Pre-Algebra Review

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Number Sets

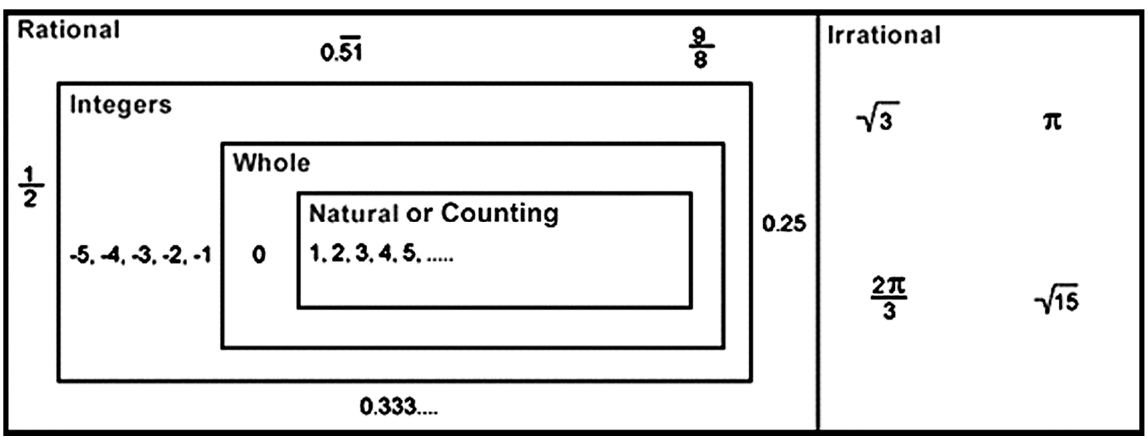
Counting numbers (also called **Natural numbers**): {1, 2, 3, 4, 5, …}
Whole numbers include the counting numbers and zero: {0, 1, 2, 3, 4, 5, …}
Integers include the whole numbers and their opposites: {… -3, -2, -1, 0, 1, 2, 3, …}

Rational numbers can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
Every rational number can be expressed as a **terminating or repeating decimal**.

Irrational numbers are all the real numbers that are *not* rational; that is, they cannot be expressed as a quotient of integers, and their decimals are non-repeating and non-terminating. **π** is an irrational number. 3.14 is an *approximation* of π, rounded to the nearest hundredth.

The **Real numbers** include all the rational and irrational numbers, and are represented by all the points that make up a number line or a coordinate axis.

The subsets of the Real numbers are demonstrated below:



Set Notation

A **set** is a collection of objects.

Roster form lists the elements of a set inside braces { }.
Examples: {2, 3, 4} or {John, Paul, George, Ringo}

In roster form, **ellipses** (…) are often used to show that a pattern continues.
Examples: The set of counting numbers can be written as {1, 2, 3,…}
The set of integers can be written as {…, –3, –2, –1, 0, 1, 2, 3,…}

A **finite set** has a certain number of elements; an **infinite set** does not.
Examples: The set of counting numbers less than 6, {1, 2, 3, 4, 5}, is finite because it has 5 elements, but the set of counting numbers, {1, 2, 3,…}, is infinite

An **empty set** (aka Null set) is a set with no elements, symbolized by {} or $\emptyset$;
Example: The set of even prime numbers greater than 2 = {}

Interval notation uses parentheses ( ) and/or brackets [ ] to name the endpoints (lower and upper bounds) of a set of all real numbers between those endpoints. These correspond to the open and closed circles in the graph of an inequality on a number line.

- A parenthesis represents an "open" endpoint ($>$ or $<$; not included in the set)
- A bracket represents a "closed" endpoint ($\geq$ or $\leq$; included in the set)

Example: (–1,5] represents all real numbers *x* such that $-1 < x \leq 5$.

If a set has no upper bound, the **infinity symbol** ∞ is used; for no lower bound, –∞ is used. In either case, since there is no endpoint, use a parenthesis.
Examples: [3,∞) for $x \geq 3$ (–∞,2) for $x < 2$ (–∞,∞) for all real numbers

Inclusive means endpoints are included (closed); **exclusive** means they are not (open).
Example: The set of integers between 3 and 10, inclusive, is [3, 10].

Set-builder notation: uses braces { } and a vertical bar | to define a set by the properties that its members must satisfy. It will often start with "{*x*|" which is read as "the set of all *x* such that."
Example: {3, 4, 5} can be written as {*x*| 3 ≤ *x* ≤ 5, where *x* is a whole number}

An alternate format of set-builder notation places the type of number after the variable.
Example: {3, 4, 5} can also be written as {*x* whole | 3 ≤ *x* ≤ 5 }

The **inequality symbols** are important: ≤ is closed (*inclusive*); < is open (*exclusive*).
Example: {3, 4, 5} can also be written as { *x* whole | 2 < *x* < 6 }

Order of Operations

Order of Operations:

1. Exponents
2. Multiplication and Division (left to right)
3. Addition and Subtraction (left to right)

Note that steps 2 and 3 are performed **left to right**.

Examples: 6 ÷ 3 × 2 results in 4 (perform the division first, going left to right)
6 – 3 + 2 results in 5 (perform subtraction first, going left to right)

Parentheses (or brackets, absolute value symbols, fraction bars, radical signs, etc.) can change the normal order of operations.

Examples: 3 × (4 + 5) forces addition to be performed before multiplication
$(2x)^2$ means square the product, not just the x

When an expression has **multiple layers** of parentheses or brackets, work from the innermost first.

Example: 2 × [9 + (5 – 2)] = 2 × [9 + 3] = 2 × 12 = 24

When entering expressions with a **fraction bar** into a calculator, be sure to enter the numerator in parentheses and the denominator in parentheses.

Example: $\frac{9+3}{3+1}$ should be entered as (9 + 3)÷(3+1), resulting in 12/4 = 3.
If parentheses are omitted, 9 + 3 ÷ 3 + 1 would result in 9 + 1 + 1 = 11.

Integers

Absolute Value: the distance that a number is from zero, written using the vertical symbols, | |

Examples: |5| is 5 |–8| is 8 |0| is 0

On the calculator, use [MATH] [NUM] [abs(] for absolute value.

