Answer Key

Geometry
Course Workbook

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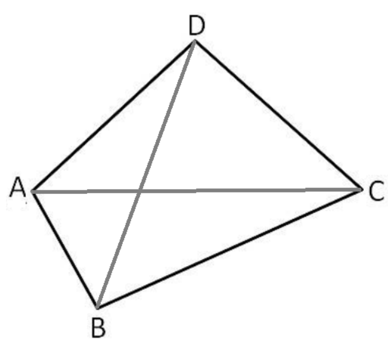
17. Diagonals $\overline{AC}$ and $\overline{BD}$.  	18. a) Triangles ABE, CDE, and BCE. b) Quadrilaterals $ABCD$, $ABCE$, and $BCDE$.
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1.2. Pythagorean Theorem

1. $5^2 + 7^2 = c^2$ $74 = c^2$ $c = \sqrt{74} \approx 8.6$	2. $24^2 + b^2 = 26^2$ $576 + b^2 = 676$ $b^2 = 100$ $b = 10$ cm
3. $x^2 + 19.5^2 = 20^2$ $x^2 + 380.25 = 400$ $x^2 = 19.75$ $x \approx 4.4$ ft.	4. $9^2 + b^2 = 18^2$ $b^2 = 243$ $b \approx 15.6$
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6. $x^2 + 7^2 = (x+1)^2$ $x^2 + 49 = x^2 + 2x + 1$ $2x = 48$ $x = 24$ Vertical bar is $x + 1 = 25$ inches	

1.3. Perimeter and Circumference

1. $x + (x+2) + (x-3)$ Perimeter is $3x - 1$	2. $4x + (x+3) + 3x - 1 = 34$ $8x + 2 = 34$ $8x = 32$ $x = 4$ $4x = 16, x + 3 = 7, 3x - 1 = 11$
3. Length of □ = diameter = 8 in. Perimeter = $\frac{1}{2} 2\pi r + l + 2w$ $= \frac{1}{2} 8\pi + 8 + 14 = 4\pi + 22 \approx 34.6$ in.	4. Width of □ = radius = 2 cm. Perimeter = $\frac{1}{2} 2\pi r + r + 2l + w$ $= \frac{1}{2} 4\pi + 2 + 8 + 2 = 2\pi + 12$ ≈ 18.3 cm.
5. $\frac{1}{2} 15\pi + 60 + 15 = 7.5\pi + 75 \approx 98.6$ ft.	6. $\frac{1}{2} 4\pi + 4 \cdot 4 = 2\pi + 16 \approx 22.3$ cm.
7. $\frac{1}{2} 12\pi + 8 + 14 = 6\pi + 22$ m	8. $\frac{1}{2} 6\pi + 4 + 10 + 6 = 3\pi + 20$
9. All sides of the polygon are 3.5. Perimeter $= 3.5 \times 7 = 24.5$ inches	10. Each side $s = 2r = 10$ Perimeter = $4\left(\frac{1}{4} 2\pi r\right) + 4s = 10\pi + 40$

11. Length of arc $SBT = \frac{1}{4}2\pi r = \frac{1}{4}12\pi = 3\pi$. Let $w = RC$, so $AR = 8 - w$. $CT = r - RC = 6 - w$ $AS = r - AR = 6 - (8 - w) = w - 2$ Since $ABCR$ is a rectangle, its diagonals are $\cong$. Diagonal RB is a radius, so $AC = RB = 6$. Perimeter $= \widehat{SBT} + CT + AS + AC = 3\pi + (6 - w) + (w - 2) + 6 = 3\pi + 10$

1.4. Area

1. $\frac{1}{2}\pi r^2 + s^2 = \frac{1}{2}36\pi + 12^2 = 18\pi + 144$	
2. $\pi r^2 + lw = 5^2\pi + (20)(10) = 25\pi + 200$	
3. $\frac{3}{4}\pi r^2 + s^2 = \frac{3}{4}\pi(4)^2 + 4^2 = 12\pi + 16$	
4. $\pi r^2 + s^2 = \pi x^2 + (2x)^2 = \pi x^2 + 4x^2$ or $(\pi + 4)x^2$	
5. $s^2 - \pi r^2 = 8^2 - 4^2\pi = 64 - 16\pi$	6. $s = \sqrt{36} = 6$ $\quad r = \frac{1}{2}s = 3$ $A = \pi r^2 = 9\pi$
7. $A = s^2 - \frac{1}{2}\pi r^2 = 6^2 - \frac{1}{2}9\pi = 36 - 4.5\pi$	8. $lw - 2\pi r^2 = (20)(10) - 2(5^2)\pi =$ $200 - 50\pi$
9. $\frac{1}{4}\pi r^2 - \frac{1}{2}bh = \frac{1}{4}(4^2)\pi - \frac{1}{2}(4)(4) =$ $4\pi - 8$	10. $lw - \frac{1}{2}bh = (12)(4) - \frac{1}{2}(9)(4) = 30$
11. $s^2 - \pi r^2 = 6^2 - 3^2\pi = 36 - 9\pi$ sq. in.	
12. $lw - \pi r^2 = (36)(15) - 4^2\pi = 540 - 16\pi \approx 490$ sq. ft. $\quad 490 \times 1.95 = \$955.50$	
13. $(80 - 70)^2 + (40 - 15)^2 = c^2$ $10^2 + 25^2 = c^2$ $725 = c^2$ $c = \sqrt{725} \approx 26.93$ $\quad$ $Area \approx 26.93 \times 6 \approx 162$ *sq. ft.*	
14. $density = \frac{population}{area}$ $27{,}785 = \frac{x}{303}$ $x \approx 8{,}419{,}000$	15. $A = 2000 \times 500 = 1{,}000{,}000$ sq. ft. $1{,}000{,}000\ ft^2 \times \frac{1\ acre}{43{,}560\ ft^2} \approx 22.957\ acres$ $d = \frac{6{,}887}{22.957} \approx 300$ trees per acre
16. a) $\frac{14.7}{100{,}000} = \frac{x}{1{,}632{,}000}$ $100{,}000x = 23{,}990{,}400$ $x \approx 240$	b) $d = \frac{240}{22.8} \approx 10.5$ per square mile

Chapter 2. Coordinate Geometry

2.1. Forms of Linear Equations

1. No. $(3) = 3(-2) + 15?$ $3 \neq 9$	2. $y = mx + b$ $y = -4x + 5$
3. $y - y_1 = m(x - x_1)$ $y + 2 = -3(x - 1)$	4. $m = \frac{-5+3}{5+2} = -\frac{2}{7}$ $y - y_1 = m(x - x_1)$ $y + 3 = -\frac{2}{7}(x + 2)$
5. $m = \frac{5-3}{8-1} = \frac{2}{7}$ $y - 3 = \frac{2}{7}(x - 1)$	6. $m = \frac{4-0}{5-(-5)} = \frac{2}{5}$ $y - 4 = \frac{2}{5}(x - 5)$
7. $-2x + y = -5$ $2x - y = 5$	8. $4y = 4\left(\frac{3}{4}x + \frac{1}{2}\right)$ $4y = 3x + 2$ $-3x + 4y = 2$ $3x - 4y = -2$

2.2. Parallel and Perpendicular Lines

1. equation (1) $2y + 2x = 6$ $2y = -2x + 6$ $y = -x + 3$	2. equation (1) $-3y = 2x + 5$ $-2x - 3y = 5$ $-2(-2x - 3y = 5)$ $4x + 6y = -10$
3. equation (2)	4. equation (1) $x - 5y = 25$ $-5y = -x + 25$ $y = \frac{1}{5}x - 5$
5. $y = -2x + 2$	6. $y = -2x$
7. $2y - x = 8$ $(-1) = \frac{1}{2}(4) + b$ $2y = x + 8$ $b = -3$ $y = \frac{1}{2}x + 4$ $m = \frac{1}{2}$ $y = \frac{1}{2}x - 3$	8. $m = -3$ $(12) = -3(-9) + b$ $b = -15$ $y = -3x - 15$
9. $3y = 6x + 3$ $(4) = -\frac{1}{2}(2) + b$ $y = 2x + 1$ $b = 5$ $m = -\frac{1}{2}$ $y = -\frac{1}{2}x + 5$	10. The slope of $y = 2x + 3$ is 2. The slope of $2y + x = 6$ is $-\frac{1}{2}$. Since the slopes are opp reciprocals, the lines are $\perp$.

11. The slope of $x + 2y = 4$ is $-\frac{1}{2}$. The slope of $4y - 2x = 12$ is $\frac{1}{2}$. Since the slopes are neither = nor opp reciprocals, the lines are neither ∥ nor ⊥.	12. $\frac{x-1}{4} = \frac{-3}{8}$ $2(x-1) = -3$ $2x - 2 = -3$ $x = -\frac{1}{2}$

2.3. Distance Formula

1. $d = \sqrt{(6+2)^2 + (-3-3)^2}$ $= \sqrt{64+36} = \sqrt{100} = 10$	2. $d = \sqrt{(8-3)^2 + (10-5)^2}$ $= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$
3. $d = \sqrt{(7+1)^2 + (4-9)^2}$ $= \sqrt{64+25} = \sqrt{89}$	4. $d = \sqrt{(1-5)^2 + (6-3)^2}$ $= \sqrt{16+9} = \sqrt{25} = 5$
5. $d = \sqrt{(7-3)^2 + (2+4)^2}$ $= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$	6. $d = \sqrt{(6-3)^2 + (-1-8)^2}$ $= \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$
7. $d = \sqrt{(4+3)^2 + (25-1)^2}$ $= \sqrt{49+576} = \sqrt{625} = 25$	8. $d = \sqrt{(146+4)^2 + (52-2)^2}$ $= \sqrt{22{,}500 + 2{,}500} = \sqrt{25{,}000} = 50\sqrt{10}$

9. $m_\perp = -5$

Equation of ⊥ line:	Solve system for x:	Solve for y:	Distance for $(6,-2)$ to $(5,3)$:
$y + 2 = -5(x - 6)$	$-5x + 28 = \frac{1}{5}x + 2$	$y = \frac{1}{5}(5) + 2$	$d = \sqrt{(5-6)^2 + (3+2)^2}$
$y = -5x + 28$	$x = 5$	$y = 3$	$= \sqrt{1+25} = \sqrt{26}$

2.4. Midpoint Formula

1. $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+6}{2}, \frac{3-3}{2}\right) = (2,0)$	2. $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3+8}{2}, \frac{5+10}{2}\right) =$ $\left(5\frac{1}{2}, 7\frac{1}{2}\right)$
3. $\left(\frac{-5+3}{2}, \frac{1+5}{2}\right) = (-1,3)$	
4. The third terms of the arithmetic sequences x: 1, 3, ... and y: −5, 5, ... are 5 and 15, so $B(5,15)$.	5. M is the midpoint of BC. Sequences x: 4, 0, ... and y: −3, −2, ... give us third terms of −4 and −1, so $C(-4,-1)$.
6. x: 1, 3.5, 6 and y: 8, 2, −4, so $R(6,-4)$	7. x: −1, 2, 5 and y: 5, 3, 1, so $(5,1)$

2.5. Perpendicular Bisectors

1. Midpoint is $\left(\frac{3+9}{2},\frac{5+17}{2}\right)=(6,11)$ $m=\frac{17-5}{9-3}=\frac{12}{6}=2 \qquad m_{\perp}=-\frac{1}{2}$ Equation of ⊥ bisector is $y-11=-\frac{1}{2}(x-6)$	2. Midpoint is $\left(\frac{-2+6}{2},\frac{3-3}{2}\right)=(2,0)$ $m=\frac{-3-3}{6+2}=\frac{-6}{8}=-\frac{3}{4} \qquad m_{\perp}=\frac{4}{3}$ Equation of ⊥ bisector is $y=\frac{4}{3}(x-2)$
3. Midpoint is $\left(\frac{2+8}{2},\frac{6+12}{2}\right)=(5,9)$ $m=\frac{12-6}{8-2}=\frac{6}{6}=1 \qquad m_{\perp}=-1$ Equation of ⊥ bisector is $y-9=-(x-5)$ $y-9=-x+5$ $y=-x+14$ y-intercept is 14	4. Midpoint is $\left(\frac{-4+2}{2},\frac{5+5}{2}\right)=(-1,5)$ The segment is horizontal with a 0 slope. Its ⊥ bisector is vertical, with an equation of $x=-1$.
5. $M=\left(\frac{4+8}{2},\frac{2+6}{2}\right)=(6,4)$ $m=\frac{6-2}{8-4}=1 \qquad m_{\perp}=-1$ $y-4=-(x-6)$	6. $M=\left(\frac{-1+7}{2},\frac{1-5}{2}\right)=(3,-2)$ $m=\frac{1-(-5)}{-1-7}=-\frac{3}{4} \qquad m_{\perp}=\frac{4}{3}$ $y+2=\frac{4}{3}(x-3)$
7. $M=\left(\frac{3+3}{2},\frac{-1+5}{2}\right)=(3,2)$ Line segment is vertical, so its ⊥ bisector is horizontal, with equation $y=2$.	8. $M=\left(\frac{0+6}{2},\frac{0+0}{2}\right)=(3,0)$ Line segment is horizontal, so its ⊥ bisector is vertical, with equation $x=3$.

2.6. Directed Line Segments

1. $k=\frac{5}{6}$ $P_x=A_x+\frac{5}{6}(B_x-A_x)=3+\frac{5}{6}(9-3)=8$ $P_y=A_y+\frac{5}{6}(B_y-A_y)=5+\frac{5}{6}(17-5)=15$ $P(8,15)$	2. $k=\frac{3}{5}$ $S_x=R_x+\frac{3}{5}(T_x-R_x)=-2+\frac{3}{5}(3+2)=1$ $S_y=R_y+\frac{3}{5}(T_y-R_y)=2+\frac{3}{5}(-8-2)=-4$ $S(1,-4)$
3. $k=\frac{3}{8}$ $P_x=L_x+\frac{3}{8}(M_x-L_x)=-2+\frac{3}{8}(6+2)=1$ $P_y=L_y+\frac{3}{8}(M_y-L_y)=3+\frac{3}{8}(-3-3)=\frac{3}{4}$ $P\left(1,\frac{3}{4}\right)$	4. $k=\frac{2}{5}$ $G_x=F_x+\frac{2}{5}(H_x-F_x)=1+\frac{2}{5}(6-1)=3$ $G_y=F_y+\frac{2}{5}(H_y-F_y)=-3+\frac{2}{5}(5+3)=\frac{1}{5}$ $G\left(3,\frac{1}{5}\right)$

Chapter 3. Polygons in the Coordinate Plane

3.1. Triangles in the Coordinate Plane

1. $m_{\overline{AB}} = \frac{11-7}{5-2} = \frac{4}{3}$ $m_{\overline{BC}} = \frac{8-11}{9-5} = -\frac{3}{4}$ $\overline{AB} \perp \overline{BC}$, so $\angle B$ is a right $\angle$. Therefore, ABC is a right $\triangle$.	2. $AB = \sqrt{(5-2)^2 + (11-7)^2} = \sqrt{25} = 5$ $BC = \sqrt{(9-5)^2 + (8-11)^2} = \sqrt{25} = 5$ $AC = \sqrt{(9-2)^2 + (8-7)^2} = \sqrt{50} = 5\sqrt{2}$ Two sides are equal in length, so $\triangle ABC$ is an isosceles $\triangle$.
3. $DE = \sqrt{(5-4)^2 + (5+2)^2} = \sqrt{50}$ $EF = \sqrt{(-1-5)^2 + (3-5)^2} = \sqrt{40}$ $DF = \sqrt{(-1-4)^2 + (3+2)^2} = \sqrt{50}$ Two sides are equal in length, so $\triangle DEF$ is an isosceles $\triangle$.	4. $\overline{JK} \perp \overline{KL}$ $m_{\overline{JK}} = \frac{6-4}{6+2} = \frac{1}{4}$, so $m_{\overline{KL}} = -4$ $\frac{-2-6}{x-6} = -4$ $-8 = -4(x-6)$ $-8 = -4x + 24$ $-32 = -4x$ $x = 8$
5. Recognize that $\overline{PR}$ is horizontal (since $y = -2$ for both endpoints). $PR = 8 - 4 = 4$ An altitude may be drawn from the opp vertex, Q, to point $S(-6, -2)$. $QS = 4 - (-2) = 6$ Therefore, the area is $\frac{1}{2}(4)(6) = 12$ square units.	6. $AB = \sqrt{(5-2)^2 + (1-2)^2} = \sqrt{10}$ $DE = \sqrt{(4-1)^2 + (-5+4)^2} = \sqrt{10}$ $BC = \sqrt{(4-5)^2 + (5-1)^2} = \sqrt{17}$ $EF = \sqrt{(3-4)^2 + (-1+5)^2} = \sqrt{17}$ $AC = \sqrt{(4-2)^2 + (5-2)^2} = \sqrt{13}$ $DF = \sqrt{(3-1)^2 + (-1+4)^2} = \sqrt{13}$ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, so $\triangle ABC \cong \triangle DEF$ by SSS.
7. $m_{\overline{RS}} = \frac{-1-7}{3+1} = -2 \quad m_{\overline{ST}} = \frac{2+1}{9-3} = \frac{1}{2}$ The slopes are opp reciprocals, so they are $\perp$ lines forming a right $\angle$ at S. Since $\angle S$ is a right $\angle$, $\triangle RST$ is a right $\triangle$.	8. To prove that $\triangle JEN$ is a right $\triangle$, prove that its legs are $\perp$ by showing their slopes are opp reciprocals: $m_{\overline{JE}} = \frac{-3-1}{-2+4} = -2 \quad m_{\overline{EN}} = \frac{-1+3}{2+2} = \frac{1}{2}$ To prove that $\triangle JEN$ is an isosceles $\triangle$, prove that it legs are $\cong$ by using the distance formula: $JE = \sqrt{(-2+4)^2 + (-3-1)^2} = \sqrt{20}$ $EN = \sqrt{(2+2)^2 + (-1+3)^2} = \sqrt{20}$

3.2. Quadrilaterals in the Coordinate Plane

1. $\left(\frac{1+10}{2}, \frac{3+7}{2}\right) = \left(\frac{11}{2}, 5\right)$

   [diagonals bisect each other, so E is the midpoint of both $\overline{AC}$ and $\overline{BD}$]

2. $m_{\overline{AB}} = \frac{-8-0}{-1+5} = -2 \quad m_{\overline{CD}} = \frac{4+4}{3-7} = -2$

   $m_{\overline{BC}} = \frac{-4+8}{7+1} = \frac{1}{2} \quad m_{\overline{AD}} = \frac{4-0}{3+5} = \frac{1}{2}$

   $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, and $\overline{AD} \perp \overline{AB}$, so all 4 ∠'s are right ∠'s.

   Therefore, $ABCD$ is a ▭.

3. $AB = \sqrt{(1+6)^2 + (0+3)^2} = \sqrt{58}$

   $BC = \sqrt{(4-1)^2 + (7-0)^2} = \sqrt{58}$

   $CD = \sqrt{(-3-4)^2 + (4-7)^2} = \sqrt{58}$

   $AD = \sqrt{(-3+6)^2 + (4+3)^2} = \sqrt{58}$

   $m_{\overline{AB}} = \frac{0+3}{1+6} = \frac{3}{7} \quad m_{\overline{BC}} = \frac{7-0}{4-1} = \frac{7}{3}$

   4 ≅ sides but not 4 right ∠'s → Rhombus

4. $AB = \sqrt{(2+5)^2 + (0+6)^2} = \sqrt{85}$

   $BC = \sqrt{(11-2)^2 + (9-0)^2} = \sqrt{162}$

   $CD = \sqrt{(4-11)^2 + (3-9)^2} = \sqrt{85}$

   $AD = \sqrt{(4+5)^2 + (3+6)^2} = \sqrt{162}$

   $m_{\overline{AB}} = \frac{0+6}{2+5} = \frac{6}{7} \quad m_{\overline{BC}} = \frac{9-0}{11-2} = 1$

   2 pairs of opp sides ≅ but not 4 right ∠'s → ▱

5. $AB = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{17}$

   $BC = \sqrt{(6-5)^2 + (-2-2)^2} = \sqrt{17}$

   $CD = \sqrt{(2-6)^2 + (-3+2)^2} = \sqrt{17}$

   $AD = \sqrt{(2-1)^2 + (-3-1)^2} = \sqrt{17}$

   $m_{\overline{AB}} = \frac{2-1}{5-1} = \frac{1}{4} \quad m_{\overline{BC}} = \frac{-2-2}{6-5} = -4$

   $\overline{AB} \perp \overline{BC}$, so ∠$B$ is a right ∠.

