pi\left(\frac{8.3}{2}\right)^3$
$\approx 183.961 + 149.693 \approx 333.65$ cm^3
$333.65 \times 50 = 16{,}682.7$ cm^3
$16{,}682.7 \times 0.697 = 11{,}627.8$ g
$11{,}627.8$ g $= 11.6278$ kg
$11.6278 \times 3.83 = \$44.53$

21. CC JAN ‘17 [36]
$V_{cylinder} =$
$\pi(26.7)^2(750) - \pi(24.2)^2(750) =$
$95{,}437.5\pi\ cm^3$
$95{,}437.5\ \cancel{cm^3} \cdot \frac{2.7\ \cancel{g}}{1\ \cancel{cm^3}} \cdot \frac{1\ \cancel{kg}}{1000\ \cancel{g}} \cdot \frac{\$0.38}{1\ \cancel{kg}} =$
\$307.62
$V_{prism} =$
$(40)^2(750) - (35)^2(750) = 281{,}250$
$281{,}250\ \cancel{cm^3} \cdot \frac{2.7\ \cancel{g}}{1\ \cancel{cm^3}} \cdot \frac{1\ \cancel{kg}}{1000\ \cancel{g}} \cdot \frac{\$0.38}{1\ \cancel{kg}} =$
\$288.56
Prisms cost less.
$307.62 - 288.56 = \$19.06$

22. CC JAN ‘18 [29]
$D = \frac{W}{V}$, so $7.95 = \frac{W}{1015}$
$W = (7.95)(1015) = 8{,}069.25$ g
$500 \times 8069.25 = 4{,}034{,}625$ g =
4034.625 kg
$4034.625 \times 0.29 \approx \$1{,}170$

23. CC JUN ‘18 [34]
$V = \pi r^2 = \pi(10)^2(18) = 1800\pi$
$1800\pi\ \cancel{in^3} \cdot \frac{1\ ft^3}{12^3\ \cancel{in^3}} \approx 1.0417\pi\ ft^3$
$1.0417\pi(95.46)(0.85) \approx 266$ lbs
$266 + 270 = 536$ lbs

24. CC AUG ‘18 [34]
Diameter of hollow is $4 - 2(0.5) = 3$.
Radius of outer sphere is 2 and radius of inner sphere is 1.5.
$V = \frac{4}{3}\pi(2^3 - 1.5^3) \approx 19.4$
$19.4 \times 1.308 \times 8 \approx 203$

25. CC JAN ‘20 [27]
$V = (8)(3)\left(\frac{1}{12}\right) = 2$
$43 \times 2 = 86$ lbs

26. CC AUG ‘22 [34]
$\frac{24}{2.94} \approx 8.16; \frac{12}{2.94} \approx 4.08; \frac{18}{2.94} \approx 6.12$
$8 \times 4 \times 6 = 192$ baseballs
$V = \frac{4}{3}\pi(1.47)^3 \approx 13.306$ cu. in.
$192 \times 13.306 \times 0.025 \approx 64$ lbs

27. CC JAN ’24 [33]
$h = \sqrt{16^2 \cdot 6^2} = \sqrt{220}$
$V = \frac{1}{3}(12)^2\sqrt{220} \approx 712$ cm^3
$712 \times 0.032 \approx 23$ oz

16.5 Lateral Area and Surface Area

1. CC JUN ‘15 [19] Ans: 2

16.6 Rotations of Two-Dimensional Objects

1. CC JUN ‘15 [1] Ans: 4
2. CC AUG ‘15 [3] Ans: 4
3. CC JUN ‘16 [1] Ans: 3
4. CC AUG ‘16 [3] Ans: 1
5. CC JUN ‘17 [18] Ans: 1
6. CC AUG ‘17 [13] Ans: 3
7. CC JAN ‘18 [10] Ans: 4
8. CC JUN ‘18 [16] Ans: 3
9. CC AUG ‘18 [3] Ans: 4
10. CC JAN ‘19 [11] Ans: 3
11. CC JUN ‘19 [3] Ans: 2
12. CC AUG ‘19 [11] Ans: 4
13. CC JUN ‘22 [8] Ans: 1
14. CC JAN ‘23 [2] Ans: 3
15. CC AUG ‘23 [7] Ans: 3
16. CC AUG ‘22 [26]
$V_{cone} = \frac{1}{3}\pi(8)^2(5) \approx 335.1$
17. CC JAN ’24 [25]
$V_{cone} = \frac{1}{3}\pi(5)^2(12) \approx 314$ cm^3