Multiplying or dividing two integers:

Same Signs → Positive result
Different Signs → Negative result

Examples: (–5)(–2) = 10 (same signs) (5)(–2) = –10 (different signs)

$\frac{-12}{-3} = 4$ (same signs) $\frac{-12}{3} = -4$ (different signs)

Double Signs Rules for adding or subtracting:

Eliminate "Double Signs" and rewrite without parentheses around integers.

Same Signs → Plus
Different Signs → Minus

$$a + (+b) = a + b$$
$$a + (-b) = a - b$$
$$a - (+b) = a - b$$
$$a - (-b) = a + b$$

Adding two integers:

Same Signs → Add the absolute values, and keep the same sign
Different Signs → Subtract the absolute values, and keep the sign of the larger

Examples: Using the *Double Signs Rule* and then adding:

(+3) + (+2)	= 3 + 2 = 5	(–3) + (–2)	= –3 – 2 = –5
(+3) + (–2)	= 3 – 2 = 1	(–3) + (+2)	= –3 + 2 = –1
(+3) – (+2)	= 3 – 2 = 1	(–3) – (+2)	= –3 – 2 = –5
(+3) – (–2)	= 3 + 2 = 5	(–3) – (–2)	= –3 + 2 = –1

Be careful about the **Order of Operations**:

If there is an *operation in parentheses*, calculate it before adding or subtracting terms.

Example: 10 – (–8 + 6) = 10 – (–2) = 10 + 2 = 12

Exponents, multiplication and division are also done before adding or subtracting terms.

Example: $5 - (-2)^2$ = 5 – 4 = 1

Adding and subtracting a series of terms:

$$2 + (-3) - (-4) + 5 + (-6) \overset{(A)}{=} 2 \underline{-3} + 4 + 5 \underline{-6} \overset{(B)}{=} 11 \underline{-9} \overset{(C)}{=} 2$$

Explanation of steps:

(A) Use the "Double Signs" rules to eliminate the parentheses.
(B) Add the positive terms *[to get 11]* and the negative (underlined) terms *[to get –9]*.
(C) Use the rule for adding two integers with different signs to get the result.

Perfect Squares

Perfect Squares: numbers that are squares of whole numbers.

The perfect squares are:

$0^2 = 0$
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
… etc.

You can also generate the list of perfect squares by starting at 0 and adding consecutive odd numbers beginning with 1:

0
$0 + 1 = 1$
$1 + 3 = 4$
$4 + 5 = 9$
$9 + 7 = 16$
$16 + 9 = 25$
$25 + 11 = 36$
$36 + 13 = 49$
$49 + 15 = 64$
$64 + 17 = 81$
$81 + 19 = 100$
$100 + 21 = 121$
$121 + 23 = 144$
… etc.

Prime Factorization

Prime numbers: whole numbers greater than 1 that have exactly two factors, 1 and itself.
The first 5 prime numbers are: 2, 3, 5, 7, 11
Composite numbers: whole numbers greater than 1 that are not prime.

Divisibility Rules: a number is divisible by
2 if the last digit is divisible by 2 (even)
3 if the *sum of the digits* is divisible by 3
5 if the last digit is divisible by 5 (0 or 5)
7 if 2 times the last digit, subtracted from the rest of the number, is divisible by 7
11 if the difference between the sums of alternate digits is divisible by 11

After 11, the divisibility rules for the rest of the primes are similar to the rule for 7, except for the value we need to multiply the last digit by, and whether this is subtracted from or added to the rest of the number. In each case, we check whether the result is divisible by *that* prime number.

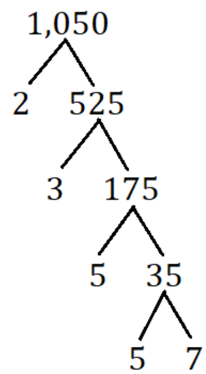
Prime to check	Multiply the last digit by	Then _____ it to/from the rest of the number
13	4	add
17	5	subtract
19	2	add
23	7	add
29	3	add
31	3	subtract

This table keeps going, but we'll stop here.

Examples: a) 161 is divisible by 7 because $16 - 2(1) = 14$, which is divisible by 7
b) 4301 is divisible by 11 because $(4 + 0) - (3 + 1) = 0$, and 0 is divisible by any number, including 11
c) 299 is divisible by 13 because $29 + 4(9) = 65$, which is divisible by 13

When the number is large, determining whether it is divisible by a prime greater than 5 may require repeating the process.
Example: To see if 85,969 is divisible by 13,
$8596 + 4(9) = 8632$
$863 + 4(2) = 871$
$87 + 4(1) = 91$, which is divisible by 13, so 85,969 is divisible by 13
[We could have gone a step further: $9 + 4(1) = 13$.*]*

Prime factorization: composite numbers can be expressed uniquely as a product of its prime factors. Use repeated division on the calculator, starting with 2. The end branches are the prime factors. For example, the prime factorization tree for 1,050 is:



1,050 is even, so divide by 2

525 is odd, but divisible by 3 (5+2+5=12)

175 is not divisible by 3, but by 5

35 is also divisible by 5

Prime factorization is $2 \cdot 3 \cdot 5 \cdot 5 \cdot 7$, or $2 \cdot 3 \cdot 5^2 \cdot 7$

Using prime factorization to reduce fractions

For example, to reduce $\frac{315}{1,050}$:

$$\frac{315}{1,050} \overset{(A)}{=} \frac{3 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 3 \cdot 5 \cdot 5 \cdot 7} \qquad \overset{(B)}{\frac{\cancel{3} \cdot 3 \cdot \cancel{5} \cdot \cancel{7}}{2 \cdot \cancel{3} \cdot \cancel{5} \cdot 5 \cdot \cancel{7}}} = \overset{(C)}{\frac{3}{2 \cdot 5}} = \overset{(D)}{\frac{3}{10}}$$

Explanation of steps:

(A) Write the prime factorization of the numerator and denominator.
(B) Cancel pairs of common factors appearing in both the numerator and denominator.
(C) Write the remaining factors (or if all factors are canceled, write 1).
(D) Multiply.