   4 ≅ sides and a right ∠ → Square

6. $m_{\overline{AB}} = \frac{4-1}{4-2} = \frac{3}{2}$

   Since $\overline{AB} \perp \overline{BC}$, $m_{\overline{BC}} = -\frac{2}{3}$.

7. $PR = \sqrt{(7+4)^2 + (-5-0)^2} = \sqrt{146}$

   $QS = \sqrt{(-1-4)^2 + (-8-3)^2} = \sqrt{146}$

   Diagonals are congruent.

   $m_{\overline{PR}} = \frac{-5-0}{7+4} = -\frac{5}{11} \quad m_{\overline{QS}} = \frac{-8-3}{-1-4} = \frac{11}{5}$

   Diagonals are ⊥.

   Midpoint of $\overline{PR}$ is $\left(\frac{-4+7}{2}, \frac{0-5}{2}\right) = \left(\frac{3}{2}, -\frac{5}{2}\right)$

   Midpoint of $\overline{QS}$ is $\left(\frac{4-1}{2}, \frac{3-8}{2}\right) = \left(\frac{3}{2}, -\frac{5}{2}\right)$

   Diagonals bisect each other.

8. Both pairs of opp sides of a ▱ are ∥.

   Parallel lines have the same slope.

   The slope of side $\overline{BC}$ is 3.

   For side $\overline{AD}$ to have a slope of 3, the coordinates of point D must be (1,3).

   $m_{\overline{AB}} = \frac{2-0}{5-0} = \frac{2}{5} \quad m_{\overline{CD}} = \frac{3-5}{1-6} = \frac{2}{5}$

   $m_{\overline{AD}} = \frac{3-0}{1-0} = 3 \quad m_{\overline{BC}} = \frac{5-2}{6-5} = 3$

9. To prove that $ABCD$ is a rhombus, show that all sides are ≅ using the distance formula:

   $AB = \sqrt{(8+1)^2 + (2+5)^2} = \sqrt{130}$

   $BC = \sqrt{(11-8)^2 + (13-2)^2} = \sqrt{130}$

   $CD = \sqrt{(2-11)^2 + (6-13)^2} = \sqrt{130}$

   $AD = \sqrt{(2+1)^2 + (6+5)^2} = \sqrt{130}$

10. To prove that $ABCD$ is a ▱, show that both pairs of opp sides of the ▱ are ∥ by showing the opp sides have the same slope:

$$m_{\overline{AB}} = \frac{5-2}{6+2} = \frac{3}{8} \qquad m_{\overline{CD}} = \frac{-3-0}{-4-4} = \frac{3}{8}$$
$$m_{\overline{AD}} = \frac{-3-2}{-4+2} = \frac{5}{2} \qquad m_{\overline{BC}} = \frac{0-5}{4-6} = \frac{5}{2}$$

Because $\frac{3}{8}$ and $\frac{5}{2}$ are not opp reciprocals, the consecutive sides of $ABCD$ are not ⊥, and $ABCD$ is not a □.

11. The length of each side of quad is 5. Since each side is ≅, quad $MATH$ is a rhombus. The slope of $\overline{MH}$ is 0 and the slope of $\overline{HT}$ is $-\frac{4}{3}$. Since the slopes are not opp reciprocals, the sides are not ⊥ and do not form rights ∠'s.
Since adjacent sides are not ⊥, quad $MATH$ is not a square.

12. $m_{\overline{AB}} = \frac{6-6}{6+5} = 0 \qquad m_{\overline{CD}} = \frac{-3+3}{-3-8} = 0$

$$m_{\overline{AD}} = \frac{-3-6}{-3+5} = -\frac{9}{2} \qquad m_{\overline{BC}} = \frac{-3-6}{8-6} = -\frac{9}{2}$$

$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ because they have equal slopes. $ABCD$ is a ▱ because opp side are ∥.
$AB = 11$ and $BC = \sqrt{85}$. $ABCD$ is not a rhombus because $AB \neq BC$.
$\overline{AB}$ and $\overline{BC}$ are not ⊥ because their slopes are not opp reciprocals. Therefore, $ABCD$ is not a □ because $\angle B$ is not a right ∠.

13. Use the midpoint formula to find $M(-5,5), N(0,3), P(2,-4), Q(-3,-2)$.
Use the slope formula to find

$$m_{\overline{MN}} = -\frac{2}{5} \qquad m_{\overline{PQ}} = -\frac{2}{5}$$
$$m_{\overline{MQ}} = -\frac{7}{5} \qquad m_{\overline{NP}} = -\frac{7}{5}$$

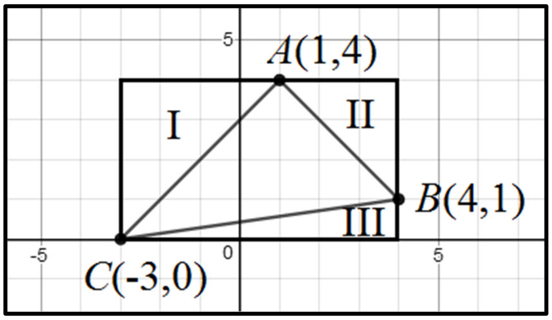
Since both opp sides have equal slopes and are ∥, $MNPQ$ is a ▱.
Use the distance formula to find $MN = \sqrt{29}$ and $NP = \sqrt{53}$.
$MN \neq NP$, so $MNPQ$ is not a rhombus since not all sides are congruent.

3.3. Perimeter and Area using Coordinates

1. $AB = \sqrt{(4-1)^2 + (1-4)^2} = \sqrt{18} = 3\sqrt{2}$
$BC = \sqrt{(-3-4)^2 + (0-1)^2} = \sqrt{50} = 5\sqrt{2}$
$AC = \sqrt{(-3-1)^2 + (0-4)^2} = \sqrt{32} = 4\sqrt{2}$
Perimeter $= 3\sqrt{2} + 5\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$

2. $AB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$
$BC = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10$
$AC = \sqrt{11^2 + (-2)^2} = \sqrt{125} = 5\sqrt{5}$
Perimeter $= 5 + 10 + 5\sqrt{5} = 15 + 5\sqrt{5}$

3. *Method 1: enclose in* ▭



Area of ▭ = 7×4 = 28

Area of I = $\frac{1}{2}(4)(4) = 8$

Area of II = $\frac{1}{2}(3)(3) = 4.5$

Area of III = $\frac{1}{2}(7)(1) = 3.5$

Area of $\triangle ABC = 28 - (8 + 4.5 + 3.5) = 12$ square units

Method 2: area of right △

$m_{\overline{CA}} = \frac{4}{4} = 1$ $m_{\overline{AB}} = -\frac{3}{3} = -1$

Slopes are opp reciprocals, so

$\overline{CA} \perp \overline{AB}$ and $\angle A$ is a right $\angle$.

From problem 1, $AC = 4\sqrt{2}$ and $AB = 3\sqrt{2}$.

Area of right $\triangle ABC = \frac{1}{2}(AC)(AB)$

$= \frac{1}{2}(4\sqrt{2})(3\sqrt{2}) = 12$ square units.

4.

$EF = \sqrt{(6-3)^2 + (10-6)^2}$

$= \sqrt{25} = 5$

$FG = \sqrt{(18-6)^2 + (5-10)^2}$

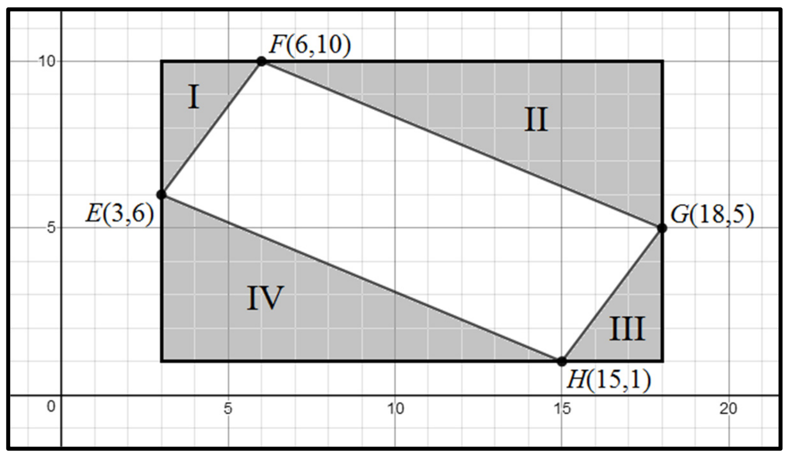
$= \sqrt{169} = 13$

Since $EFGH$ is a ▱,

$GH = EF$ and $EH = FG$.

Perimeter of $EFGH$ =

$2(5) + 2(13) = 36$



Area of ▭ $= 15 \times 9 = 135$

Area of I = Area of III = $\frac{1}{2}(3)(4) = 6$

Area of II = Area of IV = $\frac{1}{2}(5)(12) = 30$

Area of ▱$EFGH = 135 - [2(6) + 2(30)] = 63$ square units

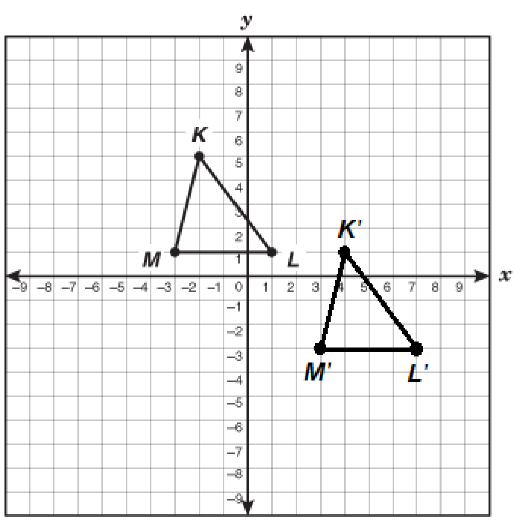
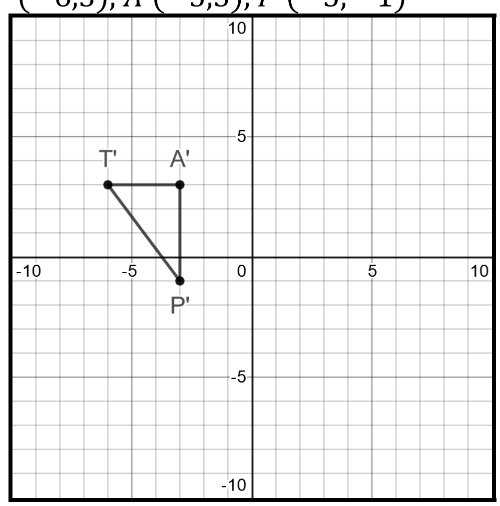
5.

vertex	*x*	*y*	*upper*	*lower*	*difference*
K	−7	−7	−14	35	−49
L	−5	2	−30	6	−36
M	3	6	−9	6	−15
N	1	−3	−7	21	−28
K	−7	−7			

Area $= \frac{|(-49) + (-36) + (-15) + (-28)|}{2} = \frac{|128|}{2} = 64$ square units.

Chapter 4. Rigid Motions

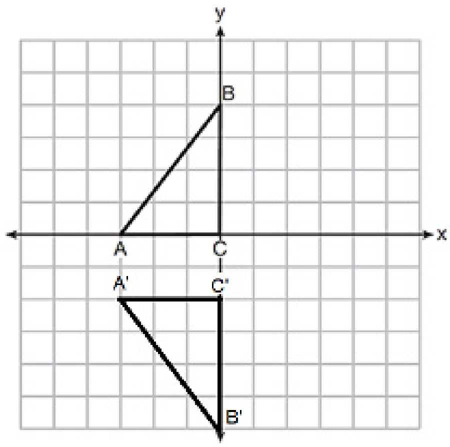
4.1. Translations

1. $(2-6, 4+1) = (-4, 5)$	2. $(-2, -2)$
3. $(x+3, y-7)$	4. $(2, -8)$
5. $(-6,6)$	6. $(x+4, y+4)$
7. $(0, -9)$	8. $T_{2,2}$ of Q is $Q'(6, -4)$
9. $R(-5,-5) \rightarrow R'(3, 0)$ is $T_{8,5}$ $U'(3, 6)$	10. $A(1,3) \rightarrow A'(4,4)$ is $T_{3,1}$ $C'(7,1)$
11. $(-5,5)$	12. $(0,1)$
13. $B(-6,4)$ and $D'(-5, -4)$	
14. 	15. $T'(-6,3)$, $A'(-3,3)$, $P'(-3, -1)$ 

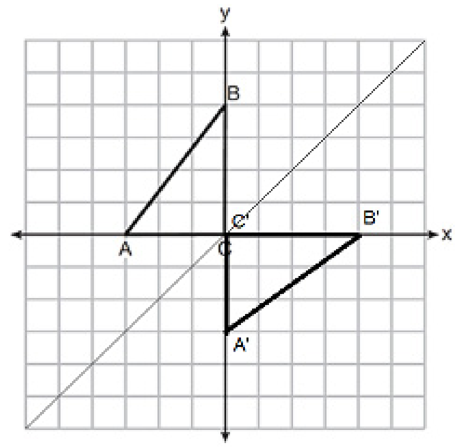
4.2. Line Reflections

1. $(2,3)$	2. $P'(-4,1)$
3. $(2,5)$	4. $(-3,4)$
5. $M'(2,8)$	6. $(-4,3)$
7. $(-2,5)$	8. $A'(0, -2)$ and $B'(4, -6)$

9.

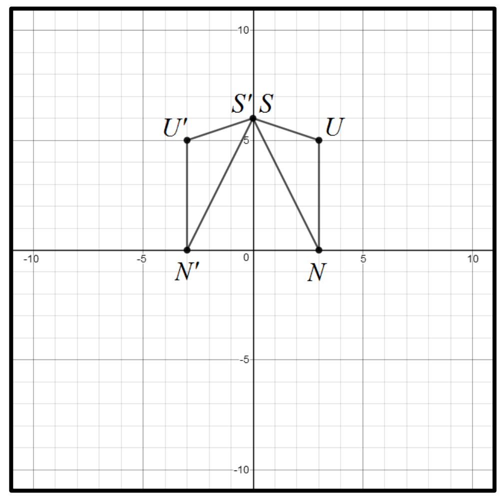


10.



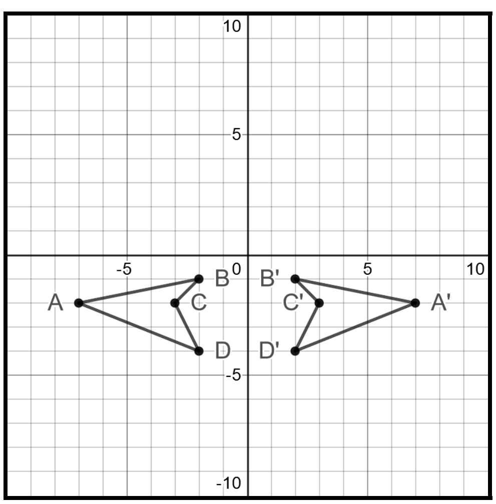
11. $X'(5,1), Y'(4,4), Z'(7,4)$

12.



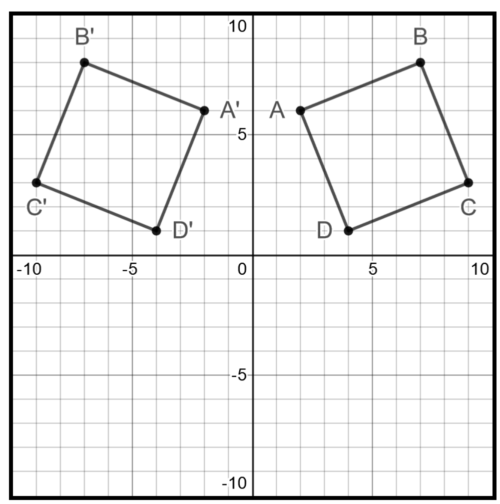
$S'(0,6), U'(-3,5), N'(-3,0)$

13.



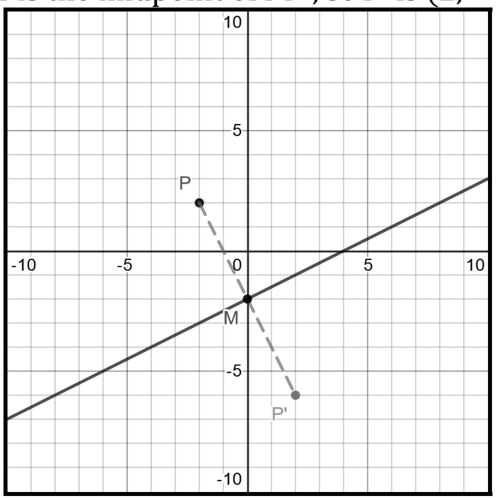
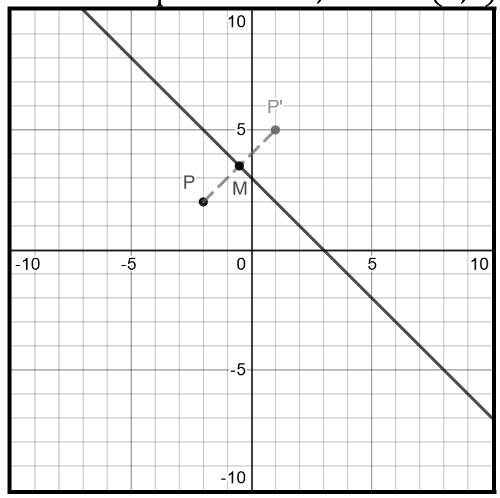
$A'(7,-2), B'(2,-1), C'(3,-2), D'(2,-4)$

14.



$AB = \sqrt{(7-2)^2 + (8-6)^2} = \sqrt{29}$

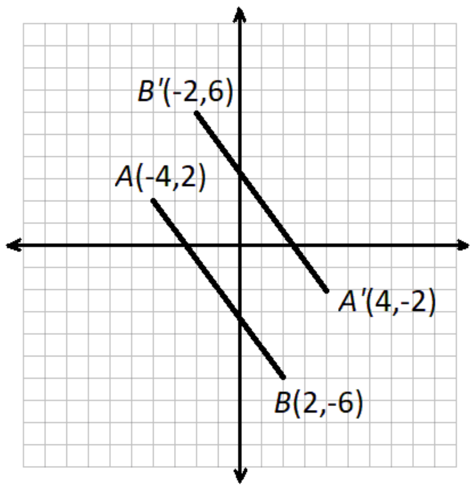
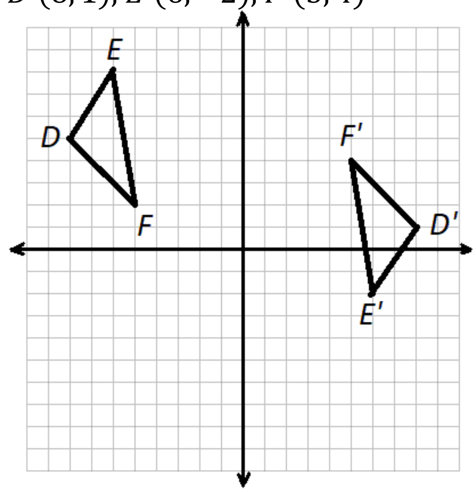
$\text{Area} = \left(\sqrt{29}\right)^2 = 29$

15. Equation of ⊥ line is $y - 2 = -2(x + 2)$, or $y = -2x - 2$ in slope-intercept form. Solving $\frac{1}{2}x - 2 = -2x - 2$ gives us $x = 0$. Substituting for x, $y = -2$, so M is $(0, -2)$. M is the midpoint of $\overline{PP'}$, so P' is $(2, -6)$. 	16. Equation of ⊥ line is $y - 2 = 1(x + 2)$, or $y = x + 4$ in slope-intercept form. Solving $-x + 3 = x + 4$ gives us $x = -\frac{1}{2}$. Substituting for x, $y = \frac{7}{2}$, so M is $\left(-\frac{1}{2}, \frac{7}{2}\right)$. M is the midpoint of $\overline{PP'}$, so P' is $(1,5)$. 