16.7 Cross Sections

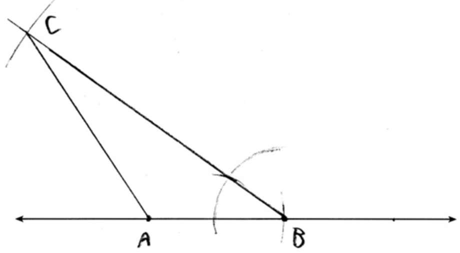
1. CC JUN ‘15 [6] Ans: 2
2. CC JAN ‘16 [1] Ans: 1
3. CC AUG ‘16 [13] Ans: 3
4. CC JAN ‘17 [23] Ans: 4
5. CC AUG ‘17 [1] Ans: 2
6. CC JAN ‘18 [5] Ans: 2

7.	CC AUG ‘18 [5]	Ans: 3
8.	CC JAN ‘20 [19]	Ans: 4
9.	CC JUN ‘22 [2]	Ans: 2
10.	CC AUG ‘22 [11]	Ans: 1
11.	CC JUN ‘23 [1]	Ans: 2
12.	CC AUG ‘23 [1]	Ans: 4
13.	CC JAN ’24 [15]	Ans: 4

CHAPTER 17. CONSTRUCTIONS

17.1 Copy Segments, Angles, and Triangles

1. CC JAN '16 [34]
   solutions vary, such as



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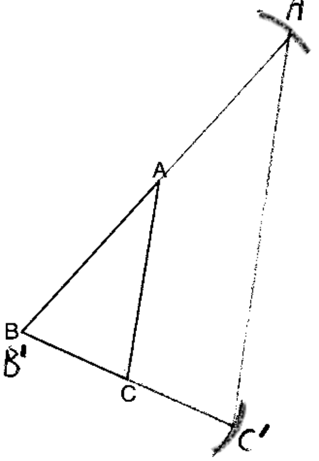
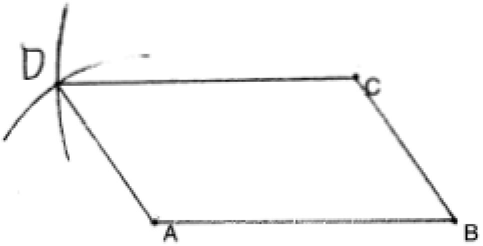
2. CC AUG '16 [32]

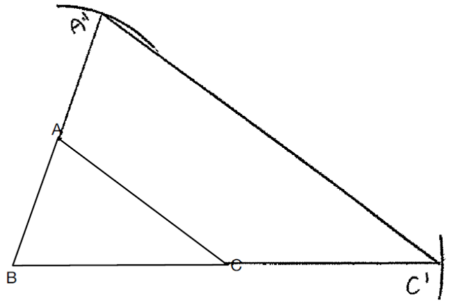


$A'C'$ is twice AC

3. CC JAN '19 [29]
   copy length of $\overline{AB}$ from C and length of $\overline{BC}$ from A for point D at intersecting arcs

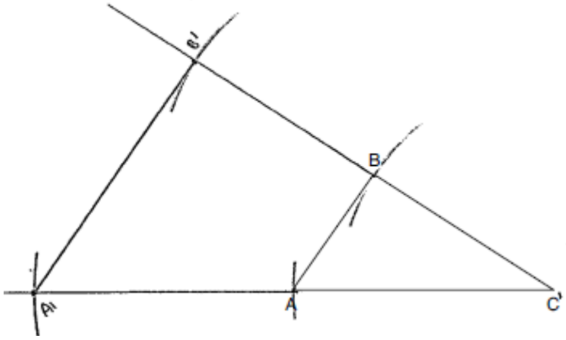


4. CC AUG '19 [32]



Yes, because a dilation preserves ∠'s.