You can check the result on a calculator: 315 ÷ 1050 [MATH] [Frac]

GCF and LCM

The **GCF (greatest common factor)** is the largest common factor of a set of numbers.
(The GCF is also known as GCD or greatest common divisor.)

Use prime factorization to find the GCF.
From the prime factorizations, the GCF is the product of the common prime factors.
Example: To find the GCF of 27 and 36, we can write the prime factorizations.
$27 = 3^3$ $\quad$ $36 = 2^2 \cdot 3^2$
Then write out the ***common*** bases *[only 3]*, to their ***lowest*** powers *[2]*.
$3^2 = 9$ is the GCF.
This method will help us later when factoring out the GCF of an algebraic expression.

You can also find the GCF using the calculator: MATH NUM gcd(27 , 36)

We can use the GCF to reduce fractions.
Example: $\frac{27}{36} = \frac{27 \div 9}{36 \div 9} = \frac{3}{4}$ Divide the numerator and denominator by the GCF.

The **LCM (least common multiple)** is the smallest common multiple of a set of numbers.
(The LCM is also known as LCD or least common denominator when we are referring to the denominators of fractions.)

Use prime factorization to find the LCM.
From the prime factorizations of the numbers, the product of all the prime factors to their highest powers will result in the LCM.
Example: To find the LCM of 27 and 36, we can write the prime factorizations.
$27 = 3^3$ $\quad$ $36 = 2^2 \cdot 3^2$
Then write out ***all*** the prime bases *[2 and 3]*, each to their ***highest*** powers.
$2^2 \cdot 3^3 = 108$ is the LCM.
We use the LCM to add fractions. We also use it to solve equations involving fractions.

You can also find the LCM using the calculator: MATH NUM lcm(27 , 36)

Place Values

Every digit in a number has a **place value**.
Example: The following chart shows the place values for the digits of the number 1,234.

thousands (10^3)	hundreds (10^2)	tens (10^1)	ones (10^0)
1,	2	3	4

This means that $1{,}234 = (1 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (4 \times 10^0)$
(Note: $10^0 = 1$)

This also holds true for the digits to the **right of the decimal point**.
Example: The place values for the digits of 0.56789 are shown below.

	tenths (10^{-1})	hundredths (10^{-2})	thousandths (10^{-3})	ten-thousandths (10^{-4})	hundred-thousandths (10^{-5})
0.	5	6	7	8	9

$$\begin{aligned} \text{So, } 0.56789 &= (5 \times 10^{-1}) + (6 \times 10^{-2}) + (7 \times 10^{-3}) + (8 \times 10^{-4}) + (9 \times 10^{-5}) \\ &= \left(5 \times \frac{1}{10}\right) + \left(6 \times \frac{1}{100}\right) + \left(7 \times \frac{1}{1{,}000}\right) + \left(8 \times \frac{1}{10{,}000}\right) + \left(9 \times \frac{1}{100{,}000}\right) \end{aligned}$$

If we place an imaginary mirror over the ones place, the names of the place values to the right of the mirror are the same as their reflections to the left except that they end in "ths".

Note that the first place to the right of the decimal point is the "tenths" place.

Also note that the powers of 10 continue to decrease into the negative powers as we move to the right of the decimal point. 10^{-n} is the reciprocal of 10^n.

Rounding

Rounding to a place left of the decimal point.

MODEL PROBLEM 1: Round 5,236,174 to the *nearest hundred thousand*.

Solution: 5,200,000

(A) Determine which digit is in the specified place. *[2 is in the hundred thousands place.]*
(B) Look one place to the right. If the next digit is at least 5, increase the digit at the specified place by one; if not, keep the digit the same.
[To the right of the 2 is a 3, so the 2 will remain unchanged.]
(C) Change each place to the right of the specified place to zeroes.

MODEL PROBLEM 2: Round 5,236,174 to the *nearest hundred*.

Solution: 5,236,200

(A) Determine which digit is in the specified place. *[1 is in the hundreds place.]*
(B) Look one place to the right. If the next digit is at least 5, increase the digit at the specified place by one; if not, keep the digit the same.
[To the right of the 1 is a 7, so the 1 will increase by one to a 2.]
(C) Change each place to the right of the specified place to zeroes.

Rounding to a place right of the decimal point.
Follow the same steps as above except step (C) is changed.

MODEL PROBLEM 3: Round 0.5374 to the *nearest hundredth*.

Solution: 0.54

(A) Determine which digit is in the specified place. *[3 is in the hundredths place.]*
(B) Look one place to the right. If the next digit is at least 5, increase the digit at the specified place by one; if not, keep the digit the same.
[To the right of the 3 is a 7, so the 3 will increase by one to a 4.]
(C) Do NOT write zeroes or any other digits after the specified place.

MODEL PROBLEM 4: Round 6.975 to the *nearest tenth*.

Solution: 7.0

(A) Determine which digit is in the specified place. *[9 is in the tenths place.]*
(B) Look one place to the right. If the next digit is at least 5, increase the digit at the specified place by one; if not, keep the digit the same. *[To the right of the 9 is a 7, so the 9 will increase by one. However, since 9+1=10 and 10 has two digits, we need to change the 9 to a 0 and then carry the 1 to the left, changing the 6 into a 7.]*
(C) Do NOT write zeroes or any other digits after the specified place.

Precision of Measurements

Real life math problems tend to involve measurements. The **accuracy** of a measurement is its nearness to the true value. The **precision** of a measurement is the degree to which its accuracy is expressed.

It is reasonable to **report measurements** only to a level of *precision* that the tool allows. A ruler marked in millimeters is more precise than a ruler that is marked only in centimeters.

Example: If someone were to weigh an item on a scale that only had markings at every tenth of a pound, it would be reasonable to give a measure of 25.7 or 25.8 pounds. We could not report a measure of 25.726 pounds.

Also, **calculations based on measurements** should only be reported to a precision based on the number of *significant digits* of the least precise measurement that is given.

To determine the **number of significant digits**, count (a) all nonzero digits, (b) all zeros to the right of the decimal point after the last nonzero digit, and (c) all zeros between significant digits. Zeros at the end of whole numbers are not considered significant digits.