4.3. Rotations

1. (2)	
2. (3)	3. (4)
4. $R_{90°}: (x, y) \rightarrow (-y, x)$ $(2, 4) \rightarrow (-4, 2)$	5. A clockwise rotation of 90° is equivalent to a counterclockwise rotation of 270°. $R_{270°}: (x, y) \rightarrow (y, -x)$ $(-2, 5) \rightarrow (5, 2)$
6. A clockwise rotation of 180° is equivalent to a counterclockwise rotation of 180°. $R_{180°}: (x, y) \rightarrow (-x, -y)$ $(-2, 1) \rightarrow (2, -1)$	7. $R_{180°}: (x, y) \rightarrow (-x, -y)$ $A'(0, 4), B'(-4, 2), C'(-5, 4), D'(-1, 6)$
8. $R_{90°}: (x, y) \rightarrow (-y, x)$ $A'(-2, 1), B'(-3, -4), C'(5, -3)$	9. $R_{90°}: (x, y) \rightarrow (-y, x)$ $P(-2,5) - C(2,3) = (-4,2)$ $(-4,2) \rightarrow (-2, -4)$ $(-2, -4) + C(2,3) = P'(0, -1)$
10. $R_{180°}: (x, y) \rightarrow (-x, -y)$ $P(3, -2) - C(2, -3) = (1,1)$ $(1,1) \rightarrow (-1, -1)$ $(-1, -1) + C(2, -3) = P'(1, -4)$	11. $R_{90°}: (x, y) \rightarrow (-y, x)$ $A(1,2) - P(2, -1) = (-1,3) \rightarrow (-3, -1) + P(2, -1) = A'(-1, -2)$ $B(-4,3) - P(2, -1) = (-6,4) \rightarrow (-4, -6) + P(2, -1) = B'(-2, -7)$ $C(-3, -5) - P(2, -1) = (-5, -4) \rightarrow (4, -5) + P(2, -1) = C'(6, -6)$

4.4. Point Reflections [NG]

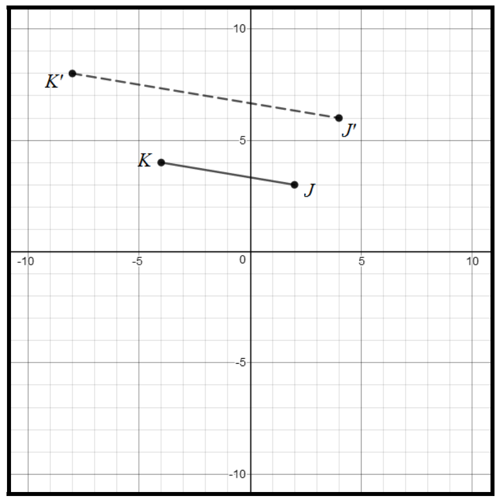
1. $K'(-4,7)$	2. (3,1)
3. $R'(-2,3), S'(-5,-1)$	4. $(2 \cdot 1 - 5, 2 \cdot (-1) - 3) = N'(-3,-5)$
5. $2 \times 6 = 12$ units	6. Use the midpoint formula: $P\left(\frac{3+7}{2}, \frac{6-2}{2}\right) = P(5,2)$
7. 	8. $D'(8,1), E'(6,-2), F'(5,4)$ 

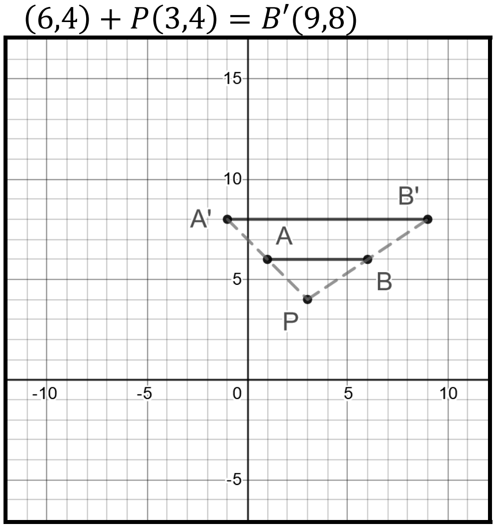
4.5. Map a Polygon onto Itself

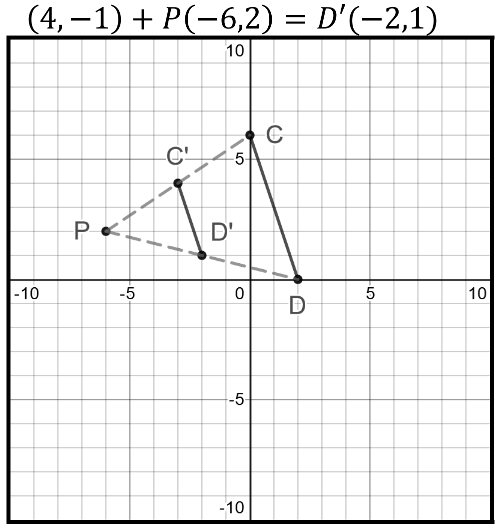
1. (3)	2. 180°
3. $\frac{360}{5} = 72°$	4. $\frac{360}{16} = 22.5°$

Chapter 5. Dilations

5.1. Dilations of Line Segments

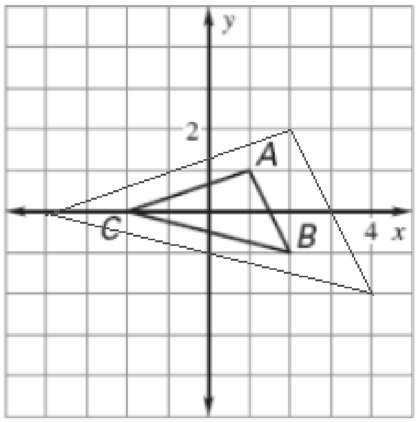
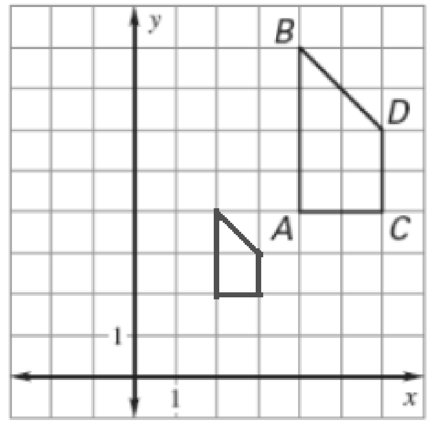
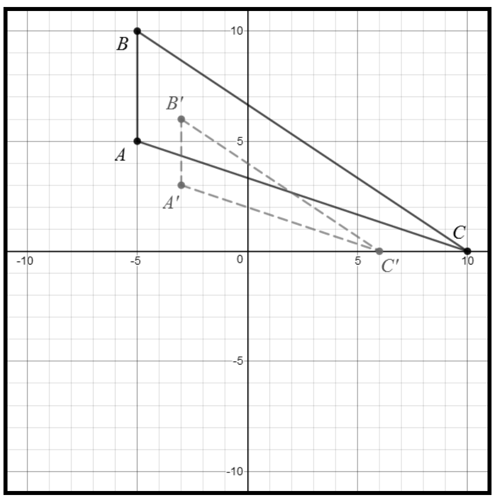
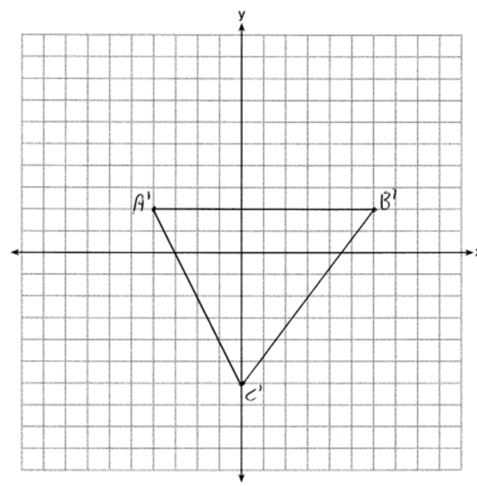
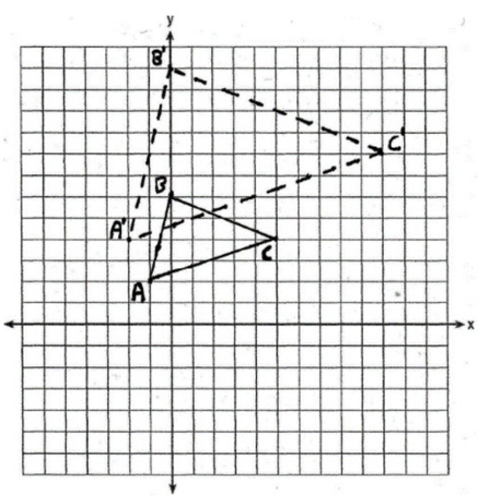
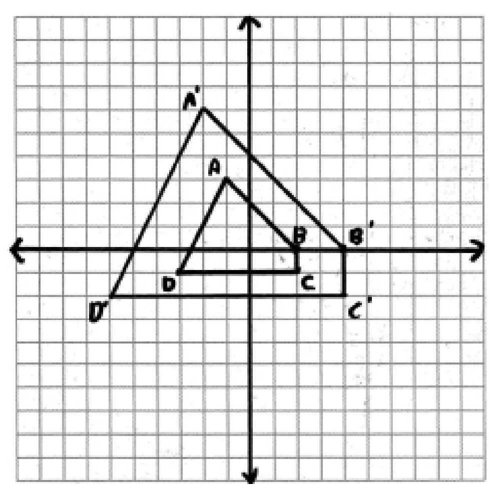
<table>
<tr><td colspan="2">1. (1)</td></tr>
<tr><td>2. $(15, -10)$</td><td>3. $(4, 12)$</td></tr>
<tr><td>4. $\left(\frac{3}{2}, -1\right)$</td><td>5. $(2, 5)$</td></tr>
<tr><td>6. $(12, 0)$</td><td>7.


</td></tr>
<tr><td>8. $A(1,6) - P(3,4) = (-2,2)$
$(-2,2) \rightarrow (-4,4)$
$(-4,4) + P(3,4) = A'(-1,8)$
$B(6,6) - P(3,4) = (3,2)$
$(3,2) \rightarrow (6,4)$
$(6,4) + P(3,4) = B'(9,8)$


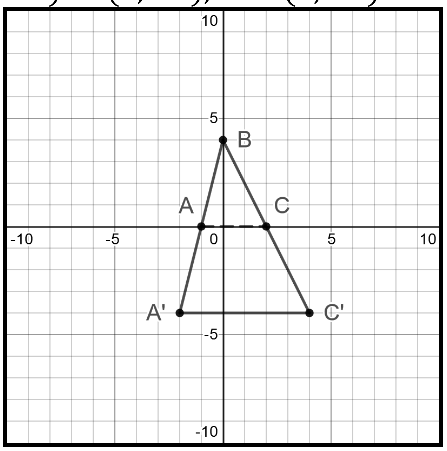
</td><td>9. $C(0,6) - P(-6,2) = (6,4)$
$(6,4) \rightarrow (3,2)$
$(3,2) + P(-6,2) = C'(-3,4)$
$D(2,0) - P(-6,2) = (8,-2)$
$(8,-2) \rightarrow (4,-1)$
$(4,-1) + P(-6,2) = D'(-2,1)$


</td></tr>
</table>

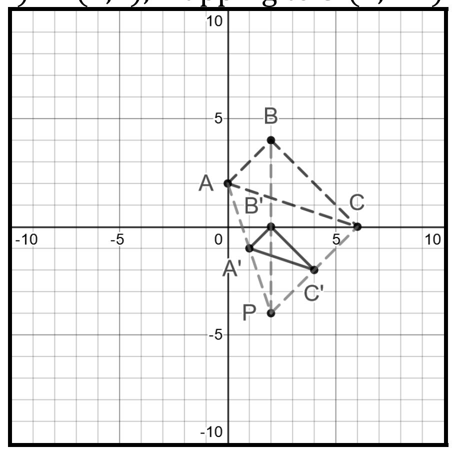
5.2. Dilations of Polygons

1. (3) $m\angle B = m\angle B'$	2. $A'(2,2), B'(3,0), C'(1,-1)$
3. 	4. 
5. 	6. 
7. 	8.  $A'(-2,6), B'(4,0), C'(4,-2), D'(-6,-2)$

9. Using $B(0,4)$ as the "new origin,"
$(-1,-4) \rightarrow (-2,-8)$, so $A'(-2,-4)$
B maps to itself, $(0,4)$
$(2,-4) \rightarrow (4,-8)$, so $C'(4,-4)$



10. Using $P(2,-4)$ as the "new origin,"
$(-2,6) \rightarrow (-1,3)$, mapping to $A'(1,-1)$
$(0,8) \rightarrow (0,4)$, mapping to $B'(2,0)$
$(4,4) \rightarrow (2,2)$, mapping to $C'(4,-2)$



11. a) $A'(-1,1), B'(4,-2), C'(3,-5), D'(-2,-2)$

b) $m_{\overline{A'B'}} = \frac{-2-1}{4+1} = -\frac{3}{5}$ $\quad m_{\overline{C'D'}} = \frac{-2+5}{-2-3} = -\frac{3}{5}$ $\quad \overline{A'B'} \parallel \overline{C'D'}$

$m_{\overline{A'D'}} = \frac{-2-1}{-2+1} = 3$ $\quad m_{\overline{B'C'}} = \frac{-5+2}{3-4} = 3$ $\quad \overline{A'D'} \parallel \overline{B'C'}$

$A'B'C'D'$ is a ▱ because both pairs of opp sides are ∥.

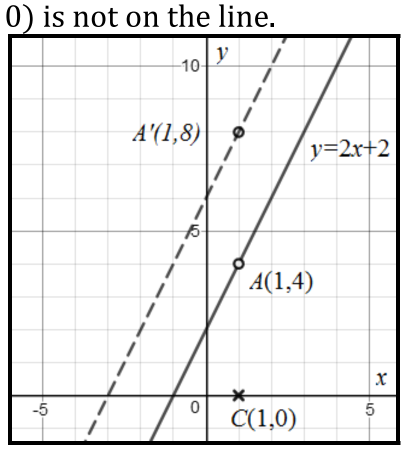
5.3. Dilations of Lines

1. $y = 3x - 20$

2. $2x + 3y = 4 \quad \rightarrow \quad y = -\frac{2}{3}x + \frac{4}{3}$
Equation of its image is $y = -\frac{2}{3}x + 4$

3. $C(1,-1)$ is a point on the line:
$(-1) = 3(1) - 4$
Therefore, the equation of the image is the same as the pre-image, $y = 3x - 4$.

4. $C(1,0)$ is not on the line.



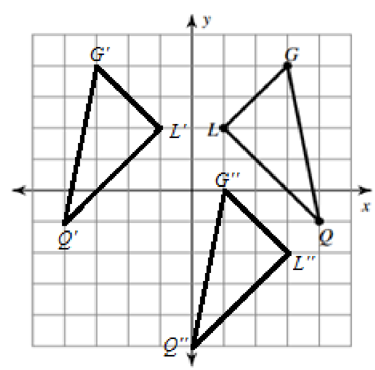
Find the point on the line where $x = 1$:
$y = 2(1) + 2 = 4$, so $A(1,4)$.
$CA = 4$, so $CA' = 2(4) = 8$ and $A'(1,8)$
$8 = 2(1) + b \quad \rightarrow \quad b = 6$
Equation of image is $y = 2x + 6$.

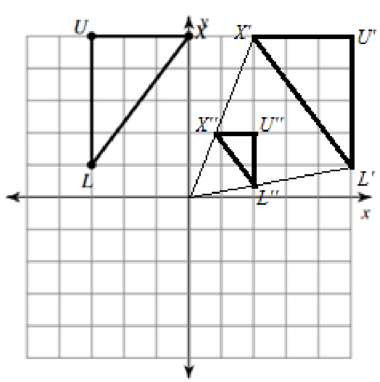
Chapter 6. Transformation Proofs

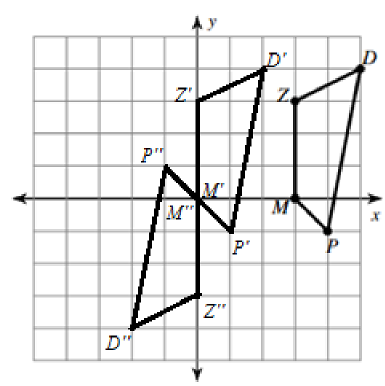
6.1. Properties of Transformations

1. a) reflection b) dilation c) translation d) rotation e) dilation	
2. (3) rotation	3. (2) opposite orientation; reflection
4. (4) $r_{y=x}$	5. (3) $r_{y\text{-}axis}$
6. (3) orientation	7. (4) D_2
8. (1) dilation	9. (3) dilation by a scale factor of $\frac{1}{2}$

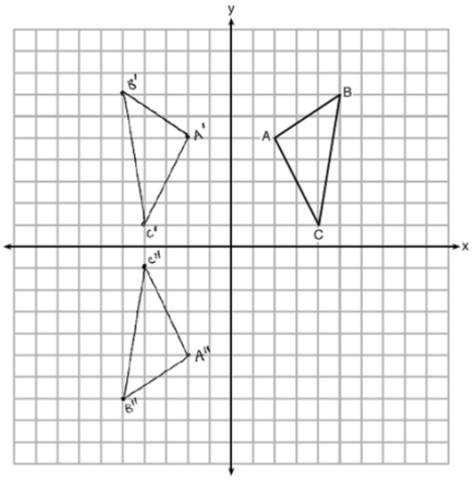
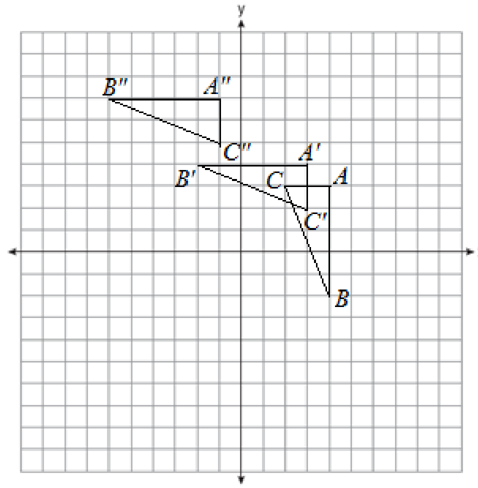
6.2. Sequences of Transformations

<table>
<tr><td colspan="2">1. (4) translation followed by a reflection</td></tr>
<tr><td>2. Rotation R_{90° with center at origin, followed by reflection r_{x-axis}</td><td>3. Rotation R_{270° around point A, followed by translation $T_{5,-2}$</td></tr>
<tr><td>4. Reflection $r_{y=\frac{1}{2}}$ followed by rotation R_{270° around the point $(1,-3)$</td><td>5.


</td></tr>
<tr><td>6.


</td><td>7.


</td></tr>
<tr><td colspan="2">8. (3) (8,12)</td></tr>
</table>

9.  $R_{180°}$	10.  $A'(3,4), B'(-2,4), C'(3,2)$ $A''(-1,7), B''(-6,7), C''(-1,5)$

11. If $\overline{AB}$ is reflected first, the coordinates are $A'(2,-6)$ and $B'(4,-2)$.
When the reflection is dilated, the coordinates are $A''(1,-3)$ and $B''(2,-1)$.
If $\overline{AB}$ is dilated first, the coordinates are $A'(1,3)$ and $B'(2,1)$.
When the dilation is reflected, the coordinates are $A''(1,-3)$ and $B''(2,-1)$.
The images are the same.

6.3. Transformations and Congruence

1. Translation so that A maps onto D.	2. Reflection over $\overline{AC}$.
3. Rotation of 180° around point J.	4. Reflection over the bisector of $\angle HLK$. *(Note that $\triangle HLK$ is isosceles.)*

5. Reflection over the vertical line passing through P, followed by the translation $T_{2,-4}$ (or vice versa). No, it is not possible because the triangles have opposite orientations and both translations and rotations preserve orientations.