5. CC AUG '22 [27]



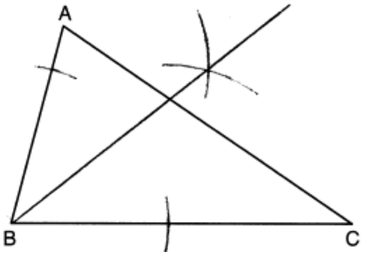
17.2 Construct an Equilateral Triangle

There are no Regents exam questions on this topic.

17.3 Construct an Angle Bisector

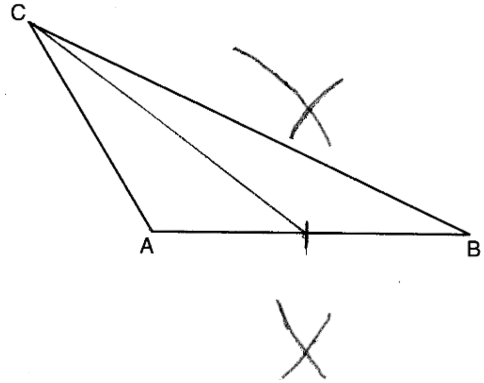
1. CC AUG '19 [29]
$\triangle ABC$ is an equilateral $\triangle$,
so $\text{m}\angle CAB = 60°$;
$\overrightarrow{AD}$ is an $\angle$ bisector, so $\text{m}\angle CAD = 30°$.

2. CC JAN '23 [25]

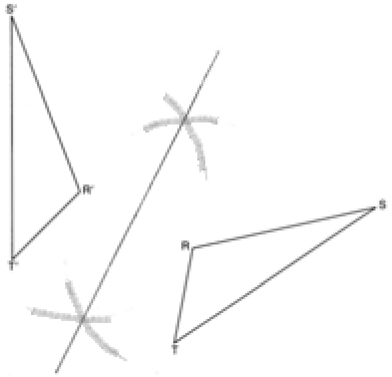


17.4 Construct a Perpendicular Bisector

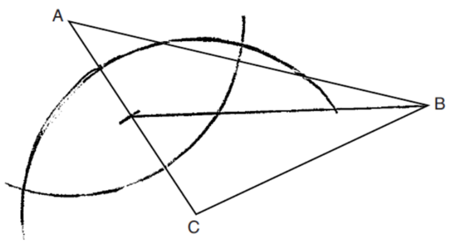
1. CC AUG '16 [28]



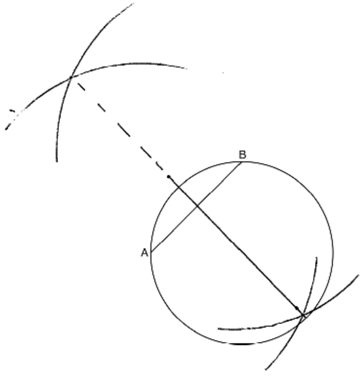
2. CC JAN '17 [25]



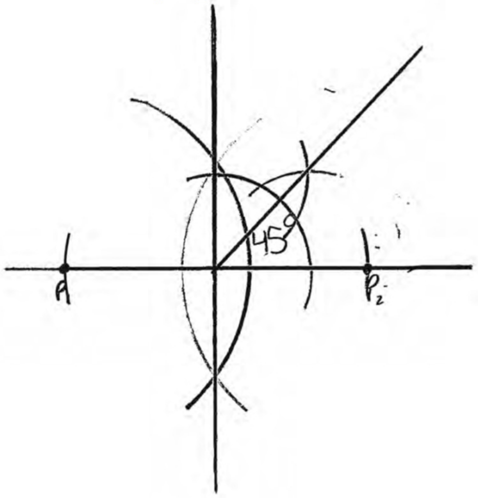
3. CC JUN '18 [29]