Examples: The number of significant digits in
31.25 is 4; 2.20 is 3; 0.00150 is 3; 3050.0 is 5; and 250,000 is 2.

When **adding or subtracting**, round to the *same place as the last significant digit* of the least precise measurement.

Example: 16.25 cm + 23.6 cm = 39.85 cm should be rounded to the nearest tenth, 39.9 cm, since 23.6 cm is the less precise measurement shown to only the units place.

When **multiplying or dividing**, the result should be rounded to the *same number of siginificant digits* as the measurement with the fewest significant digits.

Example: 16.25 cm × 23.6 cm = 383.5 cm^2 should be rounded to 384 cm^2, only three significant digits, since 23.6 cm has only three significant digits.

If it's possible to express an answer as an **exact fraction** instead of a rounded decimal, it is more accurate to do so.

Example: When dividing 17÷3, the calculator shows 5.6666667. Rather than rounding this decimal, it is more accurate to write an answer of $5\frac{2}{3}$.

Model Problem 1: *Reporting Measurements*

Victor claims to have measured the thickness of a penny with a ruler. Which of the following is most likely the value Victor measured?

(1) 0.061 in (3) 1.5 in

(2) 1.55 mm (4) 1.5 mm

Solution:
(4) 1.5mm

Explanation:
Use your knowledge of the given units and of the level of precision that the measuring tool allows. *[While (1) and (2) are both the correct measurements for the thickness of a penny, a simple ruler wouldn't be able to deliver that kind of precision. Choice (3) is clearly an incorrect measurement for the thickness of a penny, so the correct answer is (4).]*

Model Problem 2: *Significant Digits*

How many significant digits are in each of the following?

(a) 204,000 (b) 2.040 (c) 0.00204

Solution:
(a) 3
(b) 4
(c) 3

Explanation:
(a) Zeroes at the ends of whole numbers *[the last three zeroes here]* are not significant.
(b) Count zeroes to the right of the decimal point after the last nonzero digit. *[Both count.]*
(c) Don't count lead zeroes in a decimal. *[Only the last three digits are significant.]*

Model Problem 3: *Adding Measurements*

What is the sum of 6.6412 + 12.85 + 0.046 + 3.48 grams expressed to the correct number of significant digits?

Solution:
23.02 g.

Explanation:
For a sum, round to the *same place as the last significant digit* of the least precise measurement. *[The least precise measures, 12.85 and 3.48, are given to the hundredths place only, so the sum should be rounded to the same place value.]*

Model Problem 4: *Multiplying Measurements*

The dimensions of a rectangle are given as 8.7 by 3.16 inches. The area is calculated by multiplying, giving a result on the calculator of 27.492. To what place should this be rounded?

Solution:
To the units place, 27.

Explanation:
For a product, round to the *same number of significant digits* as the measurement with the fewest significant digits. *[8.7 has only two significant digits, so the result should as well.]*

Exponents

A **positive exponent** represents the number of times the base is used as a factor.

Example: $x^5 = x \cdot x \cdot x \cdot x \cdot x$ $x^1 = x$

A base with a **zero exponent** evaluates to 1: $x^0 = 1$

A base with a **negative exponent** evaluates to the reciprocal (*multiplicative inverse*) of the same base to the opposite (*positive*) exponent.

Example: x^{-3} is the reciprocal of x^3, which is $\frac{1}{x^3}$.

Addition and Subtraction Rules:
Must be like terms (same base *and* exponent): Keep the same exponent and add or subtract the coefficients

Examples: $x^5 + x^3$ cannot be combined; not like terms

$x^3 + x^3 = 2x^3$

$4x^3 - x^3 = 3x^3$

Multiplication Rule (with same base): Add the exponents

Example: $x^5 \cdot x^3 = x^{5+3} = x^8$ Why it works: $(x \cdot x \cdot x \cdot x \cdot x)(x \cdot x \cdot x) = x^8$

Division Rule (with same base): Subtract the exponents

Examples: $\frac{x^5}{x^3} = x^{5-3} = x^2$ Why it works: $\frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x \cdot x = x^2$

$\frac{x^3}{x^3} = x^{3-3} = x^0 = 1$ Why it works: $\frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = \frac{1}{1} = 1$

$\frac{x^3}{x^5} = x^{3-5} = x^{-2} = \frac{1}{x^2}$ Why it works: $\frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \frac{1}{x \cdot x} = \frac{1}{x^2}$

Power Rule: Multiply the exponent by the power

Example: $(x^5)^3 = x^{5 \cdot 3} = x^{15}$ Why it works: $(x^5)(x^5)(x^5) = x^{15}$

Rules Summary:

Operation	Exponents
Addition/Subtraction	Keep
Multiplication	Add
Division	Subtract
Raise to a Power	Multiply

A **term** is a number, a variable, or any product or quotient of numbers and variables.
A **monomial** is a single term without any variables in the denominator, such as $-2xy$.

Operations on Monomials:

a) Perform the given operation on the coefficients
b) For common bases, apply the proper rule for exponents

Examples:

$2x^2y+3x^2y=5x^2y$ Add coefficients $[2+3]$; keep variable parts $[x^2y]$

$(3a^2)(4ac)=12a^3c$ Multiply coefficients $[3\times4]$; add exponents $[a^{2+1}c]$

$\dfrac{30x^6y^3z^2}{2x^4y^3z}=15x^2z$ Divide coefficients $[30\div2]$; subtract exponents $[x^{6-4}y^{3-3}z^{2-1}]$

$(-3a^2b)^3=-27a^6b^3$ Raise coefficient to the power $[(-3)^3]$; multiply exponents $[a^{2\cdot3}b^{1\cdot3}]$

When Dividing Monomials: If the variable's **exponent is larger in the denominator** (i.e., the difference is negative), leave the variable in the denominator with a positive exponent.

Example: $\dfrac{8x^2y}{4xy^5}=\dfrac{2x}{y^4}$ y^4 remains in the denominator since $\dfrac{y}{y^5}=y^{1-5}=y^{-4}=\dfrac{1}{y^4}$

Model Problem

What is the product of $3w^2x$ and $(2w^2x^3y)^3$?