6.4. Transformations and Similarity

1. $D_{\frac{1}{2}}$ with A as the center of dilation followed by $T_{4,0}$ mapping $A \to A'$.	2. $D_{\frac{1}{2}}$ with B as the center of dilation, then a translation $T_{0,-2}$ to map $B \to B'$, and finally a reflection $r_{\overline{B'C'}}$.

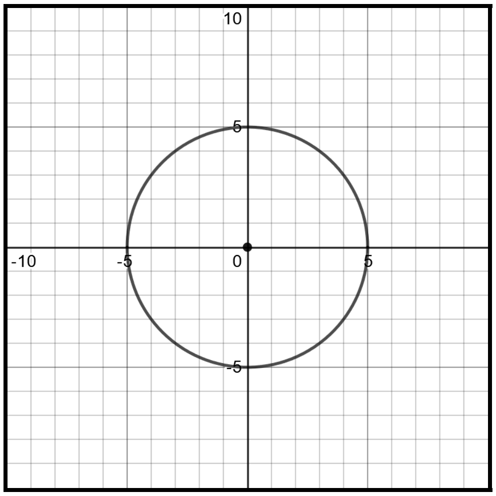
Chapter 7. Circles in the Coordinate Plane

7.1. Equation of a Circle

1. a) $(0,0)$ and $\sqrt{10}$ b) $3^2 + (-1)^2 = 10$ Yes.	2. $(2,-3)$ and 6
3. $(1,-3)$ and 3	4. $(0,7)$ and $4\sqrt{2}$
5. $r = 4$ $x^2 + y^2 = 16$	6. $r^2 = 3^2 + 4^2$ $r = 5$ $(x+3)^2 + (y-4)^2 = 25$
7. $r = 3$ $(x-1)^2 + (y+2)^2 = 9$	8. Center is at the midpoint of the diameter, $(2,-4)$. Radius is 4. $(x-2)^2 + (y+4)^2 = 16$
9. $(3,-2) \rightarrow (9,-6)$	10. $(-4,2) \rightarrow (-8,4)$ and $r = 2 \cdot 9 = 18$
11. $x^2 + 4x + y^2 = 5$ $(x^2 + 4x + 4) + y^2 = 5 + 4$ $(x+2)^2 + y^2 = 9$ Center is $(-2,0)$, radius is 3	12. $x^2 + 6x + y^2 - 4y = 12$ $(x^2 + 6x + 9) + y^2 - 4y = 12 + 9$ $(x+3)^2 + y^2 - 4y = 21$ $(x+3)^2 + (y^2 - 4y + 4) = 21 + 4$ $(x+3)^2 + (y-2)^2 = 25$ Center is $(-3,2)$, radius is 5
13. $x^2 - 16x + y^2 + 6y = -53$ $(x^2 - 16x + 64) + y^2 + 6y = -53 + 64$ $(x-8)^2 + y^2 + 6y = 11$ $(x-8)^2 + (y^2 + 6y + 9) = 11 + 9$ $(x-8)^2 + (y+3)^2 = 20$ Center is $(8,-3)$, radius is $2\sqrt{5}$	14. $x^2 - 2x = -y^2 + 10y + 1$ $x^2 - 2x + y^2 - 10y = 1$ $(x^2 - 2x + 1) + y^2 - 10y = 1 + 1$ $(x-1)^2 + y^2 - 10y = 2$ $(x-1)^2 + (y^2 - 10y + 25) = 2 + 25$ $(x-1)^2 + (y-5)^2 = 27$ Center is $(1,5)$, radius is $3\sqrt{3}$
15. Since the center of the circle is (r,r), the equation is $(x-r)^2 + (y-r)^2 = r^2$. Since point $(6,3)$ is on the circle, we can substitute this point for (x,y), giving us $(6-r)^2 + (3-r)^2 = r^2$. Now, solve for r. $36 - 12r + r^2 + 9 - 6r + r^2 = r^2$ $45 - 18r + 2r^2 = r^2$ $45 - 18r + r^2 = 0$ $r^2 - 18r + 45 = 0$ $(r-3)(r-15) = 0$ $r = \{\cancel{3}, 15\}$ Radius is 15.	

7.2. Graph Circles [NG]

1.



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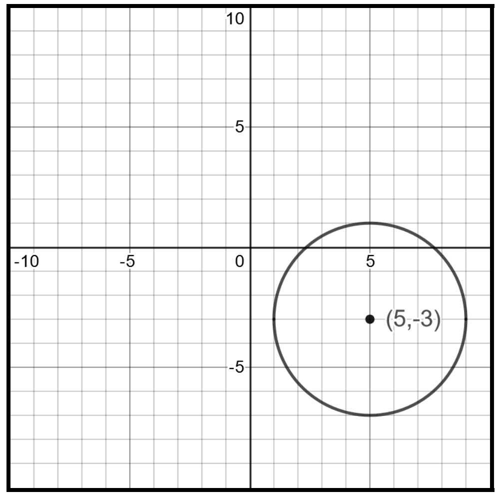

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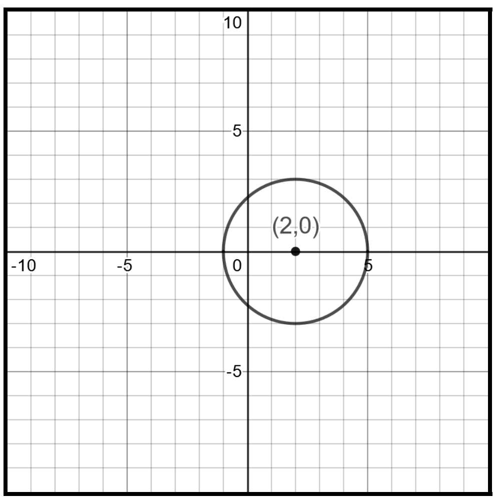

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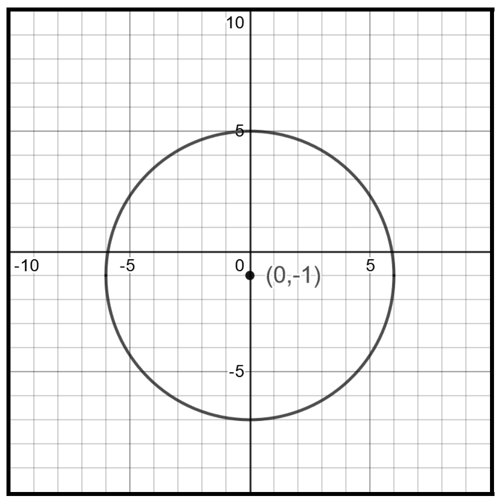
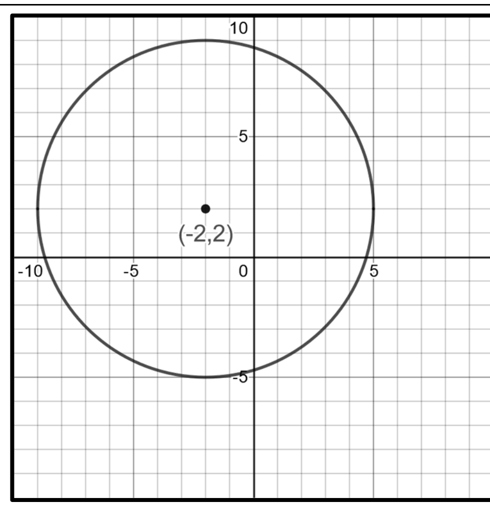


5. $x^2 + 4x + y^2 - 4y = 41$
$x^2 + 4x + 4 + y^2 - 4y = 41 + 4$
$(x + 2)^2 + y^2 - 4y = 45$
$(x + 2)^2 + y^2 - 4y + 4 = 45 + 4$
$(x + 2)^2 + (y - 2)^2 = 49$



Chapter 8. Foundations of Euclidean Geometry

8.1. Postulates, Theorems and Proofs

1. (4) vertical angles	2. (1) $\overline{AC} \cong \overline{DB}$
3. (2) $AB = CD$	4. (2) reflexive prop and subtraction prop

5.

Statements	*Reasons*
$\angle 1$ and $\angle 2$ are complementary	Given
$\angle 2$ and $\angle 3$ are complementary	Given
$\angle 1 \cong \angle 3$	Complements of the same $\angle$ are $\cong$
$m\angle 1 = m\angle 3$	Def of $\cong$ $\angle$'s

6.

Statements	*Reasons*
$\overline{PCEG}, \overline{PC} \cong \overline{GE}$	Given
$\overline{CE} \cong \overline{CE}$	Reflexive Prop
$PC = GE, \quad CE = CE$	Def of $\cong$ segments
$PC + CE = GE + CE$	Addition Prop
$PE = PC + CE, GC = GE + CE$	Segment Addition
$PE = GC$	Substitution
$\overline{PE} \cong \overline{GC}$	Def of $\cong$ segments

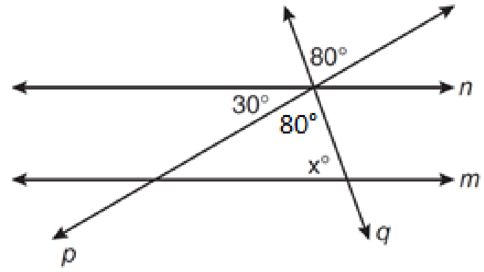
8.2. Parallel Lines and Transversals

1. (3) $\angle 4$ and $\angle 8$	2. (1) $\angle 1$ and $\angle 8$

3. (a) linear pair; supplementary
(b) vertical angles; congruent
(c) corresponding angles; congruent
(d) alternate interior angles; congruent
(e) alternate exterior angles; congruent

4. (2) consecutive interior $\angle$'s	5. (2) Alternate interior $\angle$'s are $\cong$.
6. (3) $\angle 3$ and $\angle 6$ are supplementary	7. (4) $d \parallel e$

8. $x + 20 = 2x - 10$ (alternate interior $\angle$'s are $\cong$)
$x = 30$

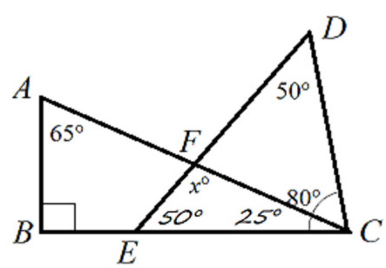
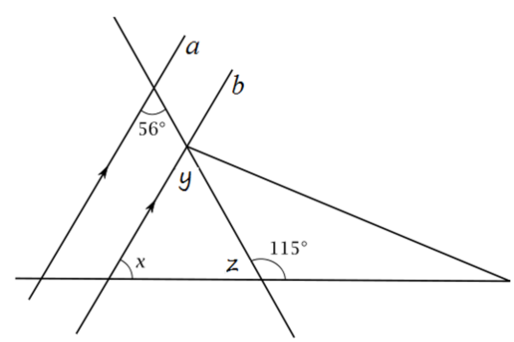
9. $m\angle EYD = 180 - 123 = 57°$ (linear pair) $m\angle AXY = m\angle EYD = 57°$ (alternate interior)	10. Vertical ∠'s are ≅, therefore:  $x = 180 - (30 + 80) = 70$
11. $a = 180 - (57 + 64) = 59°$ b = 64° c = 57° d = 180 – 64 = 116°	12. $15x - 5 = 180 - 125$ $15x - 5 = 55$ $7y + 27 = 125$ $15x = 60$ $7y = 98$ $x = 4$ $y = 14$

13.

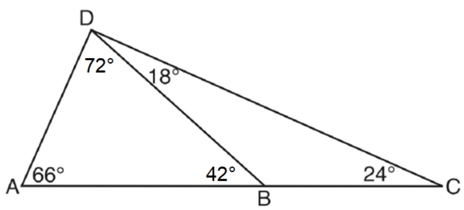
Statements	*Reasons*
∠1 and ∠3 are supplementary	Given
∠1 and ∠2 are a linear pair	Def of linear pair
∠1 and ∠2 are supplementary	Linear pairs are supplementary
∠2 ≅ ∠3	Supplements of the same ∠ are ≅
∠2 and ∠3 are alternate exterior ∠'s	Def of alternate exterior ∠'s
$m \parallel n$	If a transversal intersects two lines to form ≅ alternate exterior ∠'s, then the lines are ∥

Chapter 9. Triangles and Congruence

9.1. Angles of Triangles

1. $180-(60+40)=80$	2. $50+70=120°$
3. $\text{m}\angle ABC=180-125=55°$ $\text{m}\angle ACR=60+55=115°$	4. $60+60=120°$
5. $6x+5=3x+65$ $3x=60$ $x=20$	6. $5x+x+12+x=180$ $7x+12=180$ $7x=168$ $x=24$ ∠'s are 24°, 36°, and 120°, so △ is obtuse.
7. $\text{m}\angle J=180-(90+48)=42°$ $\text{m}\angle JMS=180-59=121°$ $\text{m}\angle JSM=180-(42+121)=17°$	8. $\text{m}\angle EHI=20°$ (vertical ∠'s) $\text{m}\angle EIH=60°$ (vertical ∠'s) $\text{m}\angle HEI=180-(20+60)=100°$
9. $\text{m}\angle RTQ=63°$ (alternate interior) $\text{m}\angle 2=180-(90+63)=27°$	10. $\text{m}\angle A=180-(90+52)=38°$ $\text{m}\angle EBC=90+38=128°$ $\text{m}\angle EBD=\frac{1}{2}\text{m}\angle EBC=64°$ $\text{m}\angle ABD=52+64=116°$ $\text{m}\angle D=180-(38+116)=26°$
11.  $\text{m}\angle \text{ACB}=180-(90+65)=25°$ $\text{m}\angle \text{DEC}=180-(80+50)=50°$ $x=180-(50+25)=105°$	12.  $y=56°$ (corresponding ∠'s) $z=65°$ (linear pair) $x=180-(56+65)=59°$
13. $\text{m}\angle ABD=180-(93+43)=44$ $x+19+2x+6+3x+5=180$ $6x+30=180$ $x=25$ $\text{m}\angle BDC=x+19=44$ Yes, because alternate interior ∠'s $\angle ABD$ and $\angle BDC$ are ≅, $\overline{AB} \parallel \overline{DC}$.	

9.2. Triangle Inequality Theorem

1. (2)	2. (4)
3. (1)	4. The sum of any two sides of a triangle must be greater than the third side. 7 + 8 < 16.
5. (2)	6. $4x + 3x - 1 + x + 3 = 34$ $8x + 2 = 34$ $x = 4$ 16, 11, and 7. Yes, because 16 < 11 + 7.
7. $\angle B, \angle A, \angle C$	8. $\overline{EF}, \overline{DE}, \overline{DF}$
9. $m\angle ABD = 42°$ (exterior $\angle$ of $\triangle CBD$) $m\angle ADB = 72°$ $(180 - 66 - 42)$ $\overline{AB}$ is the longest side and $\overline{AD}$ is the shortest side of $\triangle ABD$. 	10. $m\angle ADB = 36°$ and $m\angle CDB = 93°$. $\overline{BD}$ is the shortest side of $\triangle BCD$, but $\overline{BD}$ is not the shortest side of $\triangle ABD$. Therefore, $\overline{AB}$ is the shortest segment.

9.3. Segments in Triangles

1. (a) $\overline{BE}$ (b) $\overline{AD}$ (c) $\overline{CF}$	2. (4)
3. $6x - 6 = 90$ $6x = 96$ $x = 16$	4. $4x - 17 = 3x - 4$ $x = 13$ $m\angle XYW = 4(13) - 17 = 35$ So, $m\angle XYZ = 2(35) = 70°$
5. $4x - 8 + 6x + 13 = 90$ $10x + 5 = 90$ $x = 8.5$	
6. $x^2 + 3x = 6x + 10$ $x^2 - 3x - 10 = 0$ $(x - 5)(x + 2) = 0$ $x = 5$ (reject $x = -2$) $12y + 24 = 2(2y + 60)$ $12y + 24 = 4y + 120$ $8y = 96$ $y = 12$	
7. a) $2x + 3 = 7x - 47$ $50 = 5x$ $10 = x$ $DH = 2(10) + 3 = 23$ $DG = 2DH = 46$ b) $y^2 + 9 = 90$ $y^2 = 81$ $y = 9$ $m\angle EFH = 12y = 108°$	

9.4. Isosceles and Equilateral Triangles

1. $\frac{180-120}{2} = 30°$

2. $\frac{180-46}{2} = 67°$

3. (a) $\overline{CE}$ (b) $\overline{AF}$ (c) $\overline{BD}$

4. (3) an obtuse angle

5. $m\angle A = \frac{180-80}{2} = 50°$
$m\angle BCD = 80 + 50 = 130°$

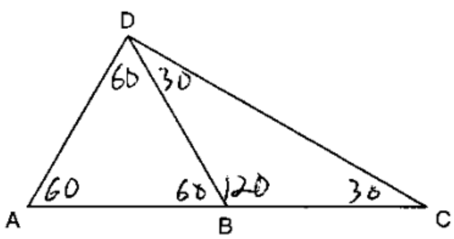
6. $m\angle LMO = 55°$
$m\angle NMO = 180 - 55 = 125°$
$m\angle N = 180 - (125 + 28) = 27°$

7. a) $60°$ (equilateral △)
b) $\frac{180-50}{2} = 65°$ (base ∠ of isosceles △)
c) $180 - (60 + 65) = 55°$ (parts of a straight ∠)
d) $65 + 50 = 115°$ (exterior ∠)
e) $60 + 60 = 120°$ (exterior ∠)
f) $60°$ (vertical ∠'s with $\angle BAC$)

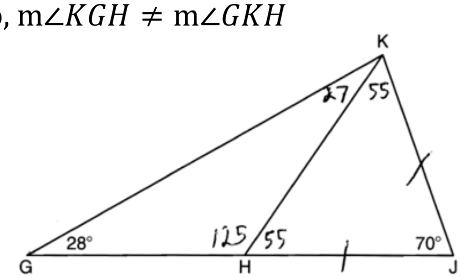
8. $m\angle QRP = \frac{180-54}{2} = 63°$
$m\angle QRS = 180 - 63 = 117°$
$x = \frac{180-117}{2} = 31.5°$

9. $m\angle EFG = 90 - 60 = 30°$
$x = 180 - (90 + 30) = 60°$

10. 30



11. No, $m\angle KGH \neq m\angle GKH$

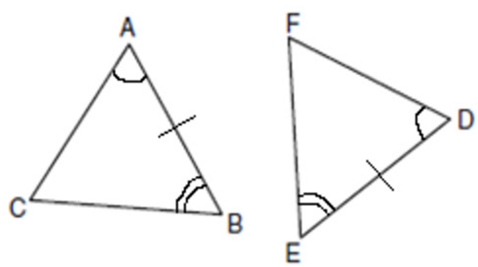
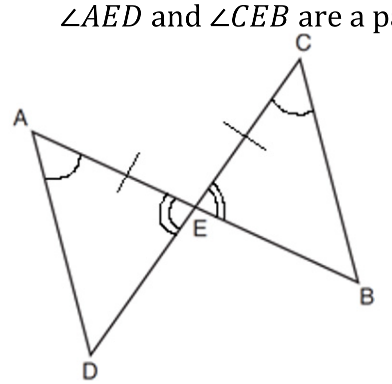
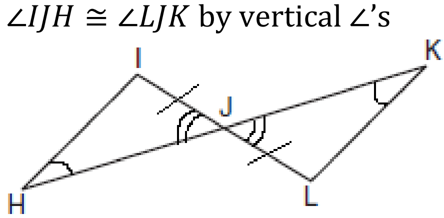
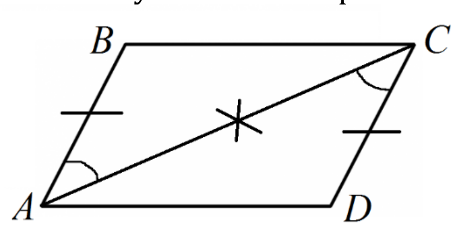


12. The altitude is also the median.
$x^2 + (x + 7)^2 = 13^2$ *[Pythagorean Thm]*
$x^2 + x^2 + 14x + 49 = 169$
$2x^2 + 14x - 120 = 0$
$2(x + 12)(x - 5) = 0$
$x = 5$ (reject $x = -12$)
Base $= 2x = 10$

13.