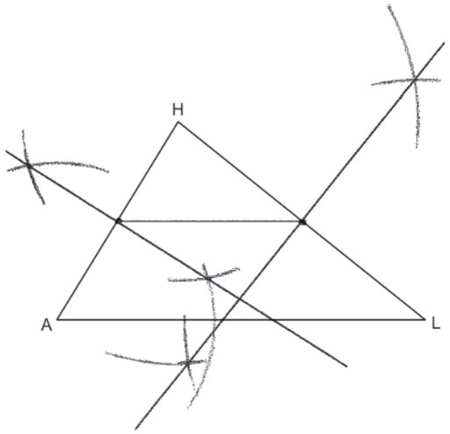
4. CC AUG '18 [25]



5. CC JAN '20 [29]

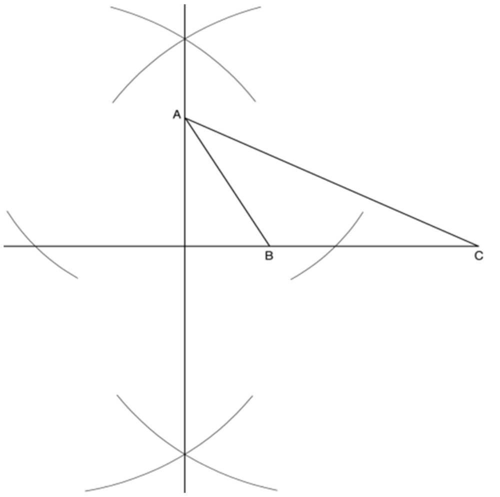


6. CC AUG ‘23 [29]

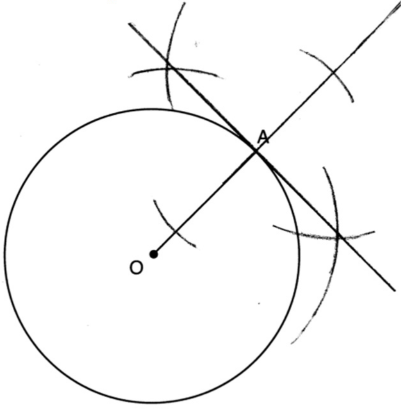


17.5 Construct Lines Through a Point

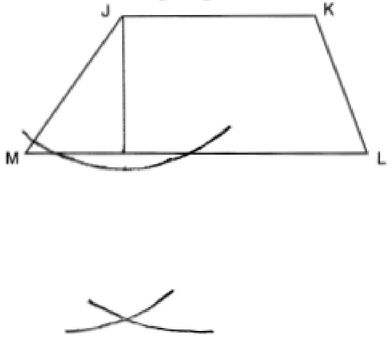
1. CC FALL ‘14 [9]
The altitude is ⊥ to the extended side.



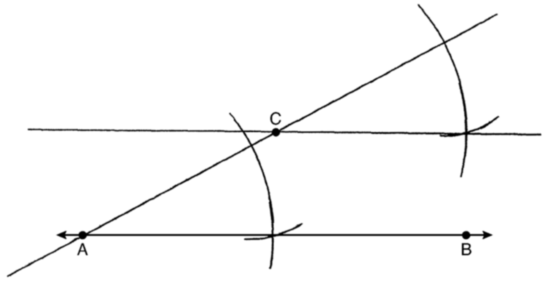
2. CC JUN ‘16 [31]
The tangent is ⊥ to $\overline{OA}$.