Solution:

$(3w^2x)(2w^2x^3y)^3=$

(A) $(3w^2x)(8w^6x^9y^3)=$

(B) $24w^8x^{10}y^3$

Explanation of steps:
Perform the operations using the normal order of operations, following the rules for exponents.

(A) *[Here, the second expression involves raising to a power, so this is performed first. Raise $2^3=8$ and then use the powers rule to multiplying the exponents by the power $w^{2\cdot3}x^{3\cdot3}y^3=w^6x^9y^3$]*

(B) *[Then multiply the monomials by multiplying the coefficients $3\times8=24$ and using the multiplication rule to add the exponents $w^{2+6}x^{1+9}y^3=w^8x^{10}y^3$]*

Scientific Notation

Scientific notation is generally used to write numbers with very large or very small absolute values. The notation uses the product of a decimal and a power of 10. The decimal must have a **single non-zero digit before the decimal point**.

To change from scientific to standard notation, the power of 10 tells you how many places to move the decimal point to the right (if positive) or to the left (if negative).
Examples:

9.3×10^{7}	Move decimal point 7 places right	93,000,000
2.9×10^{-6}	Move decimal point 6 places left	0.0000029

To enter scientific notation into the calculator, use the [2nd] [EE] keys in place of "× 10".

9.3×10^{7} is entered as: 9.3 [2nd] [EE] 7

To change a number into scientific notation:

1. Move the decimal point so that it comes **after the first non-zero digit** in the number.
2. Count how many places we would need to move the decimal point to get back to the original number. If the decimal point needs to move right, the power of 10 is positive; otherwise, the power of 10 is negative.
3. Write as the product of the decimal and the power of 10.

Changing large numbers to scientific notation:

93,000,000 9_3,000,000$_0$ 9.3×10^{7}

We'd need to move decimal point 7 places right to get back to the original number.

Changing small numbers to scientific notation:

0.0000029 0$_0$000002_9 2.9×10^{-6}

We'd need to move decimal point 6 places left to get back to the original number.

Alternatively, we can set the calculator to display in scientific notation and then enter the value. To have the calculator display all values in scientific notation:

1. Press [MODE] then the right arrow [▸] to select [Sci] and [ENTER]. Press [2nd] [QUIT].
2. Enter the number normally (for example, 0.0000029) and press [ENTER].
   The calculator displays 2.9 E -6, representing 2.9×10^{-6}.
3. *Important:* To return to standard display, press [MODE] [Normal] [ENTER] [2nd][QUIT].

To **multiply or divide** numbers in scientific notation:

1. Multiply or divide the decimals
2. Use the rules for exponents to multiply or divide the powers of 10
3. Adjust the decimal point if necessary, making sure to increase or decrease the power of 10 to compensate for the adjustment (when moving left, increase the power; when moving right, decrease the power)

Example: $\frac{(64\times10^9)}{(3.2\times10^6)}=\frac{64}{3.2}\times\frac{10^9}{10^6}=20\times10^3=2.0\times10^4$

We can also use the calculator to perform the operation:

1. Set to "Sci" display: [MODE] [Sci] [ENTER] [2nd] [QUIT]
2. Enter the expression: 64 [2nd] [EE] 9 ÷ 3.2 [2nd] [EE] 6 [ENTER]

MODEL PROBLEM 1: *CONVERTING TO SCIENTIFIC NOTATION*

Write 0.000257 in scientific notation. (*For this problem, do not use the calculator.*)

Solution: (A) (B)
2.57×10^{-4}

Explanation of steps:

(A) Move the decimal point to the right of the first non-zero digit *[after the 2]*.
(B) Count how many places we need to move the decimal point to return to the original number, and make that the power of 10. *[We need to move 4 places left, so 10^{-4}.]*

MODEL PROBLEM 2: *OPERATIONS*

The mass of a single oxygen molecule (O_2) is approximately $5.356 \text{ X } 10^{-26}$ kg. What is the approximate total mass of $5 \text{ X } 10^{20}$ oxygen molecules, written in scientific notation?

Solution:

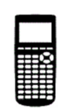
(A) $(5.356\times10^{-26})(5\times10^{20})=$

(B) $(5.356\times5)(10^{-26}\times10^{20})=$

$26.78\times10^{-6}=$

(C) 2.678×10^{-5}

Alternate Solution:
On the calculator,
[MODE] [Sci] [ENTER] [2nd] [QUIT]
5.356 [2nd] [EE] [(-)] 26 × 5 [2nd] [EE] 20 [ENTER]



Explanation of steps:

(A) Write the product or quotient *[since we are given the mass of one molecule and need to calculate the total mass of a large number of molecules, we need to find the product]*.
(B) Calculate the decimal part *[5.356 X 5 = 26.78]* and use the rules for exponents to calculate the power of 10 *[since we are multiplying powers of the same base, we add exponents: −26 + 20 = −6]*.
(C) Adjust the decimal point so that it appears after the first non-zero digit, and compensate the power of 10 accordingly *[moving the decimal point one place left means we have to increase the power of 10 by one, from −6 to −5, to compensate]*.

Fractions

Converting between fractions and decimals

To change a fraction to a decimal:
Divide the numerator by the denominator on a calculator.

Example: $\frac{1}{25}$ is entered as 1 ÷ 25 [ENTER] resulting in 0.04.

To change a decimal to a fraction:
On the calculator, using [MATH] [Frac]. If the entered value is irrational – for example, $\sqrt{2}$ – there is no equivalent fraction, so it is left as a decimal.
Example: Entering 0.04 [MATH] [Frac] results in 1/25.

Comparing fractions
You can compare two fractions by cross-multiplying and placing each cross product above the fraction whose numerator is used in the product. The fraction with the larger cross product above its numerator is the larger fraction. If the cross products are equal, the fractions are equivalent.

Which is larger, $\frac{5}{9}$ or $\frac{6}{10}$?

Solution:
(A) 50 54
(B) $\frac{5}{9} < \frac{6}{10}$

Explanation of steps:
(A) Place the cross product, 5 × 10 = 50, above the first fraction, and the cross product, 6 × 9 = 54, above the second.
(B) Since 50 < 54, the first fraction is less than the second fraction.