Statements	*Reasons*
$m\angle K = 70°$, $m\angle MLN = 55°$	Given
$m\angle JLK = m\angle MLN$	Vertical ∠'s are equal in measure
$m\angle JLK = 55°$	Substitution
$m\angle J + m\angle K + m\angle JLK = 180°$	Sum of the ∠'s of a △ is 180°
$m\angle J + 70° + 55° = 180°$	Substitution
$m\angle J = 55°$	Subtraction
$m\angle JLK = m\angle J$	Transitive prop
$\triangle JKL$ is isosceles	If two ∠'s of a △ are equal in measure, then the △ is isosceles

9.5. <u>**Triangle Congruence Methods**</u>

1. (2) by SSS	2. (1) by SAS
3. (2) $\angle A \cong \angle X$	4. (3) $\overline{JL} \cong \overline{MO}$
5. (1) $\angle A \cong \angle L$	6. $\overline{AG} \cong \overline{OL}$
7. ASA 	8. ASA $\angle AED$ and $\angle CEB$ are a pair of vertical ∠'s. 
9. AAS $\overline{IJ} \cong \overline{LJ}$ by def of bisector $\angle IJH \cong \angle LJK$ by vertical ∠'s 	10. SAS $\overline{CA} \cong \overline{AC}$ by Reflexive Prop 
11. AAS $\angle BAC \cong \angle DAC$ by def. of ∠ bisector $\overline{AC} \cong \overline{AC}$ by Reflexive Prop	12. AAS $\angle BAC \cong \angle DCA$ by alternate interior ∠'s $\overline{AC} \cong \overline{CA}$ by Reflexive Prop

9.6. Prove Triangles Congruent

1.
   3. Def of $\perp$
   6. Alternate Interior $\angle$'s Thm
   8. AAS
   9. CPCTC

2.

Statements	*Reasons*
$\overline{BE}$ and $\overline{AD}$ intersect at C, $\overline{BC} \cong \overline{EC}, \overline{AC} \cong \overline{DC}$	Given
$\angle BCA \cong \angle ECD$	Vertical $\angle$'s are $\cong$
$\triangle ABC \cong \triangle DEC$	SAS

3.

Statements	*Reasons*
$\overline{FH} \cong \overline{FI}, \overline{SH} \cong \overline{SI}$	Given
$\overline{FS} \cong \overline{FS}$	Refelexive Prop
$\triangle FHS \cong \triangle FIS$	SSS
$\angle H \cong \angle I$	CPCTC

4.

Statements	*Reasons*
$\overline{AD}$ bisects $\overline{BC}$ at E, $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$	Given
$\angle B$ and $\angle C$ are right $\angle$'s	Def. of $\perp$
$\overline{BE} \cong \overline{CE}$	Def. of segment bisector
$\angle B \cong \angle C$	Right $\angle$'s are $\cong$
$\angle AEB \cong \angle DEC$	Vertical $\angle$'s are $\cong$
$\triangle ABE \cong \triangle DCE$	ASA
$\overline{AB} \cong \overline{DC}$	CPCTC

5.

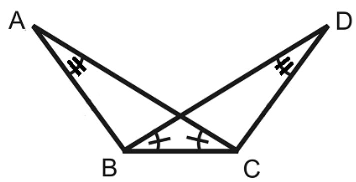
Statements	*Reasons*
$\triangle ABC, \overline{BD}$ bisects $\angle ABC, \overline{BD} \perp \overline{AC}$	Given
$\angle CBD \cong \angle ABD$	Def. of $\angle$ bisector
$\overline{BD} \cong \overline{BD}$	Reflexive Prop
$\angle CDB$ and $\angle ADB$ are right $\angle$'s	Def. of $\perp$
$\angle CDB \cong \angle ADB$	Right $\angle$'s are $\cong$
$\triangle CDB \cong \triangle ADB$	ASA
$\overline{AB} \cong \overline{CB}$	CPCTC

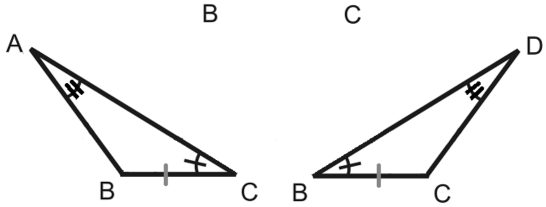
6.

Statements	*Reasons*
$\overline{AFEC}$, $\overline{AF} \cong \overline{EC}$, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	Given
$\angle 1$ is supplementary to $\angle DFC$ and $\angle 2$ is supplementary to $\angle BEA$	Linear pairs are supplementary
$\angle DFC \cong \angle BEA$	$\cong$ Supplements Thm
$\overline{EF} \cong \overline{EF}$	Refelexive Prop
$AF + EF = EC + EF$ $\overline{AE} \cong \overline{FC}$	Addition Prop
$\triangle ABE \cong \triangle CDF$	ASA

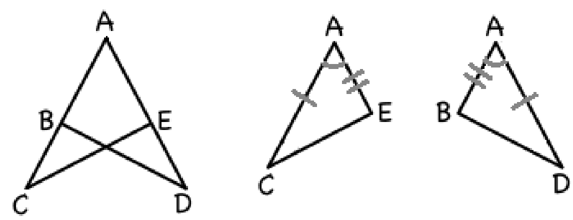
9.7. Overlapping Triangles

1. AAS
Separate out the overlapping triangles and mark the given pairs of congruent angles. Note also that the two triangles share the same side, $\overline{BC}$, which must be congruent to itself by the Reflexive Prop.





2. SAS
Separate out the overlapping triangles and mark the given pairs of congruent sides. Note also that the two triangles share the same angle, $\angle A$, which must be congruent to itself by the Reflexive Prop.



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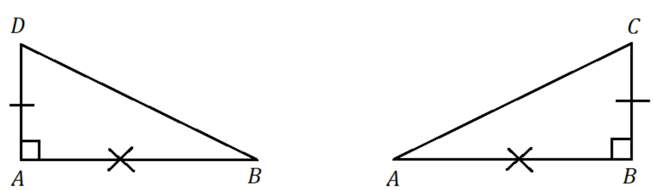
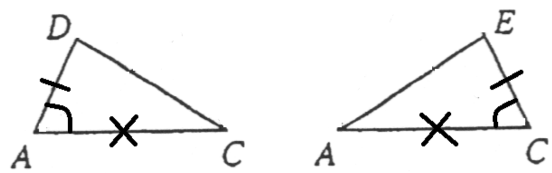


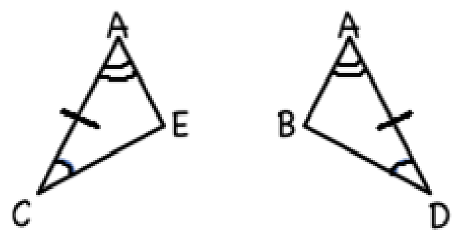
Statements		*Reasons*
$\overline{DA} \cong \overline{CB}$	(S)	Given
$\overline{DA} \perp \overline{AB}$, $\overline{CB} \perp \overline{AB}$		Given
$\angle DAB$ and $\angle CBA$ are right $\angle$'s		Def. of $\perp$
$\angle DAB \cong \angle CBA$	(A)	Right $\angle$'s are $\cong$
$\overline{AB} \cong \overline{AB}$	(S)	Reflexive Prop
$\triangle DAB \cong \triangle CBA$		SAS

4.



Statements		*Reasons*
$\overline{AB} \cong \overline{CB}$		Given
$\overline{AD} \cong \overline{CE}$	(S)	Given
$\angle DAC \cong \angle ECA$	(A)	Isosceles △ Thm
$\overline{AC} \cong \overline{AC}$	(S)	Reflexive Prop
$\triangle DAC \cong \triangle ECA$		SAS
$\angle ADC \cong \angle CEA$		CPCTC

5.



Statements		*Reasons*
$\angle C \cong \angle D$	(A)	Given
$\overline{AC} \cong \overline{AD}$	(S)	Given
$\angle A \cong \angle A$	(A)	Reflexive Prop
$\triangle ACE \cong \triangle ADB$		ASA
$\overline{CE} \cong \overline{DB}$		CPCTC

Chapter 10. Triangles and Similarity

10.1. Properties of Similar Triangles

1. m$\angle P = 25°$ and $m\angle R = 45°$, so m$\angle Q = 180 - (25 + 45) = 110°$	
2. m$\angle B = 180 - (50 + 30) = 100$ So, m$\angle X = 100$	
3. $\frac{25}{10} = \frac{AC}{6}$ $10AC = 150$ $AC = 15$	4. $\frac{50}{XY} = \frac{40}{20}$ $40XY = 1000$ $XY = 25$
5. $\frac{30}{21} = \frac{ML}{7}$ $ML = 10$	6. $\frac{9}{36} = \frac{4}{x}$ $9x = 144$ $x = 16$
7. $5x + 8x + 11x = 60$ $24x = 60$ $x = 2.5$ $5x = 5(2.5) = 12.5$	8. $\frac{15}{18} = \frac{5}{6}$

10.2. Triangle Similarity Methods

1. $\triangle QRS \sim \triangle UTS$ $\angle Q \cong \angle U$ and $\angle R \cong \angle T$ by alternate interior ∠'s formed by ∥ lines. The △s are similar by AA~. (Also, $\angle RSQ \cong \angle TSU$ by vertical ∠'s)	2. $\triangle AEB \sim \triangle CED$ $\angle A \cong \angle DCE$ and $\angle B \cong \angle CDE$ by corresponding ∠'s formed by ∥ lines. The △s are similar by AA~. (Also, $\angle E \cong \angle E$ by Reflexive Prop.)
3. (3) $\angle ACB \cong \angle DFE$ (the included ∠'s for SAS similarity)	
4. SAS	5. AA
6. (4)	

10.3. Prove Triangles Similar

1. $\angle ACB \cong \angle AED$	(Given)
$\angle A \cong \angle A$	(Reflexive Prop)
$\triangle ABC \sim \triangle ADE$	(AA~)

2. $\overline{AB} \perp \overline{BE}, \overline{DE} \perp \overline{BE}$, and $\angle BFD \cong \angle ECA$	(Given)
$\angle B$ and $\angle E$ are right $\angle$'s	(def of $\perp$ lines)
$\angle B \cong \angle E$	(right $\angle$'s are $\cong$)
$\angle BFD$ and $\angle DFE$ are supplementary and $\angle ECA$ and $\angle ACB$ are supplementary	(linear pairs)
$\angle DFE \cong \angle ACB$	($\angle$'s supplementary to $\cong$ $\angle$'s are $\cong$)
$\triangle ABC \sim \triangle DEF$	(AA~)
3. right $\angle Q$, right $\angle T$, $PQ = 6, QR = 8, ST = 3, TU = 4$	(Given)
$\angle Q \cong \angle T$	(right $\angle$'s are $\cong$)
$\frac{6}{3} = \frac{8}{4}$ *[scale of 2]*	(def of proportion)
$\frac{PQ}{ST} = \frac{QR}{TU}$	(substitution)
$\triangle PQR \sim \triangle STU$	(SAS~)
$\angle R \cong \angle U$	(CASTC)
[Alternatively, use Pythagorean Thm to find $PR = 10$ and $SU = 5$, then $\triangle PQR \sim \triangle STU$ by SSS~]	

10.4. Triangle Angle Bisector Theorem

1. $\frac{x}{4} = \frac{9}{6}$ $6x = 36$ $x = 6$	2. $\frac{2}{x-2} = \frac{5}{9}$ $5(x-2) = 18$ $5x - 10 = 18$ $5x = 28$ $x = 5.6$
3. Let $x = AB$. So, $BC = 30 - x$. $\frac{x}{30-x} = \frac{21}{24}$ $24x = 21(30-x)$ $24x = 630 - 21x$ $45x = 630$ $x = 14$	4. $\frac{x-2}{x+1} = \frac{x}{x+4}$ $(x-2)(x+4) = x(x+1)$ $x^2 + 2x - 8 = x^2 + x$ $2x - 8 = x$ $x = 8$ $P = x + (x-2) + (x+1) + (x+4)$ $= 8 + 6 + 9 + 12 = 35$
5. $6^2 + 8^2 = (DF)^2$ $100 = (DF)^2$ $DF = 10$ Let $x = EG$. By angle bisector thm, $\frac{x}{8-x} = \frac{6}{10}$ $6(8-x) = 10x$ $3^2 + 6^2 = (DG)^2$ $48 - 6x = 10x$ $45 = (DG)^2$ $48 = 16x$ $DG = \sqrt{45} \approx 6.7$ $x = EG = 3$	

10.5. Side Splitter Theorem

1. $\frac{CB}{BA} = \frac{CE}{ET}$ $\frac{3}{10-3} = \frac{6}{x}$ $3x = 42$ $x = 14$	2. $\frac{12}{x} = \frac{16}{4}$ $16x = 48$ $x = 3$
3. $\frac{3}{5} = \frac{6}{BC}$ $3BC = 30$ $BC = 10$	4. $\frac{AD}{DB} = \frac{AE}{EC}$ $\frac{8}{10-8} = \frac{12}{x}$ $8x = 24$ $x = 3$
5. $\frac{3}{9} = \frac{x}{15}$ $9x = 45$ $x = 5$	6. $\frac{EB}{25} = \frac{12}{18}$ $18EB = 300$ $EB \approx 16.7$
7. $\frac{MN}{NP} = \frac{MR}{RQ}$ $\frac{13}{8} = \frac{x}{42-x}$ $13(42 - x) = 8x$ $546 - 13x = 8x$ $546 = 21x$ $x = 26$ $MR = 26, RQ = 42 - 26 = 16$	8. $\frac{CD}{DA} = \frac{CE}{EB}$ $\frac{4}{10-4} = \frac{x+2}{4x-7}$ $4(4x - 7) = 6(x + 2)$ $16x - 28 = 6x + 12$ $10x - 40$ $x = 4$ $CE = (4) + 2 = 6$
9. $CD = AD - AC = (2x + 2) - (x - 3)$ $= x + 5$ $\frac{AB}{BE} = \frac{AC}{CD}$ $\frac{16}{20} = \frac{x-3}{x+5}$ $16x + 80 = 20x - 60$ $140 = 4x$ $35 = x$ $AC = x - 3 = 32$	10. By the side splitter thm, $\frac{9}{6} = \frac{EL}{BE}$, or $\frac{6}{9} = \frac{BE}{EL}$. By the ∠ bisector thm, $\frac{BE}{x} = \frac{EL}{15}$ $x(EL) = 15(BE)$ $x = 15 \cdot \frac{BE}{EL}$ By substitution, $x = 15 \cdot \frac{6}{9} = 10$, so $BW = 10$
11. $\frac{5}{10} = \frac{12}{x}$ 24 miles	

10.6. Triangle Midsegment Theorem

1. $x = \frac{1}{2}(24) = 12$	2. $5x = 2(2x + 2)$ $5x = 4x + 4$ $x = 4$ $DE = 2(4) + 2 = 10$ $AC = 5(4) = 20$
3. $AB = 36 \div 3 = 12$ $EF = \frac{1}{2}AB = \frac{1}{2} \cdot 12 = 6$ cm (triangle midsegment thm)	4. $BE = \frac{1}{2}BC = 6$ (def. of midpoint) $EF = \frac{1}{2}AB = 10$ ($\triangle$ midseg. thm) $AF = \frac{1}{2}AC = 8$ (def. of midpoint) Perimeter = $AB + BE + EF + AF = 44$
5. $BC = 2MN = 16$ $AC = 2ML = 10$, so $NC = \frac{1}{2}AC = 5$ $AB = 2NL = 12$, so $MB = \frac{1}{2}AB = 6$ Perimeter = $BC + NC + MN + MB = 35$	6. $ST = 2(3.5) = 7$ (midseg. thm) Let $x = RS = RT$ Perimeter $2x + 7 = 25$, so $x = 9$. $NT = \frac{1}{2}x = 4.5$ (def. of midpoint)
7. $PR = 2SU = 36$ $ST = \frac{1}{2}QR = 11$ $\angle STP$ and $\angle TSU$ are ≅ alternate interior ∠'s, so m$\angle STP = 48°$ $\angle SUR$ and $\angle TSU$ are supplementary consecutive interior ∠'s, so m$\angle SUR = 132°$	

Chapter 11. Points of Concurrency

11.1. Incenter and Circumcenter

1. (4) $\angle DBG \cong \angle EBG$	2. $m\angle REC = \frac{1}{2}(84) = 42$ $m\angle BRC = 180 - (42 + 28) = 110°$
3. a) $r = TS = 3$, so $C = 2\pi r = 6\pi$. b) $m\angle QPR = 2 \cdot 18 = 36°$ $m\angle QRP = 180 - (39 + 36) = 105°$ $m\angle TRP = \frac{1}{2} m\angle QRP = 52.5°$	4. a) $BD = 15$, $AF = 16$, and $AE = 17$ b) $AG = CG = BG = 19$ *[The circumcenter is equidistant from the three vertices of the triangle.]* c) $r = BG = 19$, so $A = \pi r^2 = 361\pi$
5. $9.4^2 + (PQ)^2 = 13.4^2$ $(PQ)^2 = 13.4^2 - 9.4^2 = 91.2$ $A = \pi r^2 = \pi \cdot (PQ)^2 = 91.2\pi \approx 286.5$ sq. units	6. Draw $\overline{CR}$. $(CR)^2 = \left(\sqrt{53}\right)^2 + 26^2 = 729.$ $CR = \sqrt{729} = 27$ Circumference $= 2\pi r = 2\pi \cdot (CR) = 54$

11.2. Orthocenter and Centroid

1. a) circumcenter c) orthocenter b) centroid d) incenter	2. (3) an obtuse triangle
3. (4) circumcenter and orthocenter	4. (1) centroid
5. a) 48 b) 16 c) 24	
6. $SP = 2PR = 24$ $TM = PT + \frac{1}{2}PT = 42$	$AT = 2AR = 40$ $PY = \frac{1}{3}AY = 9$
7. $GC = 2(FG) = 24$ cm	8. $FD = \frac{1}{2}(AF) = 3$
9. $TA = TO + OA = 10 + 2(10) = 30$	10. $BP = \frac{2}{3}(BF) = \frac{2}{3}(18) = 12$
11. $FG = \frac{1}{3}(CF) = \frac{1}{3}(24) = 8$	12. $CR = 2(RF)$ $24 = 2(2x - 6)$ $x = 9$
13. $BP = 2(PM)$ $7x + 4 = 2(2x + 5)$ $x = 2$ $PM = 2(2) + 5 = 9$	14. $QC = 2(CM)$ $5x = 2(x + 12)$ $x = 8$ $QM = QC + CM = 5(8) + (8 + 12) = 60$

Chapter 12. Right Triangles and Trigonometry

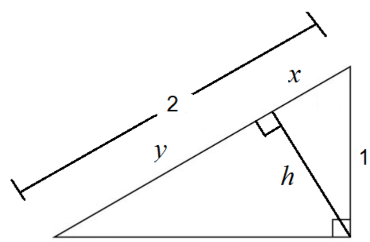
12.1. Congruent Right Triangles

1. A	2. (3)
3. Right $\triangle$s MAT and HTA (Given) $\overline{MT} \cong \overline{AH}$ (H) (Given) $\overline{AT} \cong \overline{AT}$ (L) (Reflexive Prop) $\triangle MAT \cong \triangle HTA$ (HL) $\angle M \cong \angle H$ (CPCTC)	
4. $\overline{CA} \perp \overline{AB}, \overline{ED} \perp \overline{DF}, \overline{CE} \cong \overline{BF}$ (Given) $\overline{AB} \cong \overline{ED}$ (L) (Given) $\angle A$ and $\angle D$ are right $\angle$'s (Def. of $\perp$) $\triangle ABC$ and $\triangle DEF$ are right $\triangle$s (Def. of right $\triangle$s) $\overline{EB} \cong \overline{EB}$ (Reflexive Prop) $CE + EB = BF + EB$, $\overline{CB} \cong \overline{FE}$ (H) (Addition Prop) $\triangle ABC \cong \triangle DEF$ (HL) $\overline{AC} \cong \overline{DF}$ (CPCTC)	

12.2. Equidistance Theorems

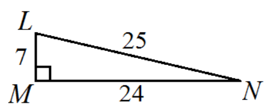
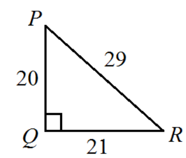
1. 12	2. 26°
3. $RS = 15$. $\triangle PSR$ is isosceles, so m$\angle R = 30°$. Therefore, m$\angle RSQ = 60°$.	4. $2x + 10 = 5x - 17$ $10 = 3x - 17$ $27 = 3x$ $x = 9$
5. $2x + 5 = 4x - 25$ $5 = 2x - 25$ $30 = 2x$ $x = 15$	6. $5x - 3 = 3x + 1$ $2x = 4$ $x = 2$ m$\angle BAC = 41(2) = 82°$ The $\perp$ bisector of the base of an isosceles $\triangle$ is also the bisector of the vertex $\angle$. Therefore, m$\angle BAD = 41°$ and m$\angle B = 49°$.
7. P is the incenter, so $\overline{PQ} \cong \overline{PR} \cong \overline{PS}$.	8. P is the circumcenter (external to $\triangle ABC$), so $\overline{PA} \cong \overline{PB} \cong \overline{PC}$.