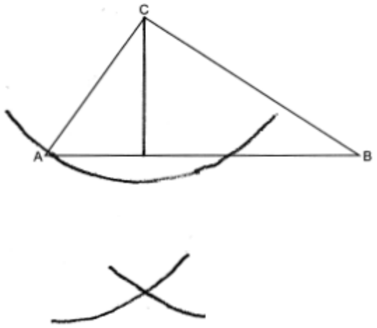
3. CC JUN ‘17 [25]



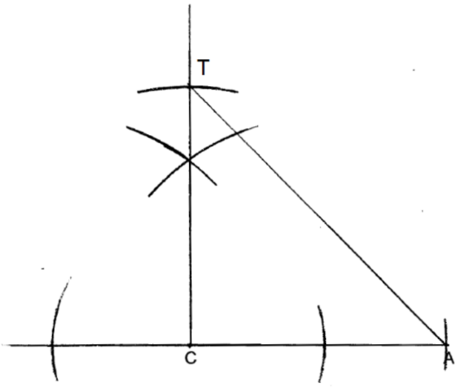
4. CC JUN ‘22 [31]



5. CC JUN ‘23 [25]

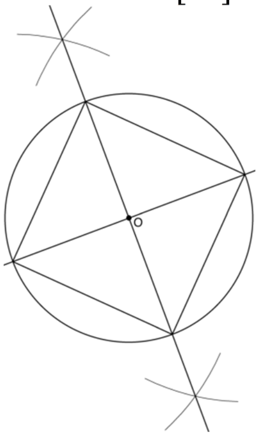


6. CC JAN ’24 [27]



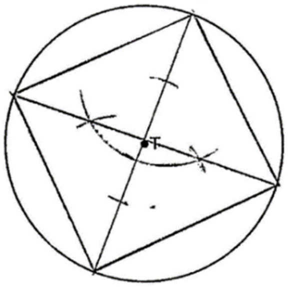
17.6 Construct Inscribed Regular Polygons

1. CC FALL ‘14 [12]

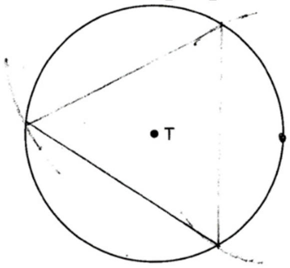


The sides of the square are four ≅ chords in the circle, so they intercept four ≅ arcs. Each arc therefore measures one-fourth of 360°, or 90°. Therefore, an arc intercepted by two adjacent sides measures $2 \times 90° = 180°$.

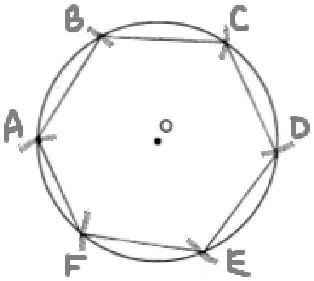
2. CC JUN ‘15 [25]



3. CC AUG ‘15 [26]

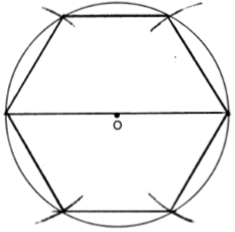


4. CC JAN ‘17 [33]

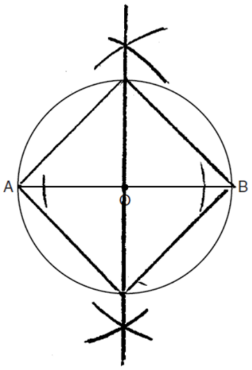


$\overline{COF}$ is a diameter, so $\angle FBC$ is an inscribed ∠ of a semicircle, and is therefore a right ∠. This means $\triangle FBC$ is a right △.

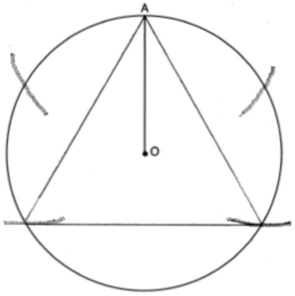
5. CC AUG ‘17 [28]



6. CC JAN ‘18 [26]



7. CC JUN ‘19 [31]



17.7 Construct Points of Concurrency

There are no Regents exam questions on this topic.

17.8 Construct Circles of Triangles

There are no Regents exam questions on this topic.