Multiplying fractions

$$\frac{5}{9}\times\frac{6}{10}\overset{(A)}{=}\frac{\cancel{5}}{\cancel{3}\cdot 3}\times\frac{\cancel{2}\cdot\cancel{3}}{\cancel{2}\cdot\cancel{5}}\overset{(B)}{=}\frac{1}{3}$$

Check on calculator: (5÷9)x(6÷10) [MATH] [Frac]

Explanation of steps:
(A) Write the prime factorization of each number. Cancel pairs of prime factors which are common to a numerator and denominator of *either* fraction.
(B) Multiply remaining factors across. If all factors are cancelled, write 1.

Dividing fractions

$$\frac{3}{10} \div \frac{1}{2} \overset{(A)}{=} \frac{3}{10} \times \frac{2}{1} \overset{(B)}{=} \frac{3}{\cancel{2} \cdot 5} \times \frac{\cancel{2}}{1} = \frac{3}{5}$$

Check on calculator: (3÷10) ÷(1÷2) [MATH] [Frac]

Explanation of steps:

(A) "Flip" the second fraction (change it to its reciprocal)
(B) Multiply, following the steps above for multiplying fractions

Adding or subtracting fractions

$$\frac{3}{10} + \frac{7}{15} \overset{(A)}{=} \frac{3}{2 \cdot 5} + \frac{7}{3 \cdot 5} \qquad \overset{(B)}{LCM = 2 \cdot 3 \cdot 5 = 30} \qquad \overset{(C)}{\frac{3 \cdot (3)}{30} + \frac{7 \cdot (2)}{30}} = \frac{9}{30} + \frac{14}{30} \overset{(D)}{=} \frac{23}{30}$$

Check on calculator: (3÷10)+(7÷15) [MATH] [Frac]

Explanation of steps:

(A) Change denominators to their prime factorizations.
(B) Find the LCM of the denominators. *[Remember to use **all** prime factors, each to their **highest** powers; see the section on GCF and LCM for details.]* This will be the denominator of the answer.
(C) Multiply each numerator by whatever prime factors are "missing" in its denominator.
(D) Add (or subtract) numerators across; the denominator stays the same.

Important Note: In all of the above examples, we could have used the calculator to find the results more quickly. However, when we learn to perform operations on algebraic fractions in Algebra II, the prime factorization methods we use here will work even with variables, so it is beneficial to learn the methods now.

Evaluate Expressions

An algebraic **expression** may contain numbers, variables, operations, and other mathematical symbols such as parentheses.
Example: $x-(y+1)$

To evaluate an algebraic expression:

1. Rewrite the expression by replacing each variable with its value in parentheses.
2. Evaluate using the correct order of operations.

The **absolute value** of a number n is the distance between n and 0, written using the vertical symbols, $|n|$. The absolute value of a positive number (or 0) is the number itself; the absolute value of a negative number is its opposite.
Examples: |5| is 5 |-8| is 8 |0| is 0
On the calculator, use [MATH] [NUM] [abs(] for absolute value.

Model Problem

What is the value of $3x-y^2$ when $x=5$ and $y=-3$?

Solution:

$3x-y^2=$

(A) $3(5)-(-3)^2=$

(B) $3(5)-9=15-9=6$

Explanation of steps:

(A) Rewrite the expression by replacing each variable with its value in parentheses.
[x with (5) and y with (–3)]

(B) Evaluate using the correct order of operations *[the exponent, then the multiplication, then the subtraction]*.

Check using the calculator:

1. Store the values of the variables: Display

   5 [STO▸][ALPHA] [X] [ENTER] $5 \rightarrow X$

   [(-)]3 [STO▸][ALPHA] [Y] [ENTER] $-3 \rightarrow Y$

2. Enter the expression:

   3 [ALPHA] [X] – [ALPHA] [Y] [x^2] [ENTER] 6

Formulas

A **formula** is an equation with a single variable on one side and an expression involving another variable (or variables) on the other.

Examples: $F = \frac{9}{5}C + 32$ finds the Fahrenheit (F) temperature for a given Celsius (C).

$A = \frac{bh}{2}$ finds the Area of a triangle (A) given its base (b) and height (h).

To evaluate a formula:

1. Substitute the given value(s) for the appropriate variable(s).
2. Evaluate the expression to find the value of the desired variable.

Model Problem

Convert −10° Celsius (C) to degrees Fahrenheit (F) using the formula, $F = \frac{9}{5}C + 32$,

Solution:

(A) (B)

$F = \frac{9}{5}(-10) + 32 = -18 + 32 = 14$ So, $-10°C = 14°F$

Explanation of steps:

(A) Substitute the given value *[–10]* for the appropriate variable *[C]*.
It is usually best to place the substituted value in parentheses.

(B) Evaluate the expression to find the value of the desired variable.

Solve Simple Linear Equations

An **equation** is a statement that one expression is equal to another. It contains an = sign.
Example: $3x - 1 = x + 5$

A **variable term** in an equation includes the variable as a factor (or the variable by itself).
A **constant term** in an equation does not include a variable factor.
Example: $3x + 5 = 35$ has a variable term [$3x$] and constant term [+5] on the left side.

The goal when solving an equation is to **isolate the variable** (transform it into *x* = *a value*).
Do this by using the reverse order of operations:

(a) add the opposite (additive inverse) of the constant term to both sides.
(b) divide both sides by (or multiply both sides by the reciprocal, or multiplicative inverse, of) the variable term's coefficient.

To check your solution:
Substitute your solution for the variable in the original equation. *It is usually best to use parentheses around the value when substituting.* Then, evaluate both sides of the equation to determine whether the solution makes the equation true.

MODEL PROBLEM 1: *ONE-STEP EQUATIONS*

Solve for *x*: $x - 6 = 12$

Solution:

$$\begin{array}{rcl} x - 6 &=& 12 \\ +6 && +6 \\ \hline x &=& 18 \end{array}$$

Check:

$$\begin{aligned} x - 6 &= 12 \\ (18) - 6 &= 12 \\ 12 &= 12 \checkmark \end{aligned}$$

Explanation:
To isolate the variable *[x]*, we need to eliminate anything else from the same side of the equation *[−6]*. We do this by performing the inverse operation *[+6]* to both sides of the equation.