12.3. Geometric Mean Theorems

1. $x^2 = 3 \cdot 9 = 27$ $x = \sqrt{27} = 3\sqrt{3}$	2. $x^2 = 12 \cdot 3 = 36$ $x = \sqrt{36} = 6$
3. $AD = 16 - 7 = 9$ $x^2 = 7 \cdot 9 = 63$ $x = \sqrt{63} = 3\sqrt{7}$	4. $x^2 = 12 \cdot 2 = 24$ $x = \sqrt{24} = 2\sqrt{6}$
5. $x^2 = 4 \cdot 7 = 28$ $x = \sqrt{28} = 2\sqrt{7}$	6. $x^2 = 8 \cdot 18 = 144$ $x = 12$
7. $6^2 = x(x+5)$ $36 = x^2 + 5x$ $x^2 + 5x - 36 = 0$ $(x+9)(x-4) = 0$ $x = 4$ (reject $x = -9$)	8. $10^2 = x(x+21)$ $100 = x^2 + 21x$ $x^2 + 21x - 100 = 0$ $(x+25)(x-4) = 0$ $x = 4$ (reject $x = -25$)
9. Let $x = AD$ $4^2 = x(x+6)$ $x^2 + 6x - 16 = 0$ $(x+8)(x-2) = 0$ $x = 2$ (reject $x = -8$) $AC = x + (x+6) = 10$	10. $x^2 = 4 \cdot 5 = 20$ $x = \sqrt{20} = 2\sqrt{5}$ $y^2 = 9 \cdot 4 = 36$ $y = \sqrt{36} = 6$ $z^2 = 9 \cdot 5 = 45$ $z = \sqrt{45} = 3\sqrt{5}$
11. $x^2 = 24 \cdot 6 = 144$ $x = \sqrt{144} = 12$ $y^2 = 30 \cdot 6 = 180$ $y = \sqrt{180} = 6\sqrt{5}$ $z^2 = 24 \cdot 30 = 720$ $z = \sqrt{720} = 12\sqrt{5}$	12.  (a) $1^2 = 2x$ $1 = 2x$ $x = \frac{1}{2}$ (b) $y = 2 - \frac{1}{2} = \frac{3}{2}$ $h^2 = xy = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$ $h = \sqrt{\frac{3}{4}}$

Chapter 13. Trigonometry

13.1. Trigonometric Ratios

1. $\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \tan A = \frac{3}{4}$	2. $\sin B = \frac{15}{17}, \cos B = \frac{8}{17}, \tan B = \frac{15}{8}$
3. $\sin S$ and $\cos R$	4. $\tan B = \frac{8}{15} \approx 0.533$
5. $\sin x = \frac{28}{53} \approx 0.528$	6. $\cos A = \frac{16}{20} = 0.8$
7. $\sin N = \frac{7}{25}$ 	8. $\tan P = \frac{21}{20}$ 
9. $30^2 + 40^2 = c^2$ $2500 = c^2$ $c = 50$ $\sin B = \frac{30}{50} = \frac{3}{5}$	

13.2. Use Trigonometry to Find a Side

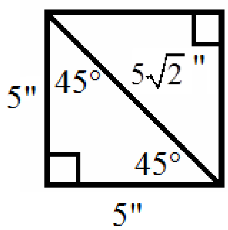
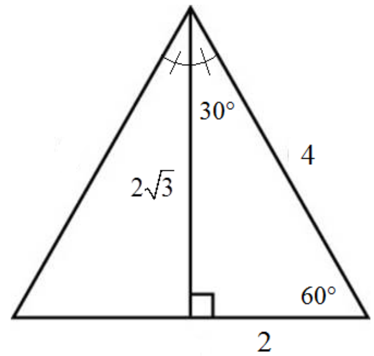
1. (3)	2. $\sin 15° = \frac{w}{10}$ $w = 10 \sin 15° \approx 2.6$
3. $\sin 32° = \frac{x}{30}$ $x = 30 \sin 32° \approx 15.9$ ft.	4. $\tan 62° = \frac{x}{15}$ $x = 15 \tan 62° \approx 28.2$ ft.
5. $\sin 57° = \frac{x}{8}$ $x = 8 \sin 57° \approx 6.7$ ft.	6. $\tan 32° = \frac{x}{25}$ $x = 25 \tan 32° \approx 15.6$ ft.
7. $\cos 65° = \frac{x}{5}$ $x = 5 \cos 65° \approx 2.1$ ft.	8. $\sin 48° = \frac{9}{x}$ $x = \frac{9}{\sin 48°} \approx 12$ ft.
9. $\tan 11° = \frac{400}{x}$ $x = \frac{400}{\tan 11°} \approx 2{,}058$ ft.	10. $\tan 58° = \frac{x}{6}$ $x = 6 \tan 58° \approx 9.60$ ft. $A = \frac{1}{2}bh \approx \frac{1}{2}(6)(9.60) \approx 28.8$ ft^2

11. $\tan 52° = \frac{50}{x}$ $x = \frac{50}{\tan 52°} \approx 39$ ft. from base to stake $\sin 52° = \frac{50}{x}$ $x = \frac{50}{\sin 52°} \approx 63$ ft. wire	12. $\sin 50° = \frac{x}{110}$ $x = 110 \sin 50° \approx 84$ ft. high $\cos 50° = \frac{x}{110}$ $x = 110 \cos 50° \approx 71$ ft. between the ropes
13. $\cos 72° = \frac{x}{10}$ $x = 10 \cos 72° \approx 3.09$ ft. ≈ 37 in. from base $\sin 72° = \frac{y}{10}$ $x = 10 \sin 72° \approx 9.51$ ft. ≈ 114 in. up wall	14. $\tan 28° = \frac{h}{200}$ (*h = cliff + lighthouse*) $h = 200 \tan 28° \approx 106.34$ ft. $\tan 18° = \frac{c}{200}$ (*c = cliff height alone*) $c = 200 \tan 18° \approx 64.98$ ft. $x = h - c \approx 106.34 - 64.98 \approx 41.4$ ft.

13.3. Use Trigonometry to Find an Angle

1. $\sin x = \frac{3}{7}$ $x = \sin^{-1}\left(\frac{3}{7}\right) \approx 25.4°$		2. $\tan A = \frac{11.2}{18.3}$ $A = \tan^{-1}\left(\frac{11.2}{18.3}\right) \approx 31.5°$		
3. $\sin x = \frac{30}{50}$	$x \approx 37°$	4. $\sin A = \frac{8}{12}$	$A \approx 42°$	
5. $\cos x = \frac{6}{28}$	$x \approx 78°$	6. $\sin A = \frac{10}{16}$ $m\angle B = 90 - m\angle A \approx 51.3°$	$m\angle A \approx 38.7°$	
7. $\tan x = \frac{420}{2000}$	$x \approx 12°$	8. $\tan x = \frac{350}{1000}$	$x \approx 19°$	
9. $12^2 + 16^2 = r^2$	$r = 20$	$s = 50 - 20 = 30$	$\sin x = \frac{16}{30}$	$x \approx 32°$
10. 36 – 28 = 8 ft	$\sin x = \frac{8}{12}$	$x \approx 41.8°$		
11. $\tan x = \frac{6}{4}$	$x \approx 56°$	$\sin 56° = \frac{b}{15}$	$b = 15 \sin 56° \approx 12$ ft.	

13.4. Special Triangles

1. 30-60-90 △ with a factor of 12.	$x = 12\sqrt{3} \approx 21$
2. $5\sqrt{2}$ inches Each side is 5 inches. 45-45-90 △ with factor of 5.  	3. 4 units The altitude of an equilateral △ is also the ∠ bisector. 30-60-90 △ with factor of 2. 

13.5. Cofunctions

1. $m\angle B = 90 - 25 = 65°$	2. $72 + x = 90$ $x = 18°$
3. $x + 15 + x - 5 = 90$ $2x + 10 = 90$ $2x = 80$ $x = 40$	4. $2x - 1 + 3x + 6 = 90$ $5x + 5 = 90$ $5x = 85$ $x = 17$
5. $x + 20 + x = 90$ $2x + 20 = 90$ $2x = 70$ $x = 35$	6. $x - 3 + 2x + 6 = 90$ $3x + 3 = 90$ $3x = 87$ $x = 29$
7. $2x + 20 + 40 = 90$ $2x + 60 = 90$ $2x = 30$ $x = 15$	8. $2x - 25 + 55 = 90$ $2x + 30 = 90$ $2x = 60$ $x = 30$

13.6. SAS Sine Formula for Area of a Triangle [NG]

1. $A = \frac{1}{2} \cdot 6 \cdot 8 \cdot \frac{1}{4} = 6$	2. $A = \frac{1}{2} \cdot 12 \cdot 15 \cdot \sin 150 = 45$
3. $A = \frac{1}{2} \cdot 7 \cdot 10 \cdot \sin 25 \approx 14.8$	4. $m\angle C = 180 - 2 \cdot 75 = 30$ $A = \frac{1}{2} \cdot 10 \cdot 10 \cdot \sin 30 = 25$
5. $A = \frac{1}{2} \cdot 14 \cdot 16 \cdot \sin 30 = 56$	6. $A = \frac{1}{2} \cdot 11 \cdot 13 \cdot \sin 70 \approx 67$
7. $A = \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 30° = 4$	8. $A = \frac{1}{2} \cdot 4 \cdot 5 \cdot \sin 59° \approx 8.6$
9. $A = \frac{1}{2} \cdot 16 \cdot 21 \cdot \sin 58 \approx 142.5$	10. $A = \frac{1}{2} \cdot 12 \cdot 8 \cdot \sin 40° \approx 30.9$
11. $A = \frac{1}{2} \cdot 12 \cdot 31 \cdot \sin 62 \approx 164.2$	12. Vertex $\angle = 180 - 2(50) = 80°$ $A = \frac{1}{2}(20.4)(20.4) \sin 80 \approx 204.9$
13. $m\angle R = 180 - (38 + 17) = 125°$ $A = \frac{1}{2}(15)(31.6) \sin 125 \approx 194$	14. $12 = \frac{1}{2} \cdot 8 \cdot b \cdot \sin 30°$ $12 = 4b \cdot \sin 30°$ $3 = b \cdot \sin 30°$ $b = \frac{3}{\sin 30°} = 6$
15. $42 = \frac{1}{2} \cdot 24 \cdot b \cdot \sin 30°$ $42 = 12b \cdot \sin 30°$ $3.5 = b \cdot \sin 30°$ $b = \frac{3.5}{\sin 30°} = 7$	16. $12 = \frac{1}{2} \cdot 6 \cdot c \cdot \sin 30°$ $12 = 3c \cdot \sin 30°$ $4 = c \cdot \sin 30°$ $c = \frac{4}{\sin 30°} = 8$

Chapter 14. Quadrilaterals

14.1. Angles of Polygons

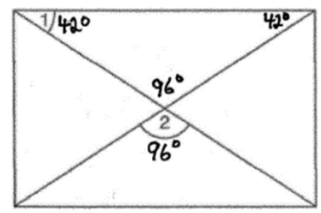
1. $\frac{5}{2}(5-3)=5$	2. $\frac{10}{2}(10-3)=35$
3. $(10-2)\cdot 180° = 1{,}440°$	4. $1{,}080° \div 8 = 135°$

5. Regular Polygons

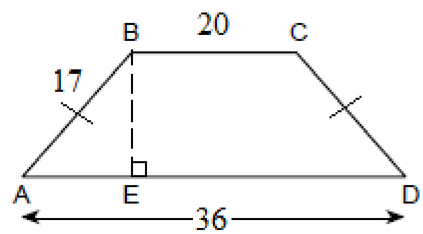
Number of sides	Measure of an exterior angle
3	120°
4	90°
5	72°
6	60°
7	≈ 51.4°
8	45°
9	40°
10	36°

14.2. Properties of Quadrilaterals

1. (4) trapezoid	2. (1) rhombus
3. (3) rhombus	4. (3) the rhombus and square
5. (2) An isosceles trapezoid has a pair of ∥ bases and a pair of ≅ legs, but may not be a ▱.	6. In a ▱, diagonals bisect each other, so $AM = 10$.
7. In a ▱, opp ∠'s are ≅, so $6x-30=4x+10$ $2x=40$ $x=20$ $m\angle A = 6(20)-30=90$	8. In a ▱, diagonals bisect each other, so $AE = 5$. In a rhombus, diagonals are ⊥, so $\triangle AEB$ is a right △ with a right ∠ at E. $(BE)^2=(AB)^2-(AE)^2$ $(BE)^2 = 8^2-5^2=39$ $BE=\sqrt{39}\approx 6.24$ $BD = 2BE \approx 12.5$
9. $AE=\frac{1}{2}AC=9 \quad BE=\frac{1}{2}BD=12$ (diagonals bisect each other) $m\angle AEB = 90°$ (⊥ diagonals) $(AB)^2 = 9^2+12^2=225$ $AB=\sqrt{225}=15$ (Pythagorean Thm.)	10. $AE=\frac{1}{2}AC=6$ $(DE)^2+6^2=10^2$ $(DE)^2=64$ $DE=8$ $DB=2\cdot DE=16$

11. $m\angle ABC = 180 - 100 = 80°$ (consecutive ∠'s are supplementary) $m\angle DBC = \frac{1}{2}80 = 40°$ (diagonals are ∠ bisectors)	12. $m\angle 1 + m\angle 2 = 180 - 120 = 60°$ (consecutive ∠'s are supplementary) $m\angle 2 = 60 - 45 = 15°$
13. Label E at the intersection of diagonals. $m\angle AEM = 90°$ (⊥ diagonals) $m\angle AMT = 90 - 12 = 78°$	14. ∠1 is a base ∠ of an isosceles △.  $m\angle 2 = 96$
15. Since $ABCD$ is a rhombus, all sides are ≅, so $AB = AD = 24$. Diagonals of a rhombus are ⊥, so △ AEB is a right △ with a right ∠ at E. $\sin 30 = \frac{BE}{24}$ $BE = 24 \sin 30 = 12$ Diagonals bisect each other, so $DE = BE = 12$	16. Since $ABCD$ is a rhombus, the diagonals are ∠ bisectors, so $m\angle ABD = \frac{1}{2} m\angle ABC = 30.$ Diagonals of a rhombus are ⊥, so △ AEB is a right △ with a right ∠ at E. $\sin 30 = \frac{18}{AB}$ $AB = \frac{18}{\sin 30} = 36$ All sides of a rhombus are ≅, so $DC = AB = 36$.

14.3. **Trapezoids**

1. a) Diagonals are ≅, so $BD = 25$. b) $m\angle D = m\angle C = 180 - 105 = 75°$.	2. $GT = \frac{40-24}{2} = 8$ $(AG)^2 = 10^2 - 8^2 = 36$ $AG = \sqrt{36} = 6$ $LF = AG = 6$ (altitudes are ≅)
3. Draw altitude $\overline{CE}$. $ED = \frac{26-12}{2} = 7$ $(CE)^2 = 25^2 - 7^2$ $CE = \sqrt{576} = 24$	4. $AE = \frac{36-20}{2} = 8$ $(BE)^2 = 17^2 - 8^2 = 225$ $BE = \sqrt{225} = 15$ 
5. Draw altitude $\overline{AT}$. △ RAT is an isosceles right △ with legs of 6. $(RA)^2 = 6^2 + 6^2$ $RA = \sqrt{72} = 6\sqrt{2}$	6. △ RST They share the same base, $\overline{RS}$, and congruent altitudes (since $\overline{VT} \parallel \overline{RS}$).

14.4. Use Quadrilateral Properties in Proofs

1. $ABCD$ is a ▱	(Given)
$\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$	(opp sides of ▱s are $\cong$)
$\overline{BD} \cong \overline{BD}$	(Reflexive Prop)
$\triangle ABD \cong \triangle CDB$	(SSS)
2.	(Givens omitted)
$\overline{BO}$ and $\overline{NR}$ bisect each other	(diagonals of a ▱ bisect each other)
$\overline{NX} \cong \overline{RX}$ and $\overline{BX} \cong \overline{OX}$	(def of bisector)
$\angle BXN \cong \angle OXR$	(vertical $\angle$'s are $\cong$)
$\triangle BNX \cong \triangle ORX$	(SAS)
3.	(Givens omitted)
$\overline{AM} \cong \overline{DM}$	(def of midpoint)
$\angle A \cong \angle D$	(all $\angle$'s in a ▭ are $\cong$)
$\overline{AB} \cong \overline{CD}$	(opp sides of a ▭ are $\cong$)
$\triangle ABM \cong \triangle DCM$	(SAS)
$\overline{BM} \cong \overline{CM}$	(CPCTC)
4.	(Givens omitted)
$\overline{FH} \cong \overline{SL}$	(opp sides of ▱s are $\cong$)
$\overline{FH} \parallel \overline{SL}$	(opp sides of ▱s are $\parallel$)
$\angle AFH \cong \angle LSG$	(alternate interior $\angle$'s thm)
$\triangle LGS \cong \triangle HAF$	(AAS)
5.	(Givens omitted)
$\overline{AB} \cong \overline{CD}$	(all sides of a square are $\cong$)
$\angle B \cong \angle C$	(all $\angle$'s of a square are $\cong$)
$\overline{EF} \cong \overline{EF}$	(Reflexive Prop)
$\overline{BE} + \overline{EF} \cong \overline{FC} + \overline{EF}$,	
so $\overline{BF} \cong \overline{EC}$	(Addition Prop)
$\triangle ABF \cong \triangle DCE$	(SAS)
$\overline{AF} \cong \overline{DE}$	(CPCTC)
6.	(Givens omitted)
$\overline{AD} \cong \overline{BC}$	(opp sides of a ▭ are $\cong$)
$\angle A \cong \angle B$	(all $\angle$'s of a ▭ are $\cong$)
$\triangle ADF \cong \triangle BCE$	(ASA)
$\overline{AF} \cong \overline{BE}$	(CPCTC)
$\overline{EF} \cong \overline{EF}$	(Reflexive Prop)
$\overline{AF} - \overline{EF} \cong \overline{BE} - \overline{EF}$,	
so $\overline{AE} \cong \overline{BF}$	(Subtraction Prop)

7.		(Givens omitted)
	$\overline{PE} \cong \overline{OE}$	(all sides of a rhombus are ≅)
	$\angle EPR \cong \angle EOR$	(opp ∠'s of a rhombus are ≅)
	$\angle SPR - \angle EPR = \angle VOR - \angle EOR$, so $\angle SPE \cong \angle VOE$	(Subtraction Prop)
	$\angle SEP \cong \angle VEO$	(vertical ∠'s are ≅)
	$\triangle SEP \cong \triangle VEO$	(ASA)
	$\overline{SE} \cong \overline{EV}$	(CPCTC)