Model Problem 2: *two-step equations*
Solve for *x*: $3x + 5 = 35$

Solution:

$$\begin{array}{lrcl} & 3x + 5 &=& 35 \\ \text{(A)} & -5 && -5 \\ \hline \text{(B)} & \dfrac{3x}{3} &=& \dfrac{30}{3} \\ & x &=& 10 \end{array}$$

Check:

$$\begin{aligned} 3(10) + 5 &= 35 \\ 35 &= 35 \checkmark \end{aligned}$$

Explanation of steps:

(A) Eliminate the constant term *[eliminate +5 by adding −5 to both sides]*.
(B) Eliminate the coefficient of the variable term *[eliminate the 3 by dividing both sides by 3]*.

Rates

A **ratio** is a comparison, by division, of two quantities.

The ratio of "*a* to *b*" can be expressed as $\frac{a}{b}$ or $a : b$.

A ratio can be **simplified** by dividing each quantity by the GCF, if the GCF > 1.

Example: The sides of a rectangle measure 40 and 15 inches. The sides are in the ratio 40:15. Since the GCF of 40 and 15 is 5, this ratio can be simplified to 8:3.

A **rate** is a special type of ratio involving two quantities measured in different units.

Example: A rate of 60 miles in 3 hours = $\frac{60 \text{ } miles}{3 \text{ } hours}$

A **unit rate** is a rate that contains a unit measure (1) in the denominator. A unit rate is usually expressed with the joining word, *per*, as in miles per gallon (*mpg*) or feet per second (*ft/sec*).

Example: $\frac{20 \text{ } miles}{1 \text{ } hour}$ = 20 *miles per hour*, or 20 *mph*

Average speed is a type of unit rate expressed as distance over time; that is, $r = \frac{d}{t}$.

From this formula, we can also derive the formulas for distance, $d = rt$, and for time, $t = \frac{d}{r}$.

A **unit price** is a unit rate that contains a price in the numerator (such as *price per pound*).

Model Problem

Samantha drives 250 miles and uses 20 gallons of gasoline. What is her vehicle's gas mileage in miles per gallon (mpg)?

Solution:

$$\frac{250 miles}{20 gallons} = \frac{250}{20} mpg = 12.5 mpg$$

Explanation:

Divide 250 ÷ 20.

Proportions

A **proportion** is an equation stating that two ratios are equal.

Example: $\frac{3}{2} = \frac{6}{4}$ $3 : 2 = 6 : 4$

When a proportion is expressed as equal fractions, **cross-multiplication** yields equal products.

Example: Since $\frac{3}{2} = \frac{6}{4}$, $2 \times 6 = 3 \times 4$

Model Problem

A computer can perform 360 instructions in 10 microseconds. How many microseconds will it take the computer to perform 288 instructions?

Solution:

(A)	(B)	(C)
$\frac{360}{10} = \frac{288}{x}$	$360x = 2880$	$x = 8$

Explanation of steps:

(A) Write a proportion from the given equal ratios or rates. Use a variable, *x*, for the unknown value. Make sure the numerators *[instructions]* and denominators *[microseconds]* are in the same units.

(B) Cross-multiply.

(C) Solve for *x*.

Percents

Changing to and from percents

A **decimal is changed to percent** by moving the decimal point 2 places to the right and adding a percent sign.
Example: $0.125 = 12.5\%$

A **fraction is changed to percent** by first changing the fraction to a decimal (by dividing on a calculator) and then changing the decimal to percent.
Example: $\frac{1}{8} = 1 \div 8 = 0.125 = 12.5\%$

A **percent is changed to a decimal** by moving the decimal point 2 places to the left and removing the percent sign.
Example: $4.5\% = 0.045$

A **percent is changed to a fraction** by first changing it to a decimal and then using the calculator's $\boxed{\text{MATH}}$ $\boxed{\text{Frac}}$ function to change the decimal to a fraction.

Example: $4.5\% = 0.045 \rightarrow \boxed{\text{MATH}}\ \boxed{\text{Frac}} \rightarrow 9/200$

Rounding percents

If asked to round a percent, the specified place refers to the number when it is written as a percent, not as a decimal.

Examples: Round $\frac{3}{8}$ to the *nearest whole percent*. $\frac{3}{8} = 0.375 = 37.5\% \approx 38\%$

Round $\frac{2}{3}$ to the *nearest tenth of a percent*. $\frac{2}{3} = 0.\overline{6} = 66.\overline{6}\% \approx 66.7\%$

Percent Problems

Percent problems can often be solved using simple algebraic *proportions*. First, state the problem in the following form:

$a \text{ is } p\% \text{ of } b.$ a is the part, b is the whole, and p is the percent

Then use the following proportion to solve for the missing value:

$$\frac{a}{b} = \frac{p}{100}$$

A **discount** is a percent that is **subtracted from** a product's original price. A **sales tax** is a percent that is **added to** a product's selling price (*after any discounts are applied*).

Model Problem 1

What is 40% of 320?

Solution:

$$\frac{a}{320} = \frac{40}{100} \qquad 100a = 12{,}800 \qquad a = 128$$

Model Problem 2

15 is 20% of what number?

Solution:

$$\frac{15}{b} = \frac{20}{100} \qquad 1500 = 20b \qquad b = 75$$

Model Problem 3

12 is what percent of 600?

Solution:

$$\frac{12}{600} = \frac{p}{100} \qquad 1200 = 600p \qquad p = 2 \text{ (Answer = 2\%)}$$

Model Problem 4

There are 234 men at a convention. This is 36% of the attendees. How many people are attending the convention?

Solution:

234 is 36% of what number?

$$\frac{234}{b} = \frac{36}{100} \qquad 23400 = 36p \qquad p = 650$$

Percent of Change

The **percent of change** (*percent increase* or *percent decrease*) =

$$\frac{\text{amount of change}}{\text{original amount}} = \frac{|\text{original} - \text{new}|}{\text{original}} \text{ written as a percent}$$

Example: Last month's rent was \$600. This month, it was increased to \$630. What was the percent of increase?

$$\frac{|600-630|}{600} = \frac{30}{600} = .05 = 5\% \textit{ increase}$$

A **percent of discount** is the percent of decrease on the price of an item that is on sale.

Example: An item originally priced at \$10.50 is sold for \$7.00. This is a $33\frac{1}{3}\%$ discount.

$$\frac{|10.50-7.00|}{10.50} = \frac{3.50}{10.50} = 0.\overline{333} = 33\tfrac{1}{3}\% \textit{ discount}$$

Model Problem

The staff at a company went from 40 to 29 employees. What is the percent decrease in staff?

Solution:

(A) (B) (C) (D)

$$\frac{|40-29|}{40} = \frac{11}{40} = 0.275 = 27.5\% \textit{ decrease}.$$

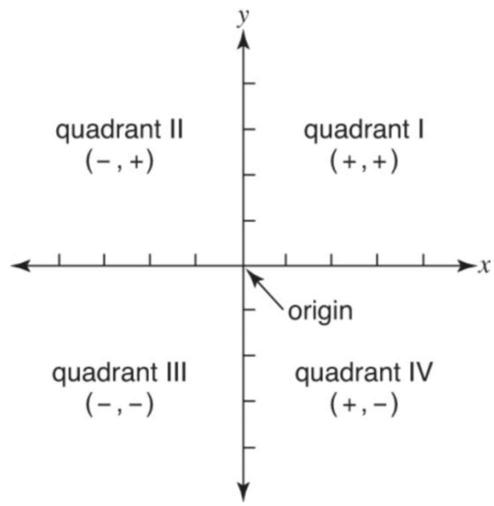
Explanation of steps:

(A) Write a fraction with the amount of change in the numerator and the original amount in the denominator.
(B) Simplify.
(C) Divide to change the fraction into a decimal.
(D) Convert the decimal to a percent by moving the decimal point two places to the right. State whether the change was an increase or decrease.

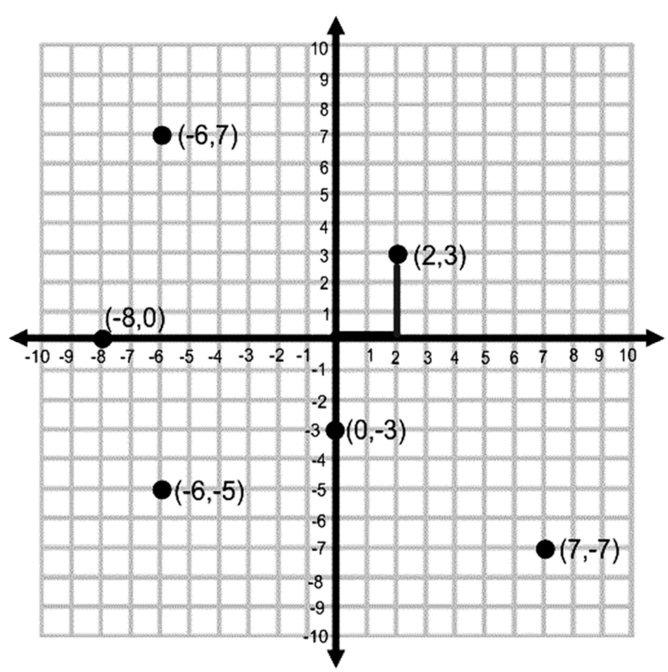
Coordinate Graphs

A coordinate plane is a two-dimensional space formed when two number lines, a horizontal ***x*-axis** and a vertical ***y*-axis**, are placed perpendicular to each other, intersecting at their respective zeroes. The point where they intersect is called the **origin**.

The axes divide the plane into four **quadrants**, numbered 1 to 4 starting at the upper right quadrant (where both the *x* and *y* values are positive) and continuing counter-clockwise.



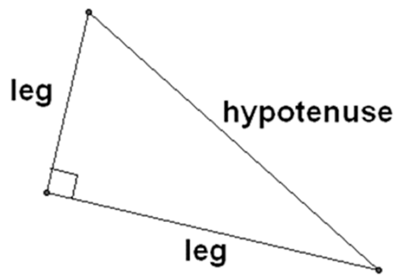
A point on the plane is represented by an **ordered pair**. An ordered pair gives a point's *x*-coordinate followed by its *y*-coordinate (always in this order). For example, find the point represented by the ordered pair, (2, 3), shown on the plane below. Use your finger to "travel" along the *x*-axis to find the *x*-coordinate, and then use your finger to "travel" up or down, parallel to the *y*-axis, to find the *y*-coordinate. The origin is represented by the ordered pair, (0, 0).



Pythagorean Theorem

In a triangle, the sum of the measures of the three angles is 180°. In a **right triangle**, one of the angles measures 90°, and the other two acute angle measures add up to 90°.

The **legs** of a right triangle are the two shortest sides. The legs are **perpendicular**; they form the right angle. The **hypotenuse** is the longest side.



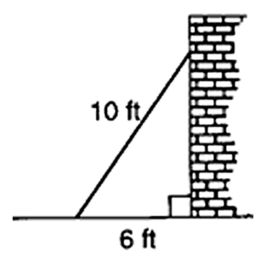
The **Pythagorean Theorem** states a relationship among the sides of any right triangle. If a and b represent the lengths of the legs, and c represents the length of the hypotenuse,

$$a^2 + b^2 = c^2$$

A set of three positive integers that can satisfy this equation is called a **Pythagorean triple**.
Examples: {3,4,5} {5,12,13} {8,15,17}

Model Problem

A wall is supported by a brace 10 feet long, as shown in the diagram to the right. If one end of the brace is placed 6 feet from the base of the wall, how many feet up the wall does the brace reach?



Solution:

(A) $a^2 + b^2 = c^2$
(B) $6^2 + b^2 = 10^2$
(C) $36 + b^2 = 100$
$b^2 = 64$
$b = \sqrt{64} = 8$ ft

Explanation of steps:

(A) Given two sides of a right triangle, use the Pythagorean Theorem to find the third side.
(B) Substitute given legs as a and b (in either order), and substitute the hypotenuse, if given, as c.
[leg a = 6 and hypotenuse c = 10]
(C) Solve for the remaining variable, and simplify the radical if possible. When taking the square root of both sides, ignore the negative square root since the length of a side must be positive.