14.5. Prove Types of Quadrilaterals

1.	Yes, one pair of opp sides are both ∥ and ≅.	2.	Yes, both pairs of opp sides are ≅.
3.	(3) Diagonals are ⊥.		

4.	$\triangle AOB \cong \triangle COD$	(Given)
	$\overline{AB} \cong \overline{CD}$	(CPCTC)
	$\angle OAB \cong \angle OCD$	(CPCTC)
	$\overline{AB} \parallel \overline{CD}$	(alternate interior ∠'s converse)
	$ABCD$ is a ▱	(quad with pair of opp sides both ∥ and ≅ → ▱)
5.	$ABCD$ is a ▱, $DF = EB$	(Given)
	$\overline{AEB} \parallel \overline{DFC}$	(opp sides of a ▱ are ∥)
	$\overline{AB} \cong \overline{DC}$	(opp sides of a ▱ are ≅)
	$\overline{AB} - \overline{EB} \cong \overline{DC} - \overline{DF}$, so $\overline{AE} \cong \overline{FC}$	(Subtraction Prop)
	$AECF$ is a ▱	(quad with pair of opp sides both ∥ and ≅ → ▱)
6.	$ABCD$ is a ▱, $CEBF$ is a rhombus	(Given)
	$\overline{BE} \cong \overline{CE}$	(all sides of a rhombus are ≅)
	$2 \cdot \overline{BE} \cong 2 \cdot \overline{CE}$	(Multiplication Prop)
	$\overline{BD} \cong 2 \cdot \overline{BE}$, $\overline{AC} \cong 2 \cdot \overline{CE}$	(diagonals of a ▱ bisect each other)
	$\overline{BD} \cong \overline{AC}$	(Substitution)
	$ABCD$ is a ▭	(if a ▱ has ≅ diagonals, then it is a ▭)
7.		(Givens omitted)
	$\angle BFA, \angle BFC, \angle DEC, \angle DEA$ are right ∠'s	(def of ⊥)
	$\angle BFA \cong \angle DEC$	(right ∠'s are ≅)
	$\overline{FE} \cong \overline{FE}$	(Reflexive Prop)
	$\overline{AE} - \overline{FE} \cong \overline{FC} - \overline{FE}$, so $\overline{AF} \cong \overline{EC}$	(Subtraction Prop)
	$\triangle BFA \cong \triangle DEC$	(AAS)
	$\overline{AB} \cong \overline{CD}$ and $\overline{BF} \cong \overline{DE}$	(CPCTC)
	$\angle BFC \cong \angle DEA$	(right ∠'s are ≅)
	$\triangle BFC \cong \triangle DEA$	(SAS)
	$\overline{AD} \cong \overline{CB}$	(CPCTC)
	ABCD is a ▱	(quad with both pairs of opp sides ≅ → ▱)

Chapter 15. Circles

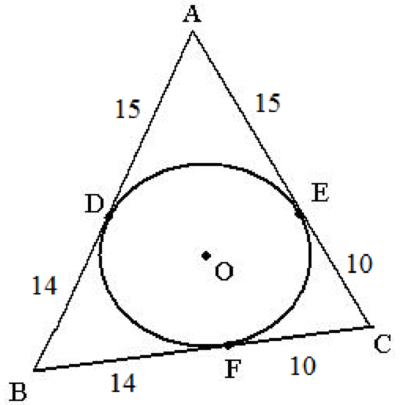
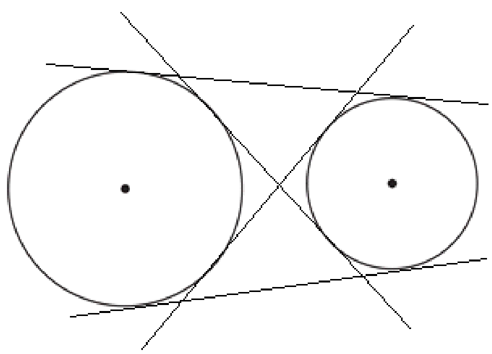
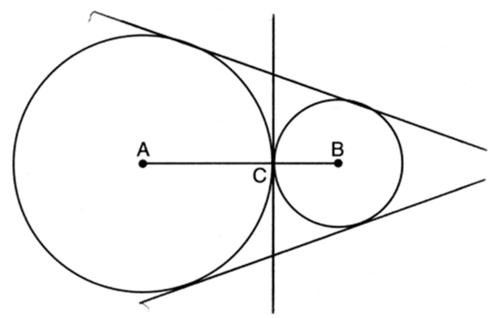
15.1. Circumference and Rotation

1. $D = 1000$ ft. $C = 2 \cdot 5\pi = 10\pi$ ft. $R = \frac{D}{C} = \frac{1000}{10\pi}$ ft ≈ 31.8 It must make at least 32 revolutions.	2. $D = 100$ ft. = 1200 in. $C = 8\pi$ in. $R = \frac{D}{C} = \frac{1200}{8\pi}$ in ≈ 47.7 47 clocks can be framed.
3. $C = 2\pi$ $D = C \cdot R = 2\pi \cdot 1100.5 \approx 6{,}914.65$ ft. $6{,}914.65\ ft \times \frac{1\ mile}{5{,}280\ ft} \approx 1.3\ miles$	4. $D = 2$ mi. = 10,560 ft. $C = 394\pi$ ft. $R = \frac{10{,}560}{394\pi}$ ft. ≈ 8.5
5. $D = 1$ mile = 5,280 ft. $C = 2\pi$ $R = \frac{5280}{3 \cdot 2\pi}$ ft. ≈ 280.1 281 rotations are needed.	

15.2. Arcs and Chords

1. (4) supplementary	2. (4) right
3. $\frac{2}{12} = \frac{1}{6}$ $\qquad \frac{1}{6}(360°) = 60°$	4. $m\angle ABC = 30°$
5. $m\angle AOC = 48°$	6. $m\,\widehat{BC} = 140°$ $m\,\widehat{AC} = 180 - 140 = 40°$
7. ∥ chords intercept ≅ arcs, so $m\,\widehat{DB} = \frac{180 - 110}{2} = 35°$	8. $\frac{180 - 80}{2} = 50°$
9. $m\angle A = 90°$, so $m\angle C = 35°$	10. $m\,\widehat{GFE} = 86 \times 2 = 172°$ $m\angle F = 180 - 86 = 94°$
11. $m\angle ADC = \frac{132 + 82}{2} = 107°$	12. $15x = 360°$ $x = 24°$ $m\,\widehat{FE} = 48°$ and $m\,\widehat{GD} = 168°$ $m\angle D = m\angle G = \frac{1}{2} m\,\widehat{FE} = 24°$ *or* $m\angle E = m\angle F = \frac{1}{2} m\,\widehat{GD} = 84°$

15.3. Tangents

1. Perimeter = 78.  	2. $\triangle ABC$ is a right $\triangle$. $(AC)^2 = 8^2 + 15^2$ $(AC)^2 = 289$ $AC = 17$ AD is a radius. $CD = AC - AD = 17 - 8 = 9$
3. $\overline{AB} \cong \overline{AC}$, so $\triangle ABC$ is an isosceles $\triangle$. $m\angle A = 180 - 2(66) = 48°$	4. $m\,\widehat{AB} = 180 - 38 = 142°$, so $m\angle AOB = 142°$
5. $m\,\widehat{RS} = 180 - 54 = 126°$.	6. The measure of minor arc DC is $360 - 246 = 114°$. $m\angle DEC = 180 - 114 = 66°$.
7. 	8. 

15.4. Secants

1. a) tangent b) secant c) diameter d) chord	2. $x = \frac{86 - 44}{2} = 21°$
3. $m\angle P = \frac{70 - 20}{2} = 25°$	4. $30 = \frac{140 - m\widehat{CD}}{2}$, so $m\,\widehat{CD} = 80°$. $m\,\widehat{DE} = 360 - (140 + 80) = 140°$.
5. $(PA)^2 = (PB)(PC)$ $(PA)^2 = (4)(16)$ $(PA)^2 = 64$ $PA = 8$	6. $(PQ)(PR) = (PS)(PT)$ $(6)(24) = (8)(PT)$ $PT = 18$

7. $(BC)(AC) = (EC)(DC)$ $(3)(12) = (EC)(9)$ $EC = 4$	8. $x^2 = 3(x+18)$ $x^2 = 3x + 54$ $x^2 - 3x - 54 = 0$ $(x-9)(x+6) = 0$ $x = 9$
9. a) Let $x = TM$. $x(x+2) = (12)(2)$ $x^2 + 2x - 24 = 0$ $(x+6)(x-4) = 0$ $x = 4$ $RT = 6 + 4 = 10$ b) $(PS)^2 = (8)(18)$ $(PS)^2 = 144$ $PS = 12$	
10. a) $m\,\widehat{ACB} = 360 - (56 + 112) = 192$ $m\,\widehat{CB} = \frac{1}{4}(192) = 48$ $m\angle CEB = \frac{56+48}{2} = 52$ b) $m\angle F = \frac{192-112}{2} = 40$ c) $m\angle DAC = \frac{112+48}{2} = 80$	
11. Because they are radii of the same circle, $BP = a$ and $BQ = a$. By the Corollary to the Intersecting Secants Thm, $(AC)^2 = AP \cdot AQ$. $b^2 = (c-a)(c+a)$ [substitute b for AC, $(c-a)$ for AP, and $(c+a)$ for AQ] $b^2 = c^2 - a^2$ [multiply the binomials] $a^2 + b^2 = c^2$ [add a^2 to both sides]	

15.5. Circle Proofs

1.

Statements		Reasons
Circle O, $\widehat{AB} \cong \widehat{AC}$		Given
$\overline{AB} \cong \overline{AC}$	(S)	If two arcs are ≅, their chords are ≅
$\overline{AO} \cong \overline{AO}$	(S)	Reflexive Prop
$\overline{OC} \cong \overline{OB}$	(S)	All radii in a circle are ≅
$\triangle AOC \cong \triangle AOB$		SSS

Alternately, $\overline{AB} \cong \overline{AC}$ could be replaced with $\angle AOC \cong \angle AOB$ (central ∠'s of ≅ arcs are ≅), and then $\triangle AOC \cong \triangle AOB$ by SAS.

2.

Statements		Reasons
Circle Q, $\overline{PQR} \perp \overline{ST}$		Given
$\overline{PQR} \cong \overline{PQR}$	(S)	Reflexive Prop
$\angle PRS$, $\angle PRT$ are right $\angle$'s		$\perp$ segments form right $\angle$'s
$\angle PRS \cong \angle PRT$	(A)	Right $\angle$'s are $\cong$
$\overline{QR}$ bisects $\overline{ST}$		If a radius is $\perp$ to a chord, it bisects the chord
$\overline{RS} \cong \overline{RT}$	(S)	Def of bisector
$\triangle PRS \cong \triangle PRT$		SAS
$\overline{PS} \cong \overline{PT}$		CPCTC

3.

Statements		Reasons
Tangents $\overline{PA}$ and $\overline{PB}$, radii $\overline{OA}$ and $\overline{OB}$, and $\overline{OP}$ intersects the circle at C.		Given
$\overline{OA} \perp \overline{PA}$ and $\overline{OB} \perp \overline{PB}$		a tangent is $\perp$ to a radius at the point of tangency
$\angle PAO$ and $\angle PBO$ are right $\angle$'s		$\perp$ lines form right $\angle$'s
$\overline{OP} \cong \overline{OP}$	(H)	Reflexive Prop
$\overline{OA} \cong \overline{OB}$	(L)	all radii in a circle are $\cong$
$\triangle AOP \cong \triangle BOP$		HL
$\angle AOP \cong \angle BOP$		CPCTC

4.

Statements		Reasons
Diameter $\overline{BOD}$, m $\widehat{BR} = 70$, m $\widehat{YD} = 70$		Given
$m\angle RDB = m\angle YBD = 35$	(A)	The measure of an inscribed $\angle$ is half its intercepted arc
$\angle BRD$ and $\angle DYB$ are right $\angle$'s		an inscribed $\angle$ of a semicircle is a right $\angle$
$\angle BRD \cong \angle DYB$	(A)	right $\angle$'s are $\cong$
$\overline{BD} \cong \overline{DB}$	(S)	Reflexive Prop
$\triangle RBD \cong \triangle YDB$		AAS

Alternately, $\overline{BR} \cong \overline{YD}$ (if two arcs are $\cong$, their chords are $\cong$), which leads to the $\triangle$s $\cong$ by HL

5.	

Statements		Reasons
Quad $ABCD$ inscribed in circle O, $\overline{AB} \parallel \overline{DC}$, diagonals $\overline{AC}$ and $\overline{BD}$		Given
$\widehat{AD} \cong \widehat{BC}$		∥ chords intercept ≅ arcs
$\angle BDC \cong \angle ACD$	(A)	inscribed ∠'s that intercept ≅ arcs are ≅
$\angle DAC \cong \angle DBC$	(A)	inscribed ∠'s that intercept the same arc are ≅
$\overline{AD} \cong \overline{BC}$	(S)	≅ chords intercept ≅ arcs
$\triangle ACD \cong \triangle BDC$		AAS

6.

Statements		Reasons
$\overline{AD}$ is a diameter, $\overline{AD} \parallel \overline{BC}$		Given
$\angle BEA \cong \angle CED$	(A)	vertical ∠'s are ≅
$\angle BAC \cong \angle CDB$	(A)	inscribed ∠'s that intercept the same arc are ≅
$\widehat{AB} \cong \widehat{DC}$		∥ chords intersect ≅ arcs
$\overline{AB} \cong \overline{DC}$	(S)	≅ chords intersect ≅ arcs
$\triangle BAE \cong \triangle CDE$		AAS
$\overline{BE} \cong \overline{CE}$		CPCTC

15.6. Arc Lengths and Sectors

1. $\frac{90}{360} = \frac{L}{30\pi}$ $L = 7.5\pi$ feet	2. $\frac{40}{360} = \frac{L}{12\pi}$ $L = \frac{4}{3}\pi$ m
3. Central ∠ = 180 – 150 = 30° $\frac{30}{360} = \frac{L}{34\pi}$ $L \approx 9$ cm	4. $\frac{\theta}{360} = \frac{14\pi}{36\pi}$ $\theta = 140°$
5. $\frac{\theta}{360} = \frac{247}{2\pi \cdot 150}$ $\theta \approx 94°$	6. $\frac{165}{360} = \frac{L}{2\pi \cdot 2.4}$ $L \approx 6.9$ meters
7. $\frac{40}{360} = \frac{S}{36\pi}$ $S = 4\pi$ sq. units	8. Central ∠ = 360 – 45 = 315° $\frac{315}{360} = \frac{S}{16\pi}$ $S = 14\pi$ sq. cm
9. Area of sector: $\frac{90}{360} = \frac{S}{100\pi}$ $S = 25\pi$ Area of △: $\frac{1}{2}(10)(10) = 50$ Area of segment = $25\pi - 50$ sq. units	10. Area of sector: $\frac{120}{360} = \frac{S}{144\pi}$ $S = 48\pi$ sq. units Area of △: $\frac{1}{2}(12)(12)sin120° = 36\sqrt{3}$ Area of segment = $48\pi - 36\sqrt{3}$ sq. units
11. $\frac{9}{360} = \frac{L}{2\pi \cdot 3954}$ $L \approx 621.1$ miles	

15.7. Radians [CC]

1. a) $45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$ rad b) $270 \cdot \frac{\pi}{180} = \frac{3\pi}{2}$ rad c) $150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$ rad	2. a) $\frac{\pi}{6} \cdot \frac{180}{\pi} = 30°$ d) $\frac{5\pi}{9} \cdot \frac{180}{\pi} = 100°$ b) $\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225°$ e) $\frac{8\pi}{5} \cdot \frac{180}{\pi} = 288°$ c) $\frac{3\pi}{5} \cdot \frac{180}{\pi} = 108°$
3. $2.5\left(\frac{180}{\pi}\right) \approx 143.2°$	4. $216\left(\frac{\pi}{180}\right) \approx 3.8$ rad
5. $65 = 5r$ $r = 13$ feet	6. $20 = 2.5r$ $r = 8$ ft.
7. $4\pi = \frac{2\pi}{3} r$ $r = 6$ ft.	8. $18 = \frac{3}{4} r$ $r = 24$ cm
9. $L = 2 \cdot 4 = 8$ inches	10. $L = \frac{\pi}{4} \cdot 12 = 3\pi \approx 9.4$ inches
11. $L = \frac{4\pi}{3} \cdot 12 = 16\pi \approx 50$ cm	12. $L = \frac{2\pi}{5} \cdot 18 = 7.2\pi \approx 23$ inches
13. $8\pi = \theta \cdot 10$ $\theta = \frac{4\pi}{5}$ rad	14. $\frac{\theta}{2\pi} = \frac{S}{\pi r^2}$ $\frac{0.97}{2\pi} = \frac{S}{81\pi}$ $S = 39.285$ cm^2

Chapter 16. Solids

16.1. Volume of a Sphere

1. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 = 972\pi$ m³	2. $r = \frac{1}{2} \cdot \frac{29.5}{\pi} = \frac{14.75}{\pi}$ $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{14.75}{\pi}\right)^3 \approx 433.5$ cu in.
3. $V_{before} = \frac{4}{3}\pi(3)^3 = 36\pi$ $V_{after} = \frac{4}{3}\pi(6)^3 = 288\pi$ $288\pi - 36\pi = 252\pi$ in³	

16.2. Volume of a Prism or Cylinder

1. $V = Bh = 6 \cdot 8 \cdot 4 = 192$ ft³	2. $V = e^3 = (1.5)^3 = 3.375$ cm³
3. $V = \pi r^2 h = \pi(4)^2(10) \approx 502.65$ in³	4. $V = \pi r^2 h = \pi(6)^2(15) \approx 1696.5$ in³
5. $V = \pi r^2 h$ $32\pi = \pi r^2(2)$ $r^2 = 16$ $r = \sqrt{16} = 4$ in.	6. $(x-2)(x+1)(2x) =$ $(x^2 - x - 2)(2x) =$ $2x^3 - 2x^2 - 4x$
7. $V = 5^3 = 125$ $V = 10^3 = 1000$ $1000 \div 125 = 8$	8. $V = \pi r^2 h$ $342 = 9\pi h$ $h = \frac{38}{\pi}$ $36 \div \frac{38}{\pi} = \frac{36\pi}{38} \approx 2.97$ 2 cans
9. $V_{larger} = Bh = 12 \cdot 30 \cdot 16 = 5760$ $V_{smaller} = Bh = 6 \cdot 12 \cdot 9 = 648$ $5760 - 648 = 5{,}112$ in³	
10. $V_{container} = (20)(15)(10) = 3{,}000$ in³ $V_{cup} = \pi(2)^2(5) = 20\pi$ in³ $3{,}000 \div 20\pi \approx 47.7$ 47 cups	
11. $V_{prism} = Bh$ $V_{cylinder} = \pi r^2 h$ $V = (5)(3.5)(7) = 122.5$ $V = \pi(2.5)^2(7) \approx 137.4$ Cylinder by 14.9 in³	

16.3. Volume of a Pyramid or Cone

1. $V = \frac{1}{3}Bh = \frac{1}{3}(10)(8)(6) = 160$ in³	2. $V = 3 \times 8 = 24$ cm³
3. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(8) = 24\pi$ in³	4. $V = \frac{1}{3}Bh$ $256 = \frac{1}{3}B(12)$ $B = 64$ $s = \sqrt{64} = 8$
5. $V = 5^3 + \frac{1}{3}(25)(6) = 175$ cm³	

6.	a) $V_{cylinder} = \pi r^2 h = \pi \cdot 5^2 \cdot 17.8 \approx 1398.0087$ cu. units $V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 5^2 \cdot 6.2 \approx 162.3156$ cu. units $V_{container} = 1398.0087 + 162.3156 \approx 1560.324$ cu. units b) Half the volume of the container $= \frac{1}{2} \cdot 1560.324 \approx 780.162$ cu. units The cone is completely filled, so subtract $780.162 - 162.316 = 617.846$. The remaining water fills $\frac{617.846}{1398.009} \approx 0.442$ of the cylinder. The water reaches .0442 of the cylinder's height, $0.442 \times 17.8 \approx 7.9$ units. From the apex of the cone, the water reaches $6.2 + 7.9 \approx 14.1$ units.

16.4. <u>Density</u>

1. $W = VD = 100\ cm^3 \cdot \frac{11.34\ g}{1\ cm^3} = 1{,}134\ g$	2. $D = \frac{W}{V} = \frac{88.6\ g}{10\ cm^3} = 8.86\ g/cm^3$
3. $D = \frac{W}{V} = \frac{94.44\ g}{12\ cm^3} = 7.87\ g/cm^3$	4. $V = \frac{W}{D} = \frac{9.8}{0.7} = 14\ mL$
5. $V = \frac{1}{3}Bh = \frac{1}{3}(24)(10) = 80\ in^3$ $W = 80\ \cancel{in^3} \cdot \frac{2\ \cancel{g}}{1\ \cancel{cm^3}} \cdot \frac{(2.54)^3\ \cancel{cm^3}}{1\ \cancel{in^3}} \cdot \frac{1\ \cancel{kg}}{1000\ \cancel{g}} \cdot \frac{2.2\ lbs}{1\ \cancel{kg}} \approx 5.8\ lbs$	

16.5. <u>Lateral Area and Surface Area</u>

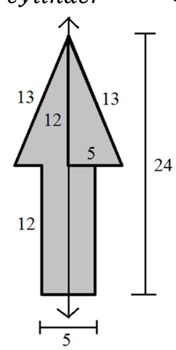
1. $SA = 2lw + 2hl + 2hw$ $= 2 \cdot 8 \cdot 6 + 2 \cdot 4 \cdot 8 + 2 \cdot 4 \cdot 6$ $= 96 + 64 + 48 = 208$ ft²	2. $SA = 2lw + 2hl + 2hw$ $= 2(3)(1.5) + 2(2)(3) + 2(2)(1.5)$ $= 9 + 12 + 6 = 27$ ft²
3. $SA = 2lw + 2hl + 2hw$ $= 2(3.0)(2.2) + 2(7.5)(3.0) + 2(7.5)(2.2)$ $= 13.2 + 45 + 33 = 91.2$ cm²	4. $SA = 2lw + 2hl + 2hw$ $= 2(5.5)(3) + 2(5.5)(6.75) + 2(3)(6.75)$ $= 33 + 74.25 + 40.5 = 147.75$ cm²
5. $V = (10)(2)(4) = 80$ cm³ $SA = 2(10)(2) + 2(10)(4) + 2(2)(4)$ $= 136$ cm²	6. $s = \sqrt[3]{64} = 4$ $SA = (6)(4)^2 = 96$ in²
7. $LA = 2\pi rh = 2\pi(6)(9) = 108\pi$ ft² $SA = 2\pi r^2 + LA = 72\pi + 108\pi = 180\pi$ ft²	8. $LA = 2\pi rh = 2\pi(5)(11) = 110\pi$ ft² $SA = 2\pi r^2 + LA = 50\pi + 110\pi = 160\pi$ ≈ 502.7 ft²
9. $2(x+3)(x-4)$ $+2(x+3)(5)$ $+2(x-4)(5)$ $2x^2 - 2x - 24$ $10x + 30$ $+\ 10x - 40$ $2x^2 + 18x - 34$	

10. $V = \pi r^2 h$, so $h = \frac{V}{\pi r^2}$

Radius r	Height h	Surface Area SA
2	$h = \frac{V}{\pi r^2} = \frac{144\pi}{4\pi} = 36$	$SA = 2\pi r^2 + 2\pi rh = 8\pi + 144\pi = 152\pi$
4	$h = \frac{V}{\pi r^2} = \frac{144\pi}{16\pi} = 9$	$SA = 2\pi r^2 + 2\pi rh = 32\pi + 72\pi = 104\pi$
6	$h = \frac{V}{\pi r^2} = \frac{144\pi}{36\pi} = 4$	$SA = 2\pi r^2 + 2\pi rh = 72\pi + 48\pi = 120\pi$

The cylinder with the radius of 4 has the least surface area.

16.6. Rotations of Two-Dimensional Objects

<table>
<tr><td>1. sphere</td><td>2. cone</td></tr>
<tr><td>3. cylinder
diameter = 10 in and height = 5 in
$V = \pi r^2 h = \pi(5)^2(5) = 125\pi$ in³</td><td>4. cylinder with a hemisphere on top
$V_{cylinder} + V_{hemisphere} =$
$\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = 27\pi + 18\pi = 45\pi$</td></tr>
<tr><td>5. 12-inch radius sphere with a 6-inch radius spherical cutout in its center
$V_{outer\ sphere} - V_{cutout} =$
$\frac{4}{3}\pi(12)^3 - \frac{4}{3}\pi(6)^3 =$
$2{,}304\pi - 288\pi = 2{,}016\pi$</td><td>6. cylinder with a cone on top
Cylinder has radius of 2.5 and height of 12.
$V_{cylinder} = \pi(2.5)^2(12) = 75\pi$


Cone has a height of $24 - 12 = 12$. Use the Pythagorean Thm to find the radius of the cone:
$r^2 + 12^2 = 13^2$, so $r = 5$
$V_{cone} = \frac{1}{3}\pi(5)^2(12)$
$= 100\pi$
$V_{object} = V_{cone} + V_{cylinder} =$
175π.</td></tr>
</table>

16.7. Cross Sections

1. (4)	2. (2)
3. (1)	4. (4)
5. (1)	6. (a) pentagon (b) rectangle

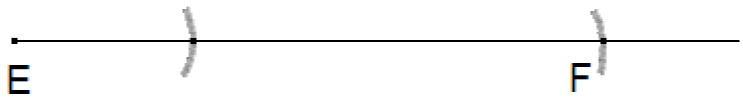
16.8. Cavalieri's Principle [CC]

1. $V = Bh = (4)(3)(6) = 72$ in³	2. $V = \pi r^2 h = \pi(2)^2(5) = 20\pi$ in³

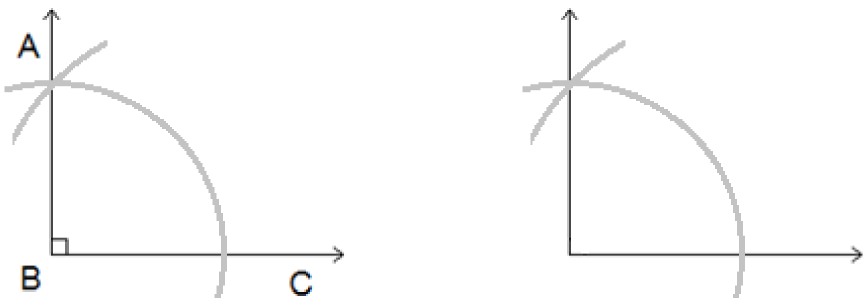
Chapter 17. Constructions

17.1. Copy Segments, Angles, and Triangles

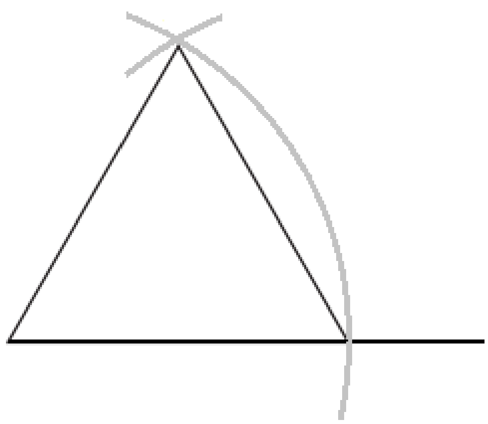
1.



2.



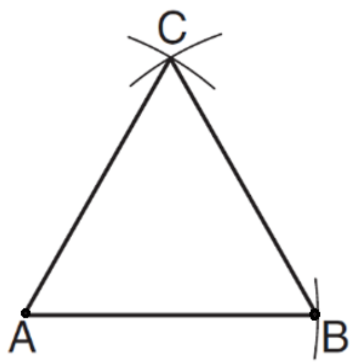
3.



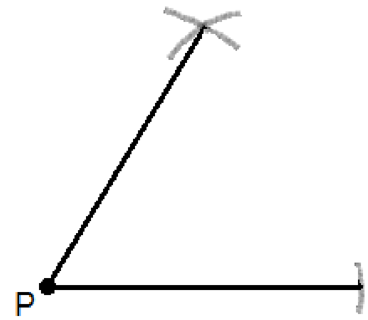
4. SAS	5. AAS

17.2. Construct an Equilateral Triangle

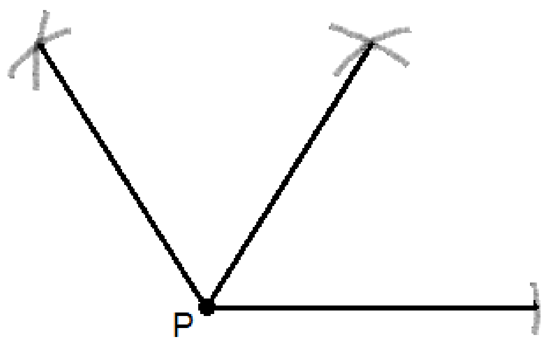
1.



2.

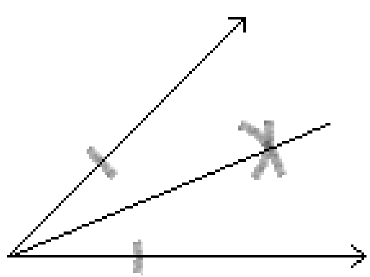


3.

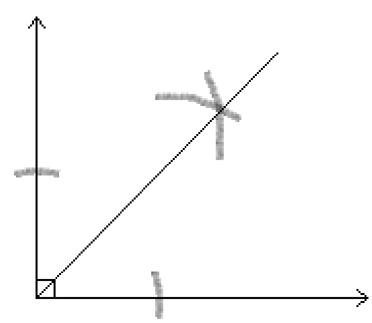


17.3. Construct an Angle Bisector

1.



2.



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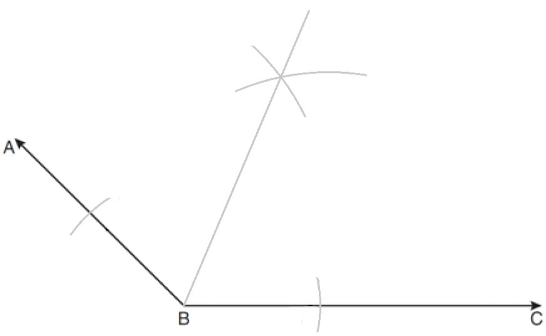
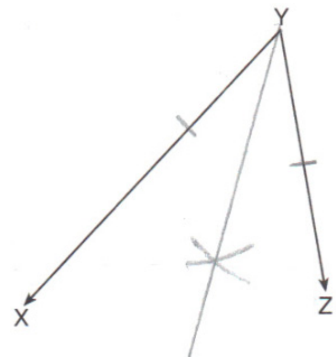
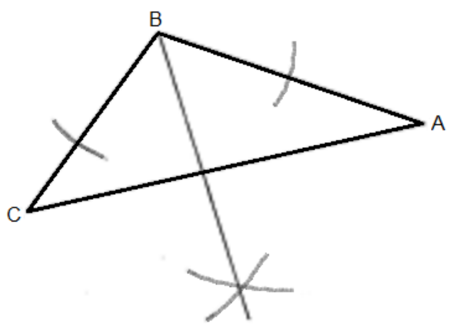


4.



5.



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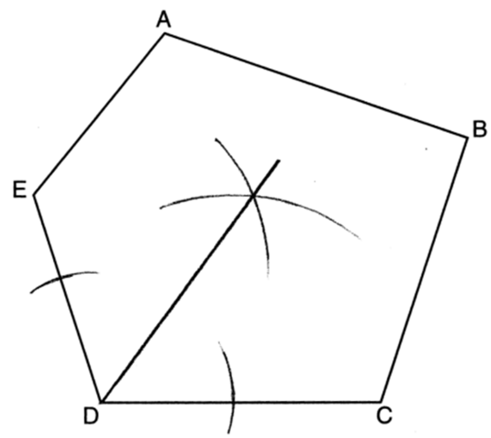
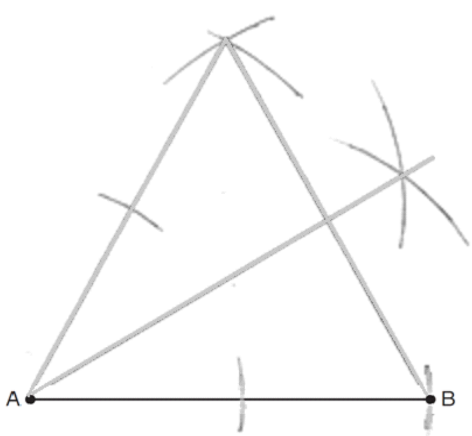
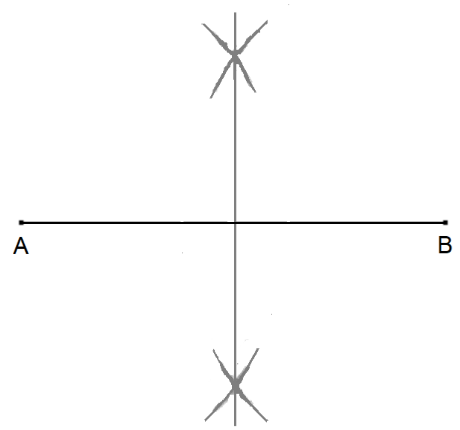


7.

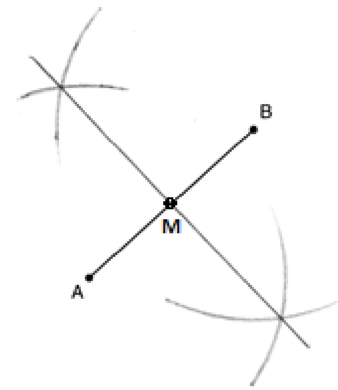


17.4. Construct a Perpendicular Bisector

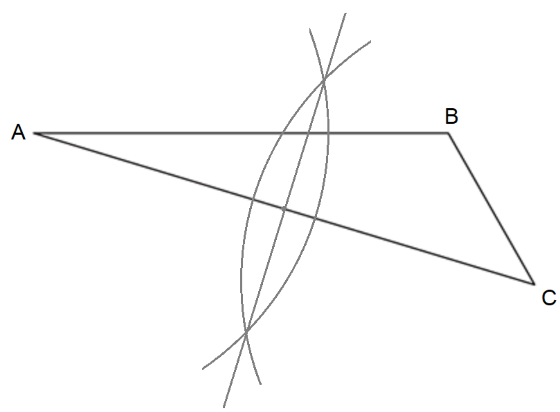
1.



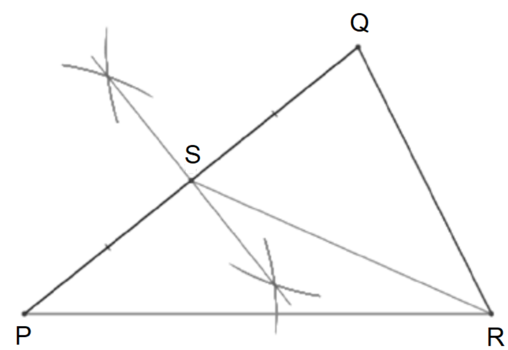
2.



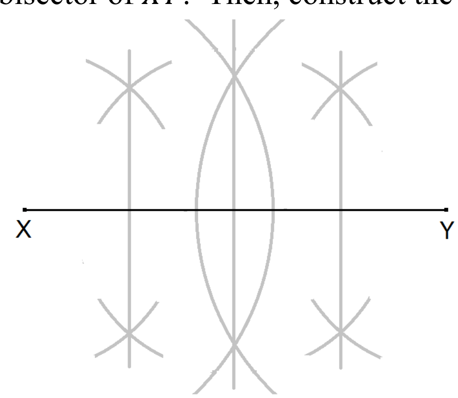
3.



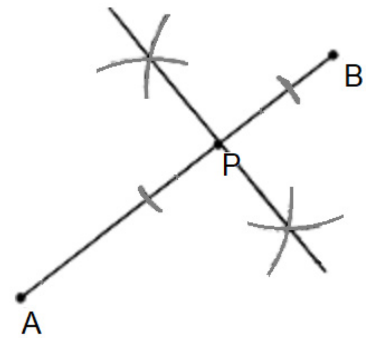
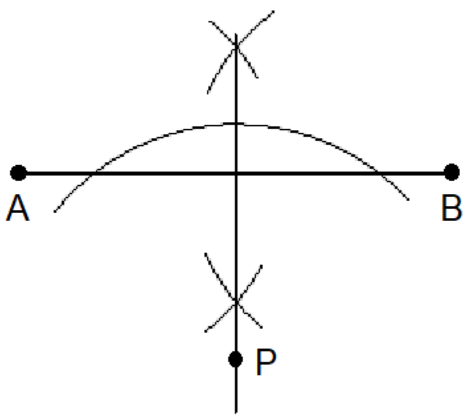
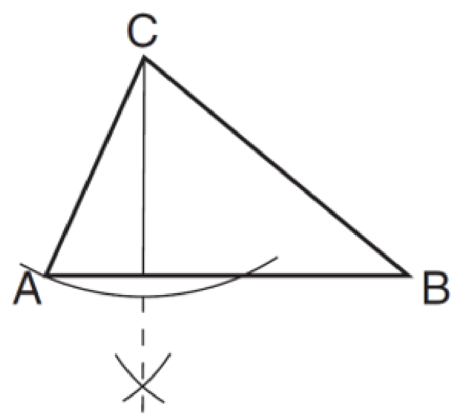
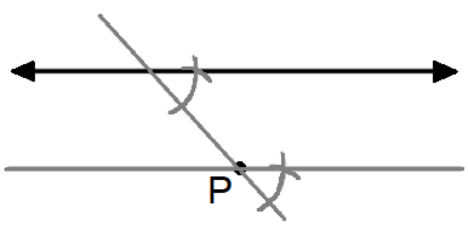
4.



5. Construct the perpendicular bisector of $\overline{XY}$. Then, construct the bisector of each half.

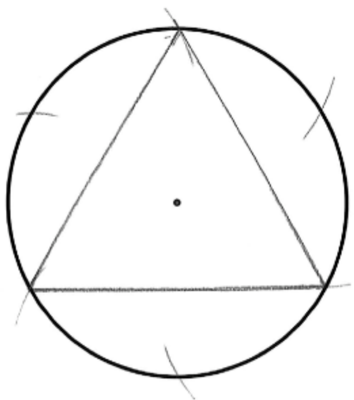


17.5. Construct Lines Through a Point

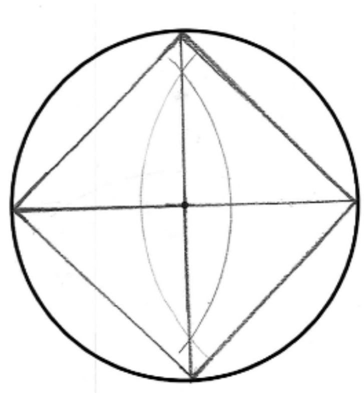
1. (2)	2. 
3. 	4. 
5. 	6. a) 5 b) 2 c) 1 d) 4 e) 3 f) 6

17.6. Construct Inscribed Regular Polygons

1.



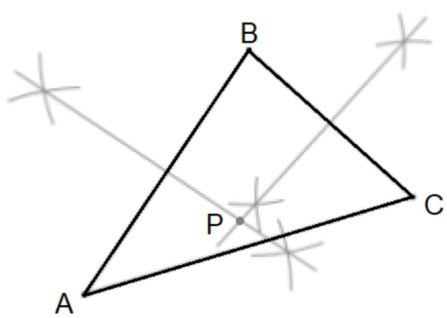
2.



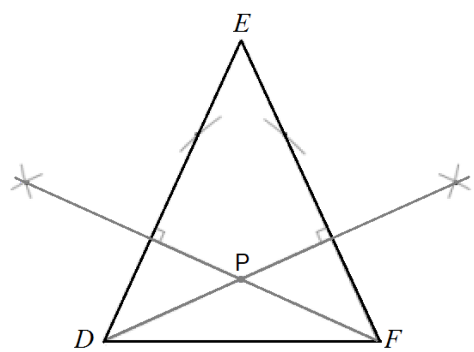
3.
a) Construct two perpendicular diameters (as if constructing an inscribed square).
b) Construct angle bisectors of the 90° central angles, forming eight 45° central angles.
c) The endpoints of the four diameters are the vertices of the regular octagon. Draw the chords between them.

17.7. Construct Points of Concurrency [NG]

1.



2.



17.8. Construct Circles of Triangles [NG]

1. (1)	2. (4)
3. a) 2 b) 1 c) 5 d) 4 e) 